An efficient integral method for capacitance extraction of multiconductor microstrip lines

Amel Boudriga Tounsi, Khaled Grayaa et Taoufik Aguili
SysCom Laboratory, Elect. Eng. Depart., Engineer School of Tunis
P.O. Box 37, 1002, le belvedere Tunis – Tunisia
ah.tounsi@planet.tn

ABSTRACT

A new integral method is developed to analyse multiconductor microstrip lines in quasi-static. The capacitance matrix of two and eight transmission lines has been successfully calculated by using our approach. The developed method is based on the generalised equivalent circuits technique and using transverse admittance operators. The obtained integral equations are solved using Galerkin's method. This approach evaluates directly the accurate determination of capacitance matrix characterised by variational form. To improve the computation efficiency and the accuracy, the unknown charges are expanded in terms of compactly-supported wavelets. Thus highly sparse matrixes are generated and a significant reduction of computational time and memory storage are obtained. Numerical results are in good agreement with those in previous publications.

1. INTRODUCTION

With the growing speed and complexity of modern digital circuits and microwave devices, crosstalk and signal distortion become the dominant factors limiting the performance of microelectronic integrated circuits. A key step to optimize electrical performance of the integrated circuits is the extraction of the self and coupling capacitances of multilayer and multiconductor interconnects. To analysis this type of structures different methods has been reported in [1]. These includes conformal transformation method [2,3], the finite difference method [4], and spectral domain approach [5].

Capacitance extraction is an important problem that has been extensively studied. Over the last decade some efforts have been made to accelerate the procedure [6] [7] [8] [9]. Ongoing research at the present time is focused on devising methods which can be applied to more general geometries than the ones considered in the past and at the same time, improving the computational efficiency and accuracy of these methods. In the recent years, within new contributions, the measured equation of invariance on surface MEI method has been used in the capacitance matrix calculation of IC interconnects [10]. This technique improved the computational efficiency through shrinking the boundary. The spectral domain approach combined with Galerkin method, variational form or iterative techniques is probably the most simple and widely used tool. This approach retains the simplicity of conventional moment methods and optimizes them by recasting all matrix elements into rapidly converting series [11].

In this paper, an original integral method is developed to calculate the capacitance matrix of multi conductor transmission lines. It is based on generalized equivalent circuit technique and using transverse admittance operators. The set of integral equations under consideration is solved by Galerkin method. Then it is demonstrated that matrix capacitance has variational form.

In first time, it is shown that this technique is simple and very efficient with an appropriate choice of basis functions such rooftop functions. In second time, a multiresolution approach is employed. Compactly-supported wavelets are used directly as trial functions to model the charge distribution on the strips.

Numerical results will be presented for two dimensional structures and compared with other available data. It is shown that the application of the Discrete Wavelet Transform (DWT) highly sparse admittance matrixes and permits a significant reduction of computational time and of memory storage.

2. FORMALISM

The studied structures are composed of a number of conductors placed on one interface. We restrict ourselves to quasi-static domain although the present method can readily be extended to few dispersive domains.

Figure 1 shows the cross section of shielded coupled micro strip lines. There are N strips with arbitrary width of W_i, air gaps of S_i between them, located at x_i (i = 1….N). The dimensions of the metallic shield are A and $B = l_1 + l_2$, the thickness of the substrate is 1_1 with dielectric constant ε_1, The dielectric substrate is assumed to be isotropic and homogeneous. Losses in the dielectric and in the conductors as well as the metal thickness are neglected. The structure is considered infinite in z direction.

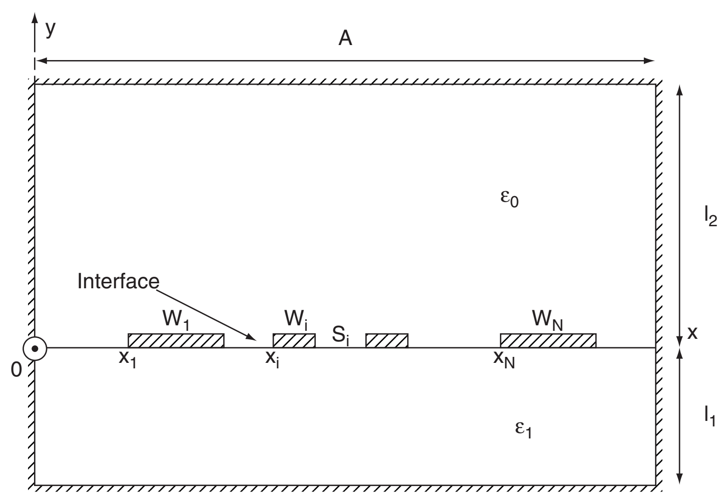

Figure 1 Cross-sectional view of multiconductor transmission lines.

2.1. PRINCIPLES

The first point of this method consists in the determination of the equivalent diagrammt of a metallized interface on the transverse plane. The aim is to express the unknown charge density of the problem considering the potential as source excitation.

In the transverse resonance method (assuming propagation in y-direction), one considers transmission plane waveguide xoy connected by adjustable sources describing the electromagnetic state of the adjustings surfaces [12]. In our work, the equation of resonance expressing the boundary conditions will be formulated by transverse admittance operators that binder the charge density ρ and the potential V [13] given by Eqn 1.

$$j\omega\rho = \hat{Y}V \tag{1}$$

The adjustable virtual sources are the trial functions describing the electromagnetic state of the problem while considering the boundary conditions. These functions are going to permit the decomposition of ρ (or V). One will have the equivalent circuit given in Figure 2 verifying:

$j\omega\rho = 0$ on the isolant domain
And the dual of $j\omega\rho$, $V' = 0$ on the metal. (2)

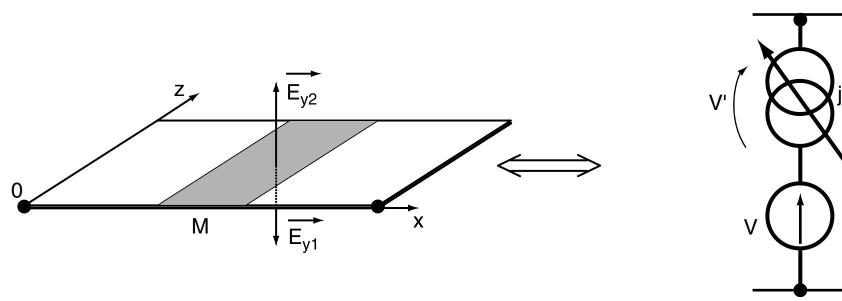

Figure 2 Interface domain and the equivalent diagram with current virtual source.

Otherwise, one can notice that this problem can be modelled considering the equivalent diagram where the virtual source translated the potential defines on the dielectric as shown in figure 3

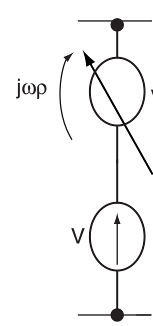

Figure 3 Equivalent diagram with potential virtual source.

2.2. DETERMINATION OF THE ADMITTANCE OPERATOR

Let E_y represents a longitudinal electric field (transverse plane xoz). according to the boundary conditions on the metal domain given by Figure 2, the charge distribution ρ is defined by the following relation:

$$\varepsilon_1 E_{y1} - \varepsilon_2 E_{y2} = \rho \tag{3}$$

One can multiply Eqn 3 by $j\omega$. So we consider:

$$j\omega\varepsilon_1 E_{y1} - j\omega\varepsilon_2 E_{y2} = j\omega\rho \tag{4}$$

In fact, we replace ρ by $j\omega\rho$ to verify the dimension equation given in Eqn 1. The current on the plane (y = 0) is generated by TM modes (f_k^{TM}) according to y-direction of the empty guide of the structure. So:

$$\vec{E}_T = -\vec{\nabla} V \tag{5}$$

Where $\vec{E}_T$ is the transverse electric field [14].

Taking in account Eqns 3, 5 and $div\vec{E} = 0$, we find for each mode designed by k, the following relation:

$$j\omega\rho_k\Big)_{i=1,2} = y_{ik} V_k \tag{6}$$

- i designes the half space 1 and 2 indicated in Figure 2
- V_k is the component potential decomposed on the TMy mode:

$$V_k = \left\langle f_k^{TM} \middle| V \right\rangle \left| f_k^{TM} \right\rangle \tag{7}$$

- Y_{ik} defined as the mode admittance given by Eqn 8:

$$y_{ik} = j\omega\varepsilon_i \frac{\left(\frac{k\pi}{A}\right)^2 + \beta^2}{p_{ik}} \tag{8}$$

In Eqn 8:

β denotes the propagation constant along the z-axis.

P_{ik} is the propagation constant along the y-axis

$$P_{ik} = \left(k\frac{\pi}{A}\right)^2 + \beta^2 \tag{9}$$

From Eqns 6, 7 and considering the contribution of all the modes f_k^{TM}(k = 1..NB), we obtain:

$$j\omega\rho = \sum_{k=1..NB} \left| f_k^{TM} \right\rangle (Y_{1k} + Y_{2k}) \left\langle f_k^{TM} \middle| V \right\rangle \tag{10}$$

Where Y_{ik} is the short-circuit admittances (i = 1,2 : top and bottom of the shielding box) on the plane y = 0:

$$Y_{ik} = y_{ik} \coth\left(P_{ik} l_i\right) \tag{11}$$

$$\hat{Y}_i = \sum_{k=1..NB} \left| f_k^{TM} \right\rangle Y_{ik} \left\langle f_k^{TM} \right| \tag{12}$$

2.3. EXPRESSION OF THE CAPACITANCE MATRIX

Taking in account the equivalent circuit in the interface y = 0 for one strip given in Figure 2, the structure given in Figure 1 can be modelled with equivalent diagram shown in Figure 4:

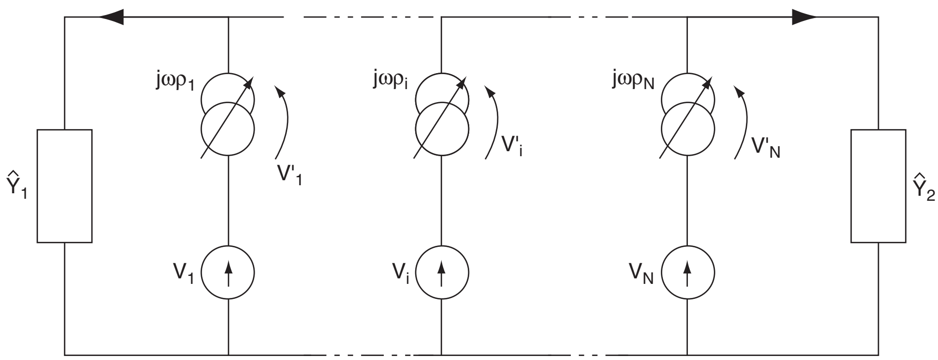

Figure 4 Equivalent circuit of the studied structure.

where :
$\hat{Y}_1$ and $\hat{Y}_2$ are the short-circuit admittances (top and bottom of the shielding box)
V_i : is the known potential on the i-th strip
V'_i : dual potential on the insulator
$j\omega\rho_i$: is the current on the interface defined by the charge distribution ρ_i on the i-th strip.
Taking in account Kirchhoff laws we obtain:

$$\sum_{i=1..N} j\omega\rho_i = \left(\hat{Y}_1 + \hat{Y}_2\right)\left(V_i + V'_i\right) \tag{13}$$

Assuming the charge distribution in terms of basis functions $g_n^{(i)}$ (n = 1 ..NE)

$$\rho_i = \sum_n x_n^{(i)} g_n^{(i)}\left(x\right) \tag{14}$$

$x_n^{(i)}$ (n = 1..NE) are the unknown coefficients to be determined, and NE is the total number of the bases. Then Eqn 13 becomes:

$$\begin{cases} j\omega\left(\hat{Y}_1 + \hat{Y}_2\right)^{-1}\left(\sum\limits_{\substack{i=1\ldots N\\ n=1\ldots NE}} x_n^{(1)} g_n^{(1)}\right) - V_1 = V'_1 \\ \vdots \\ j\omega\left(\hat{Y}_1 + \hat{Y}_2\right)^{-1}\left(\sum\limits_{\substack{i=1\ldots N\\ n=1\ldots NE}} x_n^{(i)} g_n^{(i)}\right) - V_i = V'_i \\ \vdots \\ j\omega\left(\hat{Y}_1 + \hat{Y}_2\right)^{-1}\left(\sum\limits_{\substack{i=1\ldots N\\ n=1\ldots NE}} X_n^{(N)} g_n^{(N)}\right) - V_N = V'_N \end{cases} \tag{15}$$

Galerkin's technique is applied by multiplying the equation by the basis function. A set of linear algebric equations is obtained from integral equation. The matrix form is described by Eqn 16:

$$\left(\left[M_{ij}\right]_{i,j=1..N}\right)\left[\left[X^{(j)}\right]_{j=1..N}\right]=\left[\left[V^{(j)}\right]_{j=1..N}\right] \tag{16}$$

where:

$$\left[\mathrm{X}^{(\mathrm{j})}\right]=\left[\mathrm{x}_{\mathrm{n}}^{(\mathrm{j})}\right]_{\mathrm{n}=1..\mathrm{NE}} \tag{17}$$

$$\left[\mathrm{V}^{(\mathrm{j})}\right]=\left[\left\langle \mathrm{g}_{\mathrm{m}}^{(\mathrm{j})} \middle| \mathrm{V}_{\mathrm{j}}\right\rangle\right]_{\mathrm{m}=1..\mathrm{NE}} \tag{18}$$

$$\left[\mathrm{V}'^{(\mathrm{j})}\right]=\left[\left\langle \mathrm{g}_{\mathrm{m}}^{(\mathrm{j})} \middle| \mathrm{V}_{\mathrm{j}}'\right\rangle\right]_{\mathrm{m}=1..\mathrm{NE}}=0 \tag{19}$$

$$\left[M\right]=\sum_{\substack{k=1\ldots NB\\ m,n=1\ldots NE}}\left\langle g_m^{(i)} \middle| f_k^{TM}\right\rangle \frac{1}{Y_k^1+Y_k^2}\left\langle f_m^{TM} \middle| g_n^{(j)}\right\rangle \tag{20}$$

Thus we obtain a matrix form with the unkown vector[X] :

[M][X]=[B] (21)

[M] is a matrix with size (ns,ns) with ns = N*NE

To obtain the expression of the capacitance matrix, we consider the free charge Q^i in the i-th interface :

$$j\omega Q^i=\int_{D_M}\left[g_p^i\right]^T\left[X\right]^{(i)}dx \tag{22}$$

D_M : metallic domain (i–th strip)

The elements of the capacitance matrix are given by:

$$Q^i=\sum_{j=1\ldots N} C_{ij}V_j \tag{23}$$

Therefore, we obtain a variational form of the capacitance matrix:

$$C=\left[C_{ij}\right]=\frac{1}{j\omega}\left[\left[v\right]^i\right]^T\left[\hat{Y}_{ij}\right]\left[v\right]^j \tag{24}$$

With an arbitrary excitation $V_0 = 1$ volt on the metal domains, we note:

$$[V]^j = V_j \begin{bmatrix} \langle g_p^j, 1 \rangle \\ P = 1..NE \\ j = 1..N \end{bmatrix} = V_j [v]^j \tag{25}$$

In this paper, a static model is considered then $\beta \to 0$ given in Eqn 8.

3. CHOICE OF SOURCES AND TRIAL FUNCTIONS

The modelling of a planar circuit consists of determining its behaviour by calculation and programming. Efficiency and accuracy of the computation can be improved by the choice of appropriate basis functions.

In our work different sets of basis functions have been tested, namely, wavelet bases, rooftop, U, and sinusoidal functions. In this paper, we choose compactly-supported wavelet and rooftop functions which verify the criterion established by [15], to avoid the appearance of spurious solutions, and which permit accurate results

The rooftop expansion functions approach is mostly developed for complex structures. Therefore, this approach leads to a large and dense matrix [16]. To increase the efficiency of such methods, the use of wavelet bases in multi-resolution approaches (MRA) is often proposed [17]. The use of the wavelet in L2 has two advantages: No truncation of the wavelet is needed at the boundary, and the wavelet offers a complete basis in the region of interest.

A wavelet system consists of a mother scaling function $\phi(x)$ and a mother wavelet function $\psi(x)$. These functions generate a family of basis functions $\phi_{jn}(x) = 2^{j/2}\phi(2^j x - n)$. Where n is the translation factor, and j is the resolution level [18].

An approximate of the charge distribution at a resolution k can be written as the sum of two mutually orthogonal functions namely smooth (ρ^s,macroscale) and detail (ρ^d,microscale) components. We have:

$$\rho_i(x) = \rho^s(x) + \rho^d(x) \tag{26}$$

where:

$$\rho^s(x) = P_j\rho(x) = \sum_n s_n \phi_{jn}(x), \quad s_n = \langle \rho, \phi_{jn} \rangle \tag{27}$$

$$\rho^d(x) = \sum_{m=j}^{k-1} \sum_n d_{mn}\psi_{mn}(x), \quad d_{mn} = \langle \rho, \psi_{mn} \rangle \tag{28}$$

$\langle\ \rangle$in Eqns 27, 28 denotes the inner product of L2(R) and j(k) is the reference smoothing resolution. Wavelets with N vanishing moments have the potential to sparsify dense complex matrices arising in numerical solutions to integral equations [19]. Then many matrix elements are negligible. A thresholding procedure is applied to set to zero all elements in the matrix whose magnitudes fall below the threshold. This threshold is adjusted to get an error of 1 percent or less in the results. A compression rate is defined as follows [20], [21]:

$$R(\%) = \frac{\text{NbZero}}{ns^2} \times 100 \tag{29}$$

where NbZero is the number of elements set to zero in the compression matrix and ns is the size of the linear system given in Eqn 21.

4. RESULTS

4.1. SYSTEM WITH TWO STRIP CONDUCTORS

In order to assess its effectiveness, the proposed approach is applied for a two conductors structure N = 2. The normalised parameters' structure are the same as [22]: $W_1 = W_2 = 1$, $l_1 = 1$, $s = 1$, $l_2 = 19$, $A = 20$, $\varepsilon_1 = \varepsilon_0\varepsilon_r = 10\varepsilon_0$ and $\varepsilon_2 = \varepsilon_0$ (ε_0 is the permittivity of air). Two different types of basis functions have been used to model the charge distribution; Daubechies wavelet and rooftop functions. For the first kind, we exploit the Discrete Wavelet transform DWT to compute the inner products.

A convergence study is necessary. Table 1 shows that, the results converge from NB = 1000 modes TMy and NE = 15 rooftop functions. Using Daubechies wavelet (db1), NB = 1900 modes and NE = 16 trial functions are needed to obtain the convergence.

Table 1 Convergence of scattering parameters

	Number of		R, %	Memory, Bytes	
Basis functions	**NE**	**NB**		**Full matrix**	**Compressed matrix (sparse matrix)**
Roof Top	15	1000	40	7200	6809
daubechies wavelet	16	1900	88	8192	1152

Since the developed technique uses located basis functions, the matrix given in Eqn 20 is sparse. However, a highly sparse matrix is generated when using wavelet functions. Figure 4 shows a colormap plot obtained by taking the elements of the matrix given in Eqn 20. This clearly demonstrates that the nonnegligible elements are located on or near the diagonal, which correspond to self-interactions or near-functions interactions. However, the use of wavelet functions allows a considerable reduction of nonnegligible elements.

Furthermore, as shown by table 1, for an error of 1% or less in the results, the compression rate is 40% with rooftop functions and 88% with wavelets trial functions. Thus, the use of

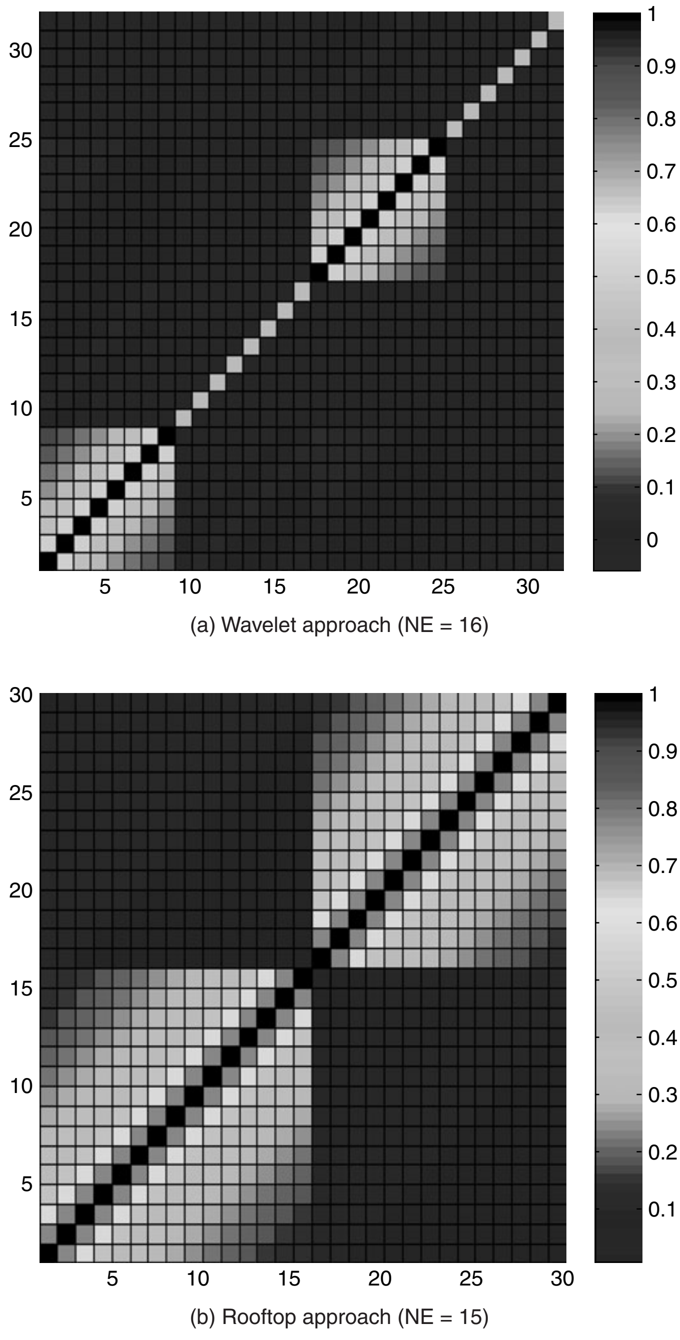

Figure 4 Colormap plot of normalized matrix M(ns,ns).

wavelets expansion decreases the computation time by a factor of three and the required memory by a factor of six.

In addition, we verify that the charge distribution computed with our approach and a compression rate of 88% satisfies the boundary conditions (figure 5).

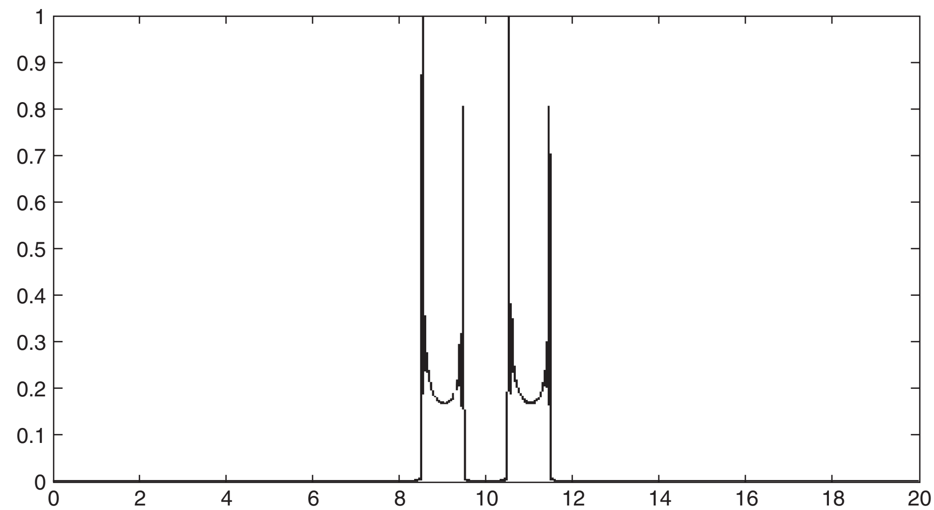

Figure 5 Normalized charge density on the x-direction (null on the insulator domain).

4.2. MULTICONDUCTORS STRUCTURE

In order to further prove the good performances of the presented method, we consider the application of the proposed approach for extracting the coupling capacitances of an eight-conductors structure.

The parameters of the structure are: N = 8 conductors, $W_1 = \dots = W_8 = 1$, $l_1 = 16$, $S_1 = \dots = S_8 = 1$, A = 175, $l_2 = 100$ and $\varepsilon_r = 12.9$.

The coefficients of the capacitance matrix (normalized to ε_0) computed with our approach are compared with those published by Oprea [23] reported by Amirhosseini in [22] and Table 2 shows that our results are in perfect agreement with those given by the references. The computing error is within 2%.

5. CONCLUSION

A new accurate quasi static integral method for extraction capacitance matrix of multiconductor transmission lines is presented. The method is based on the generalised equivalent circuits technique and using transverse admittance operators. This approach possesses the properties of rapid convergence. It has been validated by comparing our results with those published by other authors.

Taking full advantages of multiresolution analysis, it has been demonstrates that highly sparse matrix is obtained. Thus the calculation time and the memory storage are significantly reduced.

Table 2 Normalized capacitance for configuration of 8 conductors

Normalized capacitance	**C11**	**C22**	**C33**	**C44**	**C12**	**C23**	**C34**	**C45**	**C13**	**C24**
[23] reported in [22]	14,451	17,556	17,705	17,731	–6,610	–5,940	–5,875	–5,865	–1,473	–1,182
Our results	14,9	17,937	17.606	17,612	–6,448	–5.98	–5.966	–5,875	–1,485	–1,176
Normalized capacitance	**C35**	**C14**	**C25**	**C36**	**C15**	**C26**	**C16**	**C27**	**C17**	**C18**
[23] reported in [22]	–1,150	–0,647	–0,492	–0,477	–0,351	–0,261	–0,214	–0,163	–0,145	–0,137
Our results	–1,127	–0.663	–0,489	–0,467	–0,344	–0,257	–0,209	–0,158	–0,150	–0,132

REFERENCES

[1] T. Itoh, Ed., 'Planar Transmission line structures'. New York:IEEE Press, 1987.

[2] C. Wan, 'Analitically, and accurately determined quasi-static parameters of coupled microstrip lines,' IEEE Trans. MTT,Vol 44, pp.75–79, Jan.1996

[3] H. A. Wheeler, 'Transmission line properties of parallel wide strips by a conformal mapping approximation' IEEE Trans. MTT, pp. 280–289, May 1964.

[4] R.N. Simons, 'Suspended coupled slotline using double layer dielectric,' IEEE Trans. Microwave Theory Tech., vol. MTT-29, pp. 1102–1107, Oct. 1981.

[5] T. Itoh, 'Generalized spectral domain method for multiconductor printed lines and its application to turnable suspended microstrips,' IEEE Trans.Microwave Theory Tech, vol. MTT–26, pp. 983–987, Dec. 1977.

[6] A. Kouki, A. Khebir, and R. Mittra, 'Finite Element Modeling of Two-dimensional Transmission Line Strutures Using a New Asymptotic Boundary condition', IEEE MTT-S Digest, pp. 717–720, 1990.

[7] E. Drake, F. Medina and M. Horno, "Improved Quasi-TEM Spectral Domain Analysis of Boxed Coplanar Multiconductor Microstrip Lines" IEEE Trans. Microwave Theory Tech, vol. MTT-41, N°2 pp. 260–267, Feb. 1993.

[8] G. Wang, G. Pan and B.K. Gilbert, 'A hybrid Wavelet Expansion and Boundary Element Analysis for Multiconductor Transmission Lines in Multilayered Dielectric Media'. IEEE Trans. Microwave Theory Tech, vol. MTT-43, N°3 pp. 664–675, March 1995.

[9] N.D. Arora, K.V. Raol, R. Schumann, and L.M. Richardson, 'Modeling and extraction of interconnect capacitances for multilayer VLSI circuits', IEEE Trans. on Computer-Aided Design, vol. 15, 1996, pp. 58–67.

[10] Y. W. Liu, K. Lan, and K. K. Mei, 'Computation of Capacitance matrix for integrated Circuit Interconnects Using On-surface MEI Method', IEEE Microwave and guided wave letters, vol. 9, N°8, pp. 303–304, August 1999.

[11] J. L. Tsalamengas, 'Rapidly converging Spectral-Domain Analysis of Shielded Layered Finlines', IEEE Trans. Microwave Theory Tech, vol. MTT-47, N°6 pp. 805–810, June 1999.

[12] K. Grayaa, N. Hamdi, T. Aguili, A. Bouallegue, 'Full – Wave analysis of shielded planar circuits using different models of sources', IEE Proc – Microwave., Ant. Propag., August 2003.

[13] A. Boudriga Tounsi, K. Grayaa, T. Aguili "Application des ondelettes à l'étude d'une ligne microbande en quasi-statique," 18° colloque international,Optique Hertzienne et Diélectrique, pp. 318–321, OHD' 2005, Sep 2005 Hammamt,Tunisie.

[14] T. Aguili, 'Modelisation des composants S.H.F planaires par la méthode des circuits equivalents generalise', Thèse d'état, ENIT, 1999, pp. 90–100.

[15] H. Aubert, B. Souny, and H. Baudrand: 'Origin and avoidance of spurious solutions in the transverse resonance method', IEEE, Trans, 1993, MTT, pp. 450–456.

[16] T. S. Horng, W. E. Mckinzie, and N.G. Alexopoulos, 'Full-wave spectral domain analysis of compensation of microstrip discontinuities using triangular subdomain functions', IEEE, Trans. 1992, MTT-40, pp. 2137–2148.

[17] N. Guan, K. Yashiro, S. Ohkawa, 'On a choice of wavelet Bases in the wavelet transform approach,' IEEE Trans.Antennas and Propagtions, vol. 48 N 8, p1186, August 2000.

[18] I.Daubechies, ' Ten lectures on Wavelets,' SIAM, 1992.

[19] N. Soveiko, M. S. Nakhla, 'Efficient capacitance extraction computations in wavelet domain', IEEE Trans. On Circuits and Signals,Vol. 47, N°5, May 2000.

[20] R. Loison, R. Gillard and J. Citerne, 'A Multiresolution MOM Analysis of Multiport Structures Using Matched Terminations', IEEE Trans. Microwave Theory and Techniques, vol. 49, N°1, January 2001.

[21] K. Grayaa, T. Aguili, A. Bouallegue, 'Multiresolution source method for the analysis of planar microstrip structures,' Journal of Microwaves and Optoelectronics, Vol 4, N°2, Dec 2005.

[22] M. Khalaj Amirhosseini, 'Determination of capacitance and conductance matrices of lossy shielded coupled microstrip transmission lines,' Progress In Electromagnetics Ressearch, PIER 50, pp. 267–278, 2005.

[23] D. Homentcovschi, R. Oprea, 'Analytically determined quasi-static parameters of shielded or open multiconductor microstrip lines,' IEEE Trans. Microwave Theory Tech, vol. MTT-46, N°1, pp. 18–24, Jan. 1998.