Hydraulic fracturing of rock-fill dam

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ABSTRACT

The condition in which hydraulic fracturing in core of earth-rock fill dam may be induced, the mechanism by which the reason of hydraulic fracturing can be explained, and the failure criterion by which the occurrence of hydraulic fracturing can be determined, were investigated. The condition depends on material properties such as, cracks in the core and low permeability of core soil, and "water wedging" action in cracks. An unsaturated core soil and fast impounding are the prerequisites for the formation of "water wedging" action. The mechanism of hydraulic fracturing can be explained by fracture mechanics. The crack propagation induced by water pressure may follow any of mode I, mode II and mixed mode I-II. Based on testing results of a core soil, a new criterion for hydraulic fracturing was suggested, from which mechanisms of hydraulic fracturing in the core of rock-fill dam were discussed. The results indicated that factors such as angle between crack surface and direction of principal stress, local stress state at the crack, and fracture toughness KIC of core soil may largely affect the induction of hydraulic fracturing and the mode of the propagation of the crack.

Key words: earth-rock fill dam; hydraulic fracturing; crack; "water wedging" action

1. INTRODUCTION

A great number of high earth-rock fill dams are being or to be constructed in Western China where water resources are very abundant, such as *Nuozhadu* dam (261.5 m in height) on *Lancang* River in *Yunnan* Province, *ShuangJiangkou* dam (322 m in height) and *Changhe* dam (240 m in height) on *Dadu* River in *Sichuan* Province. The core soils of the earth-rock fill dams may subject to cracks, which are resulted from the arching action and/or hydraulic fracturing (Zhu and Wang [1]). Care must be taken to prevent such cracking. The engineers should decide whether the cracks are likely to extend and affect the integrity of the structure or whether they are stable and can be backfilled. Hydraulic fracturing in the soil core is a common geotechnical problem. Many investigators carried out a great deal of works on it, but the problem is far from being solved.

Previous studies have suggested different methods to determine the water pressure required to induce hydraulic fracturing. These methods may be classified into three groups. The first one is theoretical methods such as, the cylindrical or spherical cavity expansion theories in elastic or elastic-plastic mechanics (Andersen et al. [2], Lo and Kaniaru [3], and Yanagisawa and Panah [4]). The second one is empirical methods based on field or laboratory tests, such as Jaworski et al. [5], Decker and Clemence [6], and Mori and Tamura [7]. The last one is conceptional models based on laboratory tests and theories in fracture mechanics (FM), such as Murdoch [8, 9, 10].

Nevertheless, hydraulic fracturing in the core of the earth-rock fill dam is still an unsolved problem and its mechanism is not well established. The crack in the core which allows water to enter the core is prerequisite for hydraulic fracturing. Thus it is actually the propagation of the crack under water pressure. Fracture Mechanics may be used to investigate the problem. Present paper comprehensively deals with the hydraulic fracturing in the core of earth-rock fill dam, i.e. the condition of its formation, mechanism and failure criterion. Proposed failure criterion for hydraulic fracturing is tested in the core of rock-fill dam.

2. CONDITIONS

The induction of hydraulic fracturing in the soil core depends on its material properties and the “water wedging” action. The material properties are the cracks at the upstream surface of the core and the low permeability of the core soil, especially around the crack. The former allows water to enter the core rapidly, but the latter keeps the water from seeping into soil around the crack rapidly. The centrifuge model tests conducted by Shen et al. [11] proved that hydraulic fracturing could not be induced in homogeneous core without crack. The material condition is usually provided according to the study of Sherard [12].

Enough intensive “water wedging” action induced by the water in the crack is also essential for creating hydraulic fracturing. A rapid impounding as well as an unsaturated soil core, especially around the crack are prerequisites for the development of the intensive “water wedging” action.

During the construction of the earth-rock fill dam, the core soil is unsaturated and the excess pore pressure induced by the construction can dissipate gradually. Figure 1 shows the distribution of pore pressure in the core just after construction but before impounding. It indicates that a line on which pore pressure equals to zero exists in the core. The pore pressure in the area under the line is greater than zero, but that in the zone above the line is less than zero. The pore pressure of the element in the negative pore pressure area can be expressed with uniform compressive force applying on the surface of the element, and that in the positive pore pressure can be expressed with uniform tensile force.

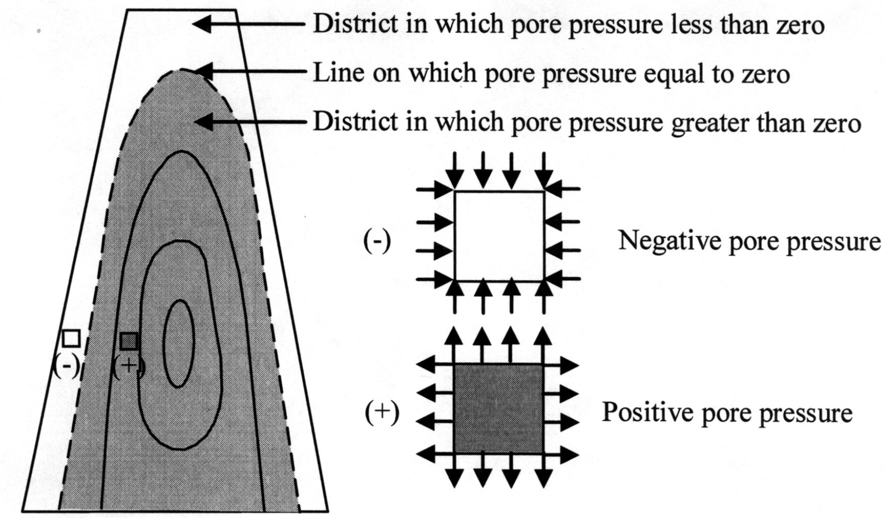
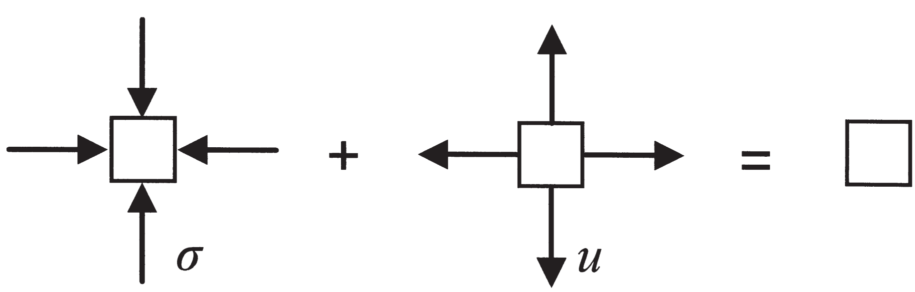
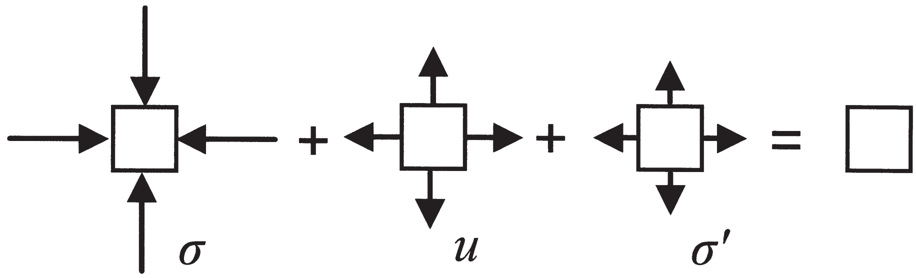


Figure 1 Distribution of pore pressure in core at the end of construction, and before impounding

In the course of impounding, the pore water pressure in the core may change because of the water pressure applying on the upstream surface of the core, and the unsaturated soil will gradually become saturated soil with the water seeping into the core. The most important difference between unsaturated and saturated soils is the difference in the change of pore water pressure after loading (Fig.2). For saturated soils, at the same time of applying load "σ" on the soil element, excess pore water pressure "u" induced by the load "σ" is equal to the load but with opposite direction as shown in Fig.2(a). However, for unsaturated soils, at the same time of applying load "σ" on the soil element, excess pore pressure "u" induced by the load "σ" is less than the applied load as shown in Fig.2(b). This is because pore air in unsaturated soil is compressed firstly with loading, and the load "σ" will induce both excess pore pressure "u" and effective stress "σ'".



(a) Saturated soil



(b)Unsaturated soil

Figure 2 Variation of pore water pressure in soils after loaded instantly (Where, σ is total load; u is pore pressure; σ' is effective stress)

2.1. "WATER WEDGING" ACTION IN UNSATURATED CORE

In the following analysis, it is assumed that the water pressure is applied instantly. The pore pressure in the core before impounding is called the initial pore pressure, expressed as "u_0", and that induced by water pressure (expressed as "p") after impounding is called the increment of pore pressure, expressed as "Δu".

The magnitude of "u_0" in unsaturated soil may be either less, equal or greater than zero. For the case of negative pore pressure "u_0", "u_0" can be expressed with a uniform compressive force applying on the surface of element, and the excess pressure "Δu" can be expressed with a uniform tensile force because of its opposite direction with "u_0". According to Fig.2(b) ("Δu" is less than the "p" in magnitude) and the superposition theorem for forces shown in Fig.3(a), the interaction force between two adjacent elements at the tip of the crack can be determined while the water pressure is applying on the surface of the crack. It shows that the resultant interaction force between the two elements, namely gradient water pressure and noted "G_w", will make them moving away from each other. Therefore, the "water wedging" action can be induced easily in the case. Figure 3(b) shows the case with positive "u_0" in unsaturated core. It indicates that "water wedging" action can also form in the crack. By comparing the two cases shown in Fig.3(a) and Fig.3(b), it is found that the intensity of "water wedging" action will be strengthen by reducing "u_0" in the unsaturated core. This also indicates that the probability of hydraulic fracturing in negative "u_0" area is higher than that in positive "u_0" zone of the core.

2.2. "WATER WEDGING" ACTION IN SATURATED CORE

The value of "u_0" in saturated soil is usually greater than zero, therefore both "u_0" and "Δu" induced by the water pressure during impounding can be expressed with the uniform tensile force applying on the surface of the soil element. The induced interaction force between elements at the tip of the crack can be obtained from the effective stress theory for saturated soil shown in Fig.2(a) and the superposition theorem shown in Fig.4. It shows that the interaction force between the two elements cannot be changed by the impounding because the intensity of "Δu" is equal to the water pressure "p". "Water wedging" action cannot be induced in the saturated core, and hydraulic fracturing is therefore unlikely if the core is under saturated state.

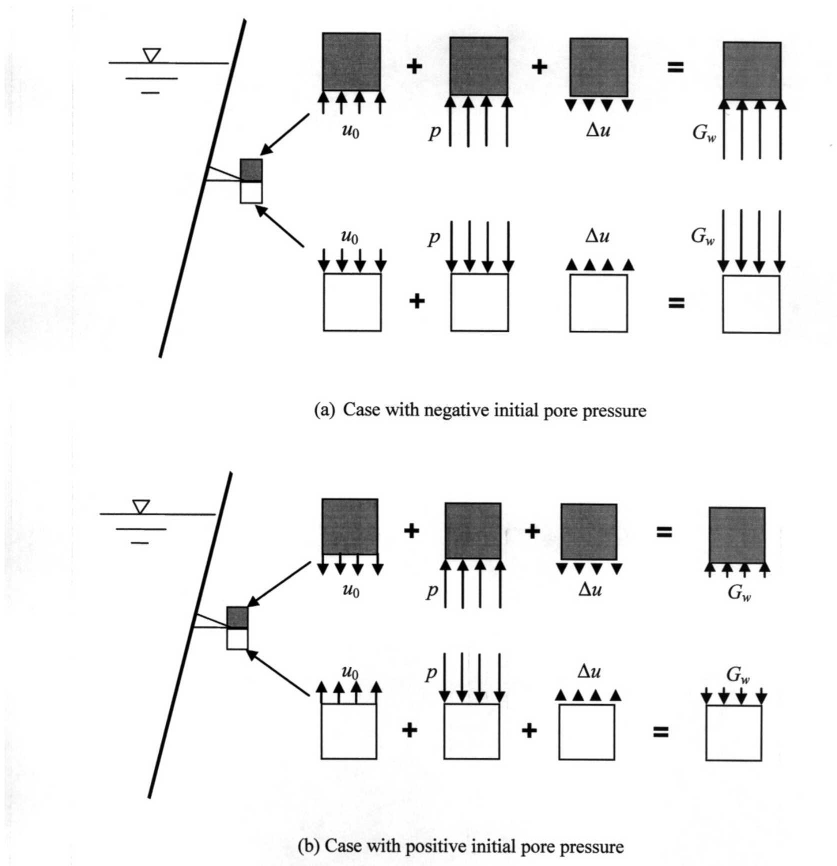


Figure 3 Hydraulic fracturing in unsaturated core (Where, u_0 is initial pore pressure, p is water pressure in crack, Δu is increment of pore pressure, and G is gradient water pressure

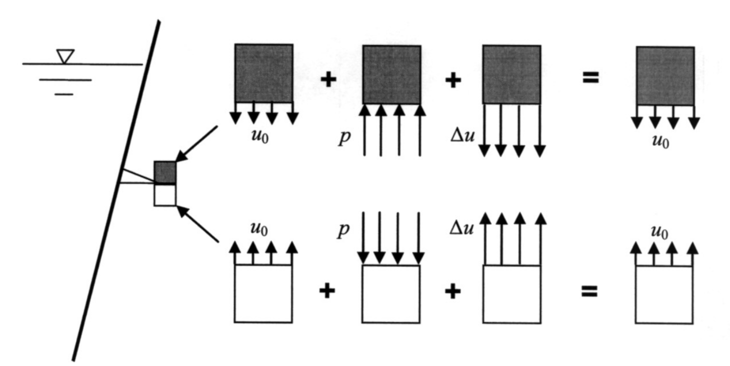


Figure 4 Hydraulic fracturing in saturated core

3. MECHANICAL MECHANISM

There are mainly two views to the mechanical mechanism of hydraulic fracturing, either tensile failure or shear failure of the core soil. Mohr-coulomb failure criterion is usually adopted as the shear failure criterion.

Computing results of the two-dimensional (2D) or three-dimensional (3D) finite element method (FEM) for many actual earth-rock fill dams indicate that the tensile stress state does not exist in the core soil at the end of construction or impounding stage. This means that if both the view of the tensile failure and the results of FEM are true, the problem of hydraulic fracturing should be ignored in most of actual dams. However, it is always investigated as a special topic in the design stage of earth-rock fill dams especially that with a vertical soil core, because the engineers believe the problem is not so simple.

As mentioned previously, the crack leading water to enter the core is needed for hydraulic fracturing, and the course of hydraulic fracturing can be regarded as the propagation of the crack under water pressure. This is actually one of the main research topics in FM, and the mechanical mechanism and criterion of hydraulic fracturing should be explained and established according to theories in FM.

The water entering the core along the crack may not only induce the water pressure applied on its surface, but also soften the soil around the crack. If "water wedging" action is also induced by the water pressure, the nominal stress near the tip of the crack may change. In terms of the theory in FM, if only the intensity of the nominal stress near the tip reaches its critical value, the crack will spread. Therefore, the mechanical mechanism of hydraulic fracturing is that "water wedging" action changes the intensity of nominal stress near the tip of the crack. If there is no "water wedging" action, there will be no hydraulic fracturing.

4. FAILURE CRITERION

Since the mechanism of hydraulic fracturing can be explained in terms of FM, a reasonable failure criterion should be established based on the FM theory, because the singularity of stress state at crack tip is always ignored in those criteria based on tensile or shear strength.

4.1. SIMPLIFICATION OF CRACK

The crack existing at the upstream surface of the core can give way for water to enter the core, and is therefore a material condition for inducing hydraulic fracturing. It is usually a local crack like that in Fig.5(a), on where, "x", "y" and "z" are the directions pointing at left abutment along a horizontal line parallel to dam axis, downward stream perpendicular to dam axis and upward vertical, respectively. The local crack is called 3D crack in FM. The calculating method for the 3D crack is much more complicated than that for 2D crack in FM. It is therefore necessary to simplify the 3D local crack to the 2D crack as shown in Fig.5(b). The simplification of local crack does actually reduce the ability of the core soil to resist to the fracture failure, and will increase the probability of hydraulic fracturing if this one is used to investigate the problem. It can easily be accepted in engineering in terms of the safety of dams.

4.2. CRITERION

FEM has been widely used in simulating stresses and strains of the earth-rock fill dam during construction and impounding. This should be considered at establishment of the criterion for determining hydraulic fracturing. The earth-rock fill dam is usually simplified as a plane strain problem in the analysis of FEM. The local 3D crack has been simplified as the 2D crack in the investigation for hydraulic fracturing. Thus, the criterion of hydraulic fracturing should also be established based on the plane strain condition.

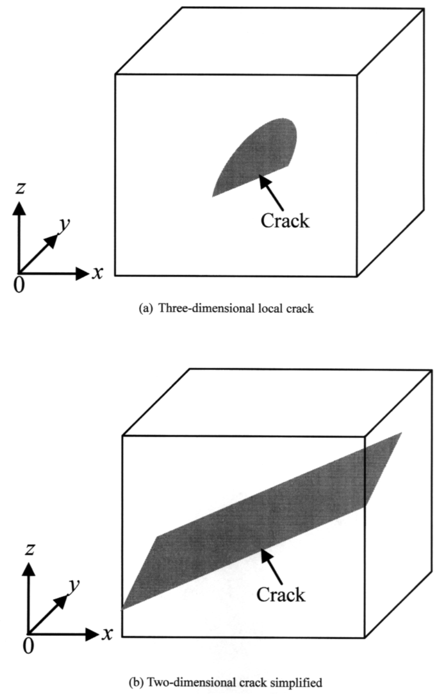


Figure 5 Crack at upstream surface of core

Under the plane strain condition, the crack propagation may follow one of mode I, mode II and mixed mode I-II. Because the stress state in the core, especially near the upstream surface of the core, is very complex, mode I oversimplifies the crack propagation for the hydraulic fracturing. It is also not reasonable to assume mode II as no pure shear stress state exists. Hence, the criterion for hydraulic fracturing should be investigated based on mixed mode I-II because the crack spreading may be induced by the combination of normal stress perpendicular to crack face and shear stress parallel to crack face (Vallejo [13]).

In concrete or rock, the crack cannot spread only under any or combination of states of tensile and shear stresses, but under any or combination of states of compressive and shear stresses. However, in soil, it is not clear whether the crack spread under the states of compressive stress or combination of compressive and shear stresses because of the lack of testing data. It is reasonable to assume the stress states inducing hydraulic fracturing include pure tensile stress, pure shear stress and combination of tensile and shear stresses considering the actual stress state in the core.

So far, a number of fracture failure criteria have been proposed for describing the failure behavior of materials following mixed mode I-II. Three of the most typical and famous criteria are the maximum circumferential stress theory suggested by Erdogan and Sih [14], energy release rate theory by Hussain et al. [15] and strain energy density factor theory by Sih [16]. To examine whether any of the three theories can be used as the criterion of hydraulic fracturing, the fracture behavior of a silty clay which is the core material of *Nuozhadu* earth-rock fill dam in Western China was investigated (Wang et al. [17]). The testing results indicated that all the three theories are not suitable for the silty clay, but linear elastic fracture mechanics (LEFM) and a simple expression as given in Eq. (1) are suitable:

$$\left(\frac{K_I}{K_{IC}}\right)^2+\left(\frac{K_{II}}{K_{IC}}\right)^2=1 \tag{1}$$

where K_{IC} is mode I fracture toughness of the core soil, and K_I and K_{II} are stress intensity factors of mode I and mode II cracks, respectively.

J integral proposed by Rice [18] is a parameter indicating the intensity of nominal stress, and it is a constant for different integral route. For 2D crack under elasticity and yield only in small range situations, the value of J integral is equal to that of the energy release rate G, i.e.:

$$J=G=-\frac{\partial\Pi}{\partial a} \tag{2}$$

where ∂a is spreading length of crack, and $\partial\prod$ is reduced energy of elastic system.

According to the relationship between energy release rate G and stress intensity factor K for mixed mode I-II crack under plane strain condition (Anderson [19]), J can be rewritten as follows:

$$J=\frac{1-\nu^2}{E}\left(K_I{}^2+K_{II}{}^2\right) \tag{3}$$

where E and v are Young's modulus and Poisson's ratio of material, respectively.

The value of G or J can be obtained by FEM (Hellen [20], Delorenzi [21], Hamoush and Salami [22], Wang and Zhu [23]). Considering the testing results as shown in Eq.(1), a new criterion for hydraulic fracturing is proposed as follows:

$$\sqrt{K_I{}^2+K_{II}{}^2}=K_{IC} \tag{4}$$

where $\sqrt{K_I{}^2+K_{II}{}^2}$ can be obtained from Eq.(3).

5. DISCUSSION

5.1. CORE WITH A TRANSVERSE CRACK

The 2D crack simplified from 3D local crack can distribute in any plane intersecting with the upstream surface of the core (Fig.5(b)). Each of them may spread if enough intensive "water wedging" action affects the crack after impounding. This leads to some uncertainties in analyzing the probability of hydraulic fracturing. However, two special types should be paid close attention, one is transverse crack, and the other is vertical crack. This section discusses the probability of hydraulic fracturing of the former.

The stress state at upstream surface of the core apart from the abutment can be expressed as that shown in Fig.6(a). It shows that only normal stresses apply on the two vertical planes perpendicular to the upstream surface of the core, and both normal and shear stresses apply on the other four planes, which are two horizontal planes and two vertical planes parallel with the upstream surface of the core. The transverse crack under 3D stress state shown in Fig.6(a) can be simplified as the plane strain crack shown in Fig.6(b).

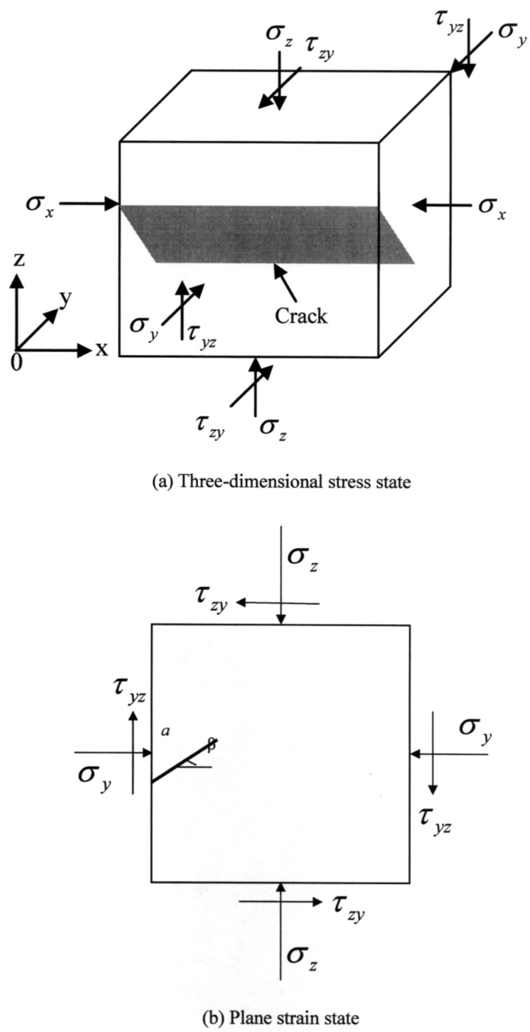


Figure 6 Transverse crack and its stress state

The normal and shear stresses applying on the crack face in Fig.6(b) can be expressed as Eqs.(5) and (6):

$$\sigma_n = \frac{1}{2}\left[\left(\sigma_z + \sigma_y\right) + \left(\sigma_z - \sigma_y\right)\cos 2\beta\right] - \tau_{yz}\sin 2\beta \tag{5}$$

$$\sigma_t = \frac{1}{2}\left(\sigma_z - \sigma_y\right)\sin 2\beta + \tau_{yz}\cos 2\beta \tag{6}$$

where σ_n and σ_t are the normal stress and shear stresses applying on the crack face, respectively; σ_y is the normal stress applying on the vertical planes parallel to upstream surface of the core; σ_z is the normal stress applying on the horizontal planes; τ_{yz} is the shear stress applying on the vertical planes parallel to upstream surface of the core; β is the slope angle of the crack face; τ_{zy} is shear stress applying on the horizontal planes, equal to τ_{yz} (Fig.6); σ_x is the normal stress applying at the vertical planes perpendicular to upstream surface of the core; and a is length of the crack.

The stress intensity factors K_I and K_{II} at the tip of the crack can be expressed as follows (Anderson [19]):

$$K_I = -F_1 \sigma \sqrt{\pi a} \tag{7}$$

$$K_{II} = F_2 \tau \sqrt{\pi a} \tag{8}$$

where F_1 and F_2 are coefficients of correction for K_I and K_{II} respectively.

While impounding, water pressure will apply on the upstream surface of the core, and apply on the crack faces if water affects the crack at the same time. Water pressure applying on the crack faces may induce "water wedging" action analyzed previously. For the convenience of analysis, it is assumed that the intensity of "water wedging" action is equal to the water pressure applying on the crack faces. The values of both the water pressures applying on the upstream surface and that on the crack faces are also taken equal. The influence of the initial pore pressure "u_0" and the increment of pore pressure "Δu" of the core soil on "water wedging" action can also be neglected. Therefore, the intensity of the water pressure can be expressed as:

$$p = \gamma_w H \tag{9}$$

where p is intensity of water pressure, γ_w is unit weight of water, and H is water head in the crack.

5.1.1. Calculation of K_I

The effective normal stress "σ" developed on the crack faces after impounding can be obtained as:

$$\sigma = \frac{1}{2}\left[(\sigma_z + \sigma_y + \gamma_w H) + (\sigma_z - \sigma_y - \gamma_w H)\cos 2\beta\right] - \tau_{yz} \sin 2\beta - \gamma_w H \tag{10}$$

Substituting Eq.(10) into Eq.(7), the K_I is calculated as:

$$K_I = -F_1 \left\{ \frac{1}{2}\left[(\sigma_z + \sigma_y + \gamma_w H) + (\sigma_z - \sigma_y - \gamma_w H)\cos 2\beta\right] - \tau_{yz} \sin 2\beta - \gamma_w H \right\} \sqrt{\pi a} \tag{11}$$

5.1.2. Calculation of K_{II}

(i) Case of open crack

In the case of open crack or if the shear strength of the crack neglected, the effective shear stress applying on the crack face can be expressed as follows from Eqs.(6) and (9).

$$\sigma_t' = \frac{1}{2}(\sigma_z - \sigma_y - \gamma_w H)\sin 2\beta + \tau_{yz} \cos 2\beta \tag{12}$$

where σ_t' is effective shear stress applying on the crack.

Substituting Eq.(12) into Eq.(8), the equation for calculating K_{II} can be written as follows:

$$K_{II} = F_2\left\{\frac{1}{2}\left(\sigma_z - \sigma_y - \gamma_w H\right)\sin 2\beta + \tau_{yz}\cos 2\beta\right\}\sqrt{\pi a} \tag{13}$$

(ii) Case of close crack

For the case of close crack, if the shear strength of the crack cannot be neglected, the effective shear stress applying on the crack face can be obtained from the following equation:

$$\tau = \sigma_t' - \tau^* \tag{14}$$

where τ is the effective shear stress applying on the crack face; τ^* is a shear stress induced by the ability of the crack to resist shear deformation, and it is called reverse shear stress here because of its opposite direction to σ_t. The expression of τ^* may be given by:

$$\begin{cases} \tau^* = \sigma_t' & (\sigma \geq 0,\ \sigma_t' \leq \tau_f) \\ \tau^* = \tau_f & (\sigma \geq 0,\ \sigma_t' > \tau_f) \\ \tau^* = 0 & (\sigma < 0) \end{cases} \tag{15}$$

The shear strength of the crack, τ_f, will reduce with water entering. The present paper uses c_1 and φ_1 to express cohesion and internal friction angle of the crack before water entering respectively, and c_2 and φ_2 to express ones after water entering. Then, τ_f can be obtained from Eq. (16):

$$\tau_f = \sigma \tan\varphi_2 + c_2 \tag{16}$$

Combining Eqs.(14), (15) and (16), the effective shear stress applying on the crack face after water entering can be rewritten as follows:

$$\tau = \begin{cases} 0 & (\sigma \geq 0,\ \sigma_t' \leq \sigma\tan\varphi_2 + c_2) \\ \sigma_t' - \sigma\tan\varphi_2 - c_2 & (\sigma \geq 0,\ \sigma_t' > \sigma\tan\varphi_2 + c_2) \\ \sigma_t' & (\sigma < 0) \end{cases} \tag{17}$$

where σ_t' and σ can be obtained from Eqs. (12) and (10) respectively.

Substituting Eqs.(12), (10) and (17) into Eq.(8), the equation for K_{II} can be rewritten as follows:

$$K_{II}=\begin{cases}0 & \text{£¨}\ H \le H_1\text{£©}\\ F_2\left\{\left[\frac{1}{2}(\sigma_z-\sigma_y-\gamma_w H)\sin 2\beta+\tau_{yz}\cos 2\beta\right]-\right. & \\ \left.\left[\frac{1}{2}\left[(\sigma_z+\sigma_y+\gamma_w H)+(\sigma_z-\sigma_y-\gamma_w H)\cos 2\beta\right]-\tau_{yz}\sin 2\beta-\gamma_w H\right]\tan\varphi_2-c_2\right\} & \\ \times\sqrt{\pi a} & \\ & \text{£¨}\ H_1 < H \le H_2\text{£©}\\ F_2\left[\frac{1}{2}(\sigma_z-\sigma_y-\gamma_w H)\sin 2\beta+\tau_{yz}\cos 2\beta\right]\sqrt{\pi a} & \text{£¨}\ H > H_2\text{£©}\end{cases} \tag{18}$$

where

$$H_1=\frac{(\sigma_z+\sigma_y)+(\sigma_z-\sigma_y)\cos 2\beta-2\tau_{yz}\sin 2\beta}{\gamma_w\left[(1+\cos 2\beta)-\sin 2\beta/\tan\varphi_2\right]}-\frac{(\sigma_z-\sigma_y)\sin 2\beta+2\tau_{yz}\cos 2\beta-2c_2}{\gamma_w\left[\tan\varphi_2(1+\cos 2\beta)-\sin 2\beta\right]} \tag{19}$$

$$H_2=\frac{(\sigma_z+\sigma_y)+(\sigma_z-\sigma_y)\cos 2\beta-2\tau_{yz}\sin 2\beta}{\gamma_w(1+\cos 2\beta)} \tag{20}$$

If , $H_1 \ge H_2$, H_1 is taken as H_2.

5.1.3. Calculation of $(K_I^2+K_{II}^2)^{0.5}$

For the case of open crack or negligible shear strength of the crack, $(K_I^2+K_{II}^2)^{0.5}$ which is used to estimate hydraulic fracturing can be obtained by combining Eqs.(11) and (13). And for the case of close crack, $(K_I^2+K_{II}^2)^{0.5}$ can be obtained by combining Eqs.(11) and (18).

Figure 7 shows all the stress intensity factors discussed in the previous paragraphs. It indicates that K_I is not affected by shear strength of the crack, and the influence of the crack shear strength on K_{II} exists only at the water head less than H_0. The influence of the crack shear strength on $(K_I^2+K_{II}^2)^{0.5}$ becomes negligible at water head greater than H_0. Therefore, in investigating the hydraulic fracturing in transverse cracks, it is noted that the influence of crack shear strength does not exist. The figure also indicates that both K_I and K_{II} are not equal to zero at water head greater than H_0 in different cases. Therefore the crack propagation for hydraulic fracturing may follow mixed mode I-II.

5.1.4. Dangerous crack angle

Given the stress state in Fig.6(b) is $\sigma_z = 2\sigma_y = 4\tau_{yz}$, the value of $(K_I^2+K_{II}^2)^{0.5}$ for different slope angles of crack under the case of open crack can be obtained (Fig.8). It shows that the water head H_0, at which the values of $(K_I^2+K_{II}^2)^{0.5}$ of the cracks with different slope angles are identical, is acting with increase of water head. The slope angle of the first spreading crack is near to 90° for a water head less than H_0 because the value of $(K_I^2+K_{II}^2)^{0.5}$ is maximum. The vertical crack parallel to upstream surface of the core does not have large menace even in the case of the induction of hydraulic fracturing. At water head greater than H_0, the slope angle of the first spreading crack is zero degree (horizontal crack). It will seriously affect the safety of the dam because a cross-core crack may form in the case of the induction of hydraulic fracturing. Therefore, the dangerous crack is horizontal crack in all transverse cracks.

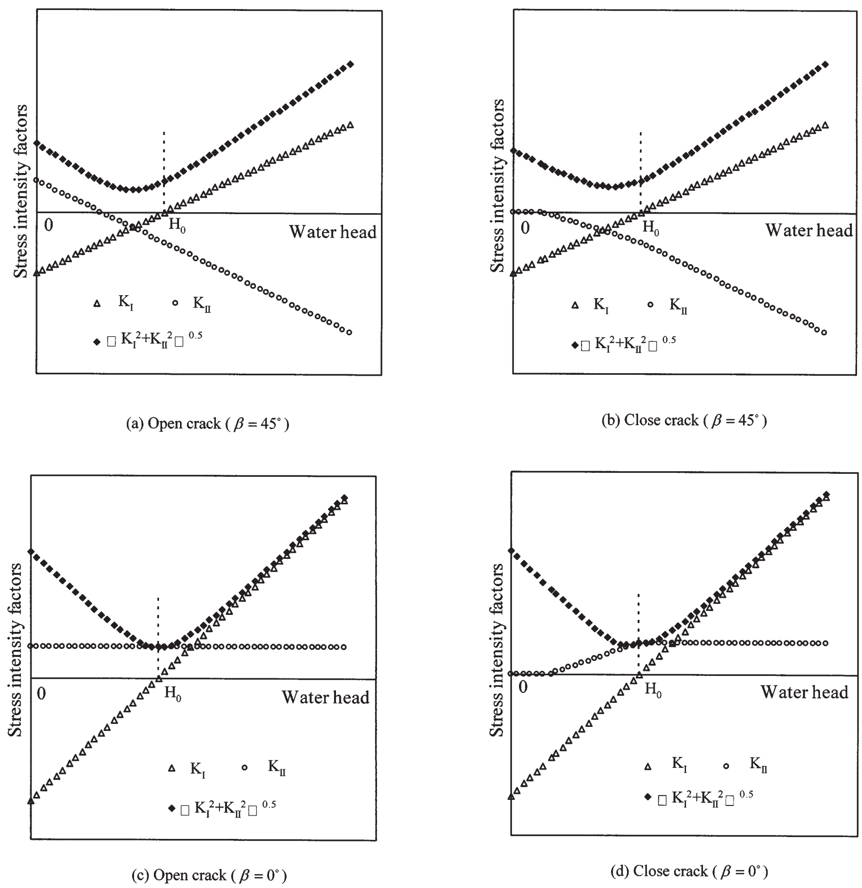


Figure 7 Stress intensity factors at tip of transverse crack

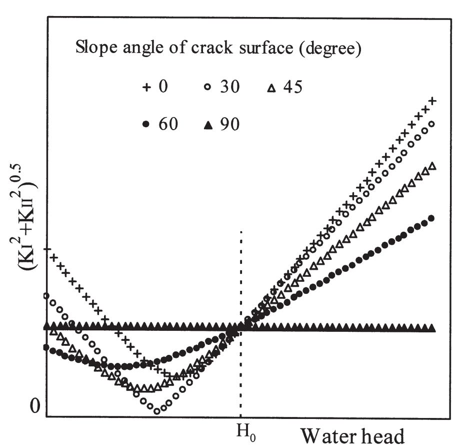


Figure 8 Variation of $(K_I^2+K_{II}^2)^{0.5}$ with slop angle of transverse crack

5.2. CORE WITH A VERTICAL CRACK

The other type of special crack is vertical crack. The vertical crack under 3D stress state shown in Fig.9(a) can be simplified as the plane strain crack shown in Fig.9(b).

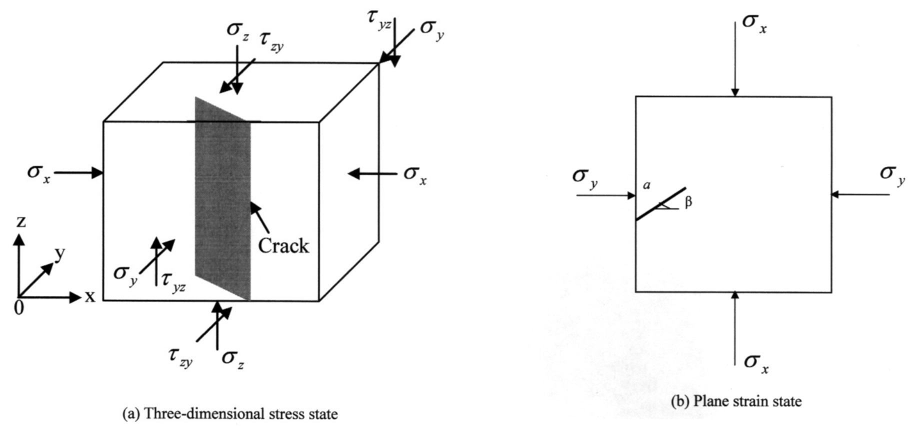


Figure 9 Vertical crack and its stress state

The normal and shear stresses applying on the crack face in Fig.9(b) can be expressed as follows:

$$\sigma_n = \frac{1}{2}\left[(\sigma_x + \sigma_y) + (\sigma_x - \sigma_y)\cos 2\beta\right] \tag{21}$$

$$\sigma_t = \frac{1}{2}(\sigma_x - \sigma_y)\sin 2\beta \tag{22}$$

When slope angle of crack β is equal to zero, the crack propagation will follow mode I. It will follow mixed mode I-II if the angle β is between zero and 90°. The stress intensity factors K_I and K_{II} can also be obtained from Eqs.(7) and (8). The intensity of the water pressure can also be expressed as Eq.(9). Using the same procedure as mentioned in the previous sections, K_I and K_{II} at the tip of crack after impounding can be obtained as follows.

The equation to calculate K_I is given by:

$$K_I = -F_1\left\{\frac{1}{2}\left[(\sigma_x + \sigma_y + \gamma_w H) + (\sigma_x - \sigma_y - \gamma_w H)\cos 2\beta\right] - \gamma_w H\right\}\sqrt{\pi a} \tag{23}$$

For the case of open crack or neglecting the crack shear strength, K_{II} can be obtained from the following equation:

$$K_{II} = F_2\left\{\frac{1}{2}(\sigma_x - \sigma_y - \gamma_w H)\sin 2\beta\right\}\sqrt{\pi a} \tag{24}$$

For the case of close crack, if the shear strength of the crack cannot be neglected, K_{II} can be expressed as follows:

$$K_{II}=\begin{cases}0 & £¨\ H \le H_1£© \\ F_2\left\{\left[\frac{1}{2}\left(\sigma_x-\sigma_y-\gamma_w H\right)\sin 2\beta\right]- \right. & \\ \left.\left[\frac{1}{2}\left[\left(\sigma_x+\sigma_y+\gamma_w H\right)+\left(\sigma_x-\sigma_y-\gamma_w H\right)\cos 2\beta\right]-\gamma_w H\right]\tan\varphi_2-c_2\right\}\sqrt{\pi a} & \\ & £¨\ H_1 < H \le H_2£© \\ F_2\left[\frac{1}{2}\left(\sigma_x-\sigma_y-\gamma_w H\right)\sin 2\beta\right]\sqrt{\pi a} & £¨\ H > H_2£©\end{cases} \tag{25}$$

where

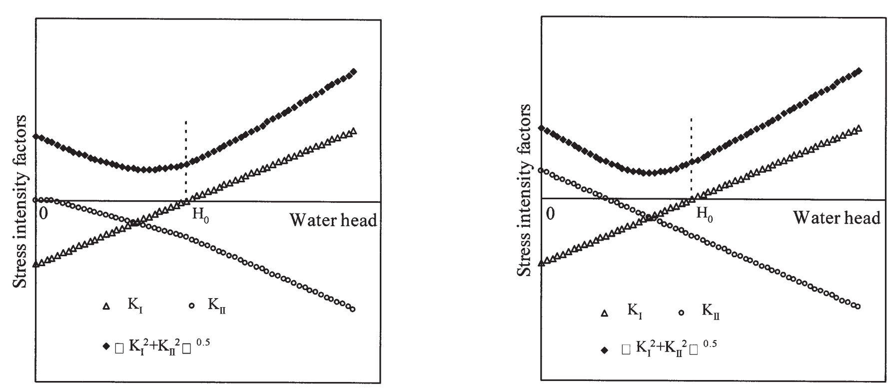
$$H_1=\frac{\left(\sigma_x+\sigma_y\right)+\left(\sigma_x-\sigma_y\right)\cos 2\beta}{\gamma_w\left[\left(1+\cos 2\beta\right)-\sin 2\beta/\tan\varphi_2\right]}-\frac{\left(\sigma_x-\sigma_y\right)\sin 2\beta-2c_2}{\gamma_w\left[\tan\varphi_2\left(1+\cos 2\beta\right)-\sin 2\beta\right]} \tag{26}$$

$$H_2=\frac{\left(\sigma_x+\sigma_y\right)+\left(\sigma_x-\sigma_y\right)\cos 2\beta}{\gamma_w\left(1+\cos 2\beta\right)} \tag{27}$$

If $H_1 \ge H_2$, H_1 is taken as H_2.

Values of $(K_I^2+K_{II}^2)^{0.5}$ for the case of open crack or neglect the shear strength of the crack can be obtained by combining Eqs.(23) and (24). For the case of close crack, it can be obtained by combining Eqs.(23) and (25).

Variations of stress intensity factors with water head shown in Fig.10 indicate that the shear strength of the crack does not have any influence on K_I, but has some influence on K_{II} only at water head less than H_0 (for which K_I is equal to zero). Since hydraulic fracturing can be induced only at water head greater than H_0, the value of $(K_I^2+K_{II}^2)^{0.5}$ used to estimate the induction of hydraulic fracturing is not affected by the shear strength of the crack.



(b) Close crack ($\beta = 45°$)

(a) Open crack ($\beta = 45°$)

Figure 10 Stress intensity factors at tip of vertical crack

To determine the direction of dangerous crack in all vertical cracks, the stress state in Fig.9(b) is assumed as $\sigma_x = 2\sigma_y$. Figure 11 shows the variation of $(K_I^2+K_{II}^2)^{0.5}$ with water head for the cracks with different slope angles under the stress state. It indicates that the water head H_0 making the same value of $(K_I^2+K_{II}^2)^{0.5}$ for the cracks with different slope angles is also existent in the vertical crack. The slope angle of the first spreading crack is near 90° at water head less than H_0, and is equal to zero degree at water head greater than H_0. The latter will seriously affect the safety of the dam because a cross-core vertical crack may form in the case of the induction of hydraulic fracturing. Therefore, the dangerous crack is the vertical crack perpendicular to the upstream surface of the core in all vertical cracks. The vertical crack is called as cross-vertical crack here. It is note that K_{II} at the tip of the cross-vertical crack is equal to zero from the Fig.11 or Eqs.(22), (24) or (25). This means that dangerous crack propagation may follow mode I.

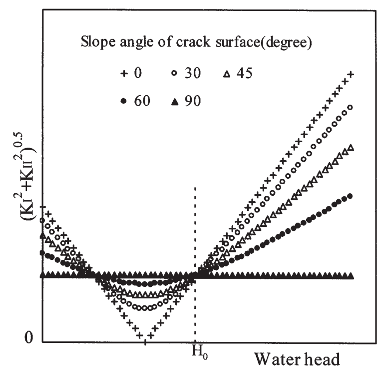


Figure 11 Variation of $(K_I^2+K_{II}^2)^{0.5}$ with slop angle of vertical crack

5.3. STRIKE-DIP OF CRACK SPREADING EASIEST

The analyses described above have indicated that the dangerous crack is the cross-vertical crack in all vertical cracks, and the horizontal crack in all transverse cracks. It is of some interests to discuss which one is more dangerous for the safety of dams.

For the two cracks, the stress states in Figs.6(b) and 9(b) can be simplified as those in Fig.12.

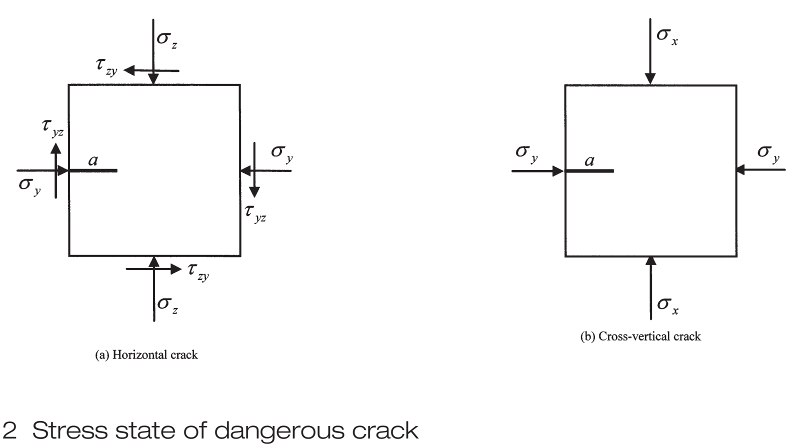


Figure 12 Stress state of dangerous crack

The previous analyses indicated that the shear strength of the crack does not have any influence either on K_I or, for a water head greater than that making $K_I = 0$, on K_{II}. The value of $(K_I^2+K_{II}^2)^{0.5}$ used to estimate the induction of hydraulic fracturing is also not affected by the shear strength of the crack. Therefore, this section will discuss the case where the crack shear strength is neglected.

For the horizontal crack shown in Fig.12(a), the calculations of K_I and K_{II} can be simplified as follows:

$$\left.\begin{aligned} K_I &= -F_1(\sigma_z - \gamma_w H)\sqrt{\pi a} \\ K_{II} &= F_2 \tau_{yz} \sqrt{\pi a} \end{aligned}\right\} \tag{28}$$

For the cross-vertical crack shown in Fig.12(b), the calculations of K_I and K_{II} are:

$$\left.\begin{aligned} K_I &= -F_1(\sigma_x - \gamma_w H)\sqrt{\pi a} \\ K_{II} &= 0 \end{aligned}\right\} \tag{29}$$

The stress state at the upstream surface of the core can be expressed as Eq.(30) for most actual dams at the end of construction before impounding.

$$\left.\begin{aligned} &\sigma_z > \sigma_x > \sigma_y > 0 \\ &\tau_{yz} = \tau_{zy} \neq 0 \end{aligned}\right\} \tag{30}$$

It is well known that the intensity of the arching action of the stress in the core has large influence on the stress state. The intensity of the arching action will change with any variation of dam materials or dam structure (Zhu and Wang [1]). The arching action affects the stress state from two aspects, one is to reduce vertical normal stress σ_z, and the other is to increase shear stress τ_{yz}. For the same point in the core, the influence of the arching action can be expressed as follows:

$$\left.\begin{aligned} &\sigma_x = const \\ &-\Delta\sigma_z = 2\Delta\tau_{yz} \end{aligned}\right\} \tag{31}$$

where $\Delta\sigma_z$ and $\Delta\tau_{yz}$ are the increments of vertical normal stress σ_z and shear stress τ_{yz} induced by the arching action respectively. The negative sign "-" means reducing, and σ_x equal to a constant means no influence of the arching action on the stress applying on the plane perpendicular to the upstream surface of the core.

Based on Eq.(31), four stress states, in which the normal stress σ_x is equal to each other, are assumed as follows:

$$\left.\begin{aligned} &\sigma_z = 2\sigma_x = 4\tau_{yz} \\ &\sigma_z = 1.67\sigma_x = 2.5\tau_{yz} \\ &\sigma_z = 1.33\sigma_x = 1.6\tau_{yz} \\ &\sigma_z = \sigma_x = \tau_{yz} \end{aligned}\right\} \tag{32}$$

Figure 13 shows the results of $(K_I^2+K_{II}^2)^{0.5}$ from Eqs.(28) and (29) for the horizontal crack and the cross-vertical crack under the four stress states respectively. In the figure, H_v0 and H_h0 are the water heads making K_I equal to zero at the tips of the cross-vertical crack and the horizontal crack respectively. H_0 is the water head at which the values of $(K_I^2+K_{II}^2)^{0.5}$ are equal to each other for both the cross-vertical crack and the horizontal crack.

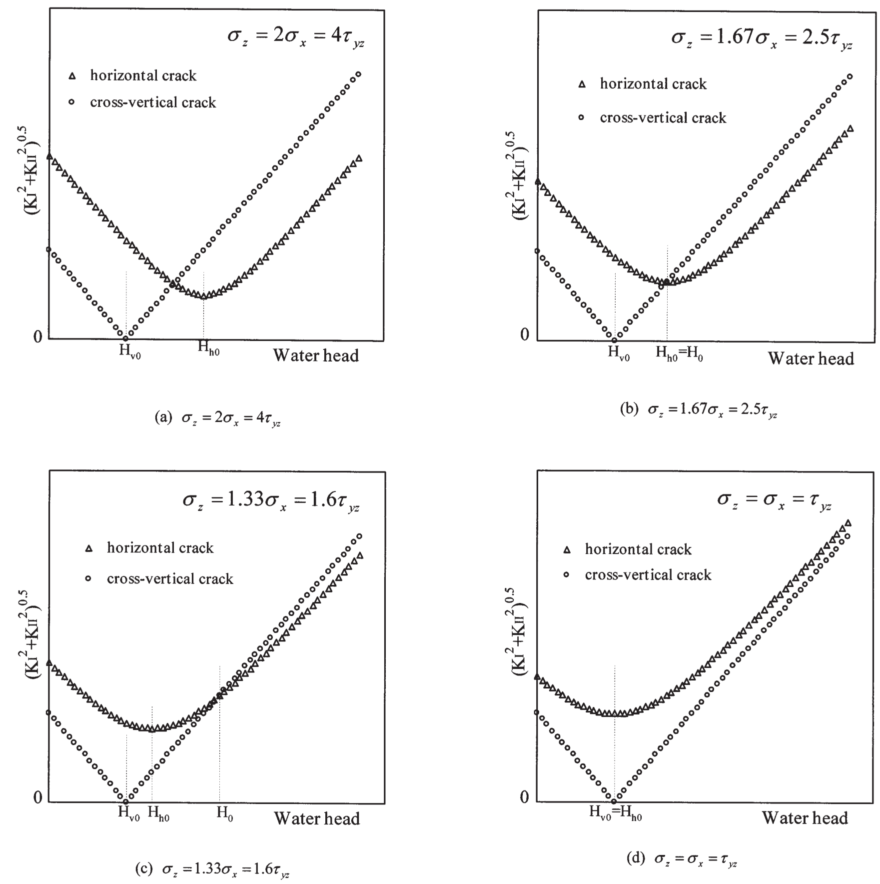


Fig.13. $(K_I^2+K_{II}^2)^{0.5}$ of horizontal and cross-vertical cracks

Figure 13(a) shows that the probability of hydraulic fracturing in the cross-vertical crack is always greater than that in the horizontal crack under the stress state $\sigma_z = 2\sigma_x = 4\tau_{yz}$, and the propagation of the crack may follow mode I. Figure 13(b) shows that under the stress state $\sigma_z = 1.67\sigma_x = 2.5\tau_{yz}$, the probability of hydraulic fracturing in the cross-vertical crack is more than that in the horizontal crack, except for the water head H_{h0} or H_0 at which the probability for both the two crack are equal. Moreover, the crack spreading may follow mode I except for the water head H_{h0} or H_0 at which the crack spreading may follow mode I or mode II. Figure 13(c) shows that under the stress state $\sigma_z = 1.33\sigma_x = 1.6\tau_{yz}$, the hydraulic fracturing may be induced in the cross-vertical crack firstly at water head between H_{v0} and H_{h0} and greater than H_0. The hydraulic fracturing may be induced in the horizontal crack firstly at water head between H_{h0} and H_0, and the crack propagation may follows mode I at water head between H_{v0} and H_{h0} and greater than H_0 and mixed mode I-II at water head between H_{h0} and H_0. In addition, Fig. 13(d) shows that the chance of hydraulic fracturing in the horizontal crack is always more than that in the cross-vertical crack under the stress state $\sigma_z = \sigma_x = \tau_{yz}$, and the crack propagation may follow mixed mode I-II.

Therefore, the stress state has much influence on the induction of hydraulic fracturing. The strike-dip of the crack inducing hydraulic fracturing and the crack propagation may change with the stress state. In actual analysis of the problem of hydraulic fracturing, it is necessary to investigate the probability in both of the two dangerous cracks according to the criterion shown in Eq.(4) at the same time. Both of the stress state and K_{IC} of the core soil affect the induction of hydraulic fracturing and the mode of the propagation of the crack.

6. CONCLUSIONS

Hydraulic fracturing in the core of the earth-rock fill dam is a very complicated and unsolved problem. This paper investigated the condition of its formation, mechanism and failure criterion. “Water wedging” action is the mechanical source that induces hydraulic fracturing, but it can only form in unsaturated core soil. The most dangerous stage of hydraulic fracturing should be the incipient impounding period. A failure criterion for hydraulic fracturing was proposed based on the fracture testing results of a silty clay (Wang et al. [17]) and the theories in LEFM. Proposed failure criterion for hydraulic fracturing was tested in the core of rock-fill dam. The results indicate that factors such as angle between crack surface and direction of principal stress, local stress state at the crack, and fracture toughness K_{IC} of core soil may largely affect the induction of hydraulic fracturing and the mode of the propagation of the crack. The easiest spreading and dangerous crack in all transverse cracks at the upstream surface of the core under water pressure is horizontal crack, and that in all vertical cracks is cross-vertical crack. The comparison of the two major cracks does not allow to determine which is more dangerous, because the stress state at the crack has large influence on the induction of hydraulic fracturing and the mode of the crack propagation.

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NOTATION

a	length of crack
c_1	cohesion of crack before water entering
c_2	cohesion of crack after water entering
E	Young’s modulus of material
F_1	coefficient of correction for K_I
F_2	coefficient of correction for K_{II}
G	energy release rate
G_w	gradient water pressure
H	water head in crack
J	J integral
K	stress intensity factor
K_I	stress intensity factors of mode I crack

K_{IC}	fracture toughness of core soil
K_{II}	stress intensity factors of mode II crack
p	water pressure
u	excess pore water pressure in soil element
u_0	initial pore pressure
x	direction pointing at left abutment along a horizontal line parallel to dam axis
y	direction pointing at downward stream perpendicular to dam axis
z	direction pointing at upward vertical
v	Poisson's ratio of material
β	slope angle of crack face
γ_w	unit weight of water
φ_1	internal friction angle of crack before water entering
φ_2	internal friction angle of crack after water entering
σ	load applying on soil element, effective normal stress developed on crack faces
σ'	effective stress in soil element
σ_n	normal stress applying on crack face
σ_t	shear stresses applying on crack face
σ_t'	effective shear stress applying on crack
σ_x	normal stress applying at vertical planes perpendicular to upstream surface of core
σ_y	normal stress applying on vertical planes parallel to upstream surface of core
σ_z	normal stress applying on horizontal planes
τ	effective shear stress applying on crack face
τ^*	reverse shear stress
τ_f	shear strength of crack
τ_{yz}	shear stress applying on vertical planes parallel to upstream surface of core
τ_{zy}	shear stress applying on horizontal planes
Δu	increment of pore pressure
$\Delta\sigma_z$	increments of vertical normal stress σ_z
$\Delta\tau_{yz}$	increments of shear stress τ_{yz}
∂a	spreading length of crack
$\partial\Pi$	reduced energy of elastic system

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