Ultrasound-Driven Fluid Motion – Modelling Approach

D Rubinetti*, D A Weiss
Institute of Thermal and Fluid Engineering, University of Applied Sciences and Arts - Northwestern Switzerland

ABSTRACT
Fluid motion induced by ultrasound is an effect that arises from the attenuation of sound waves in a fluid. This phenomenon allows for a series of applications in industry. To achieve a significant effect in practice, high-intensity acoustics is required, which can solely be realized using the characteristics of ultrasound. Its high-frequency behaviour on one side is confronted with the nearly steady-state nature of the fluid flow on the other side.

The present study proposes a numerical modelling approach to further investigate and identify development potential. The coupling of acoustics and fluid flow poses a challenging multiphysics problem, its treatment requires an appropriate handling of transient quantities on the different frequency scales. Basically the force triggering a low frequency fluid flow arises based on the time average of quantities varying on a high frequency scale.

The analysis includes an estimation of acting terms by dimensionless relations as well as a verification by means of a simplified test-case. The concept presented is numerically stable and appropiate. It can be adapted to related applications involving sound-driven fluid motion.

1. INTRODUCTION
Ultrasound-driven fluid motion occurs when a fluid absorbs sound waves and responds accordingly. This effect is used for a remarkable range of technologies of different scales from medical handheld devices up to the stirring of molten metal [1]. Numerical modeling of this effect and its underlying driving mechanisms grant access to a deeper understanding and identification of improvement potential. Merging the highly transient nature of ultrasound with the quasi-stationary characteristics of fluid flow poses a challenging multiphysics problem. In this study an approach to couple high-frequency physics (ultrasound acoustics) and low-frequency physics (fluid flow) is presented showing the key points to obtain a successful convergence by means of dimensionless analysis.

2. PHYSICAL MODEL
To link high-frequency acoustics and low-frequency fluid dynamics the crucial point is to separate the frequency domains. In order to bridge the segregated domains, an expression for the slowly varying driving force is needed as a function of the quickly varying ultrasound quantities. This is done by averaging over time-scales which are large in comparison to the

*Corresponding Author: donato.rubinetti@fhnw.ch

ultrasonic time scale, but still sufficiently small as compared to the fluid mechanical time scale. This of course requires that the two frequency domains can indeed be separated in such a way at all. As a result, the following term is included into the momentum balance of the fluid motion [2]:

$$\boldsymbol{F} = -\frac{1}{2}\boldsymbol{\nabla}\overline{\boldsymbol{v_a}\cdot\boldsymbol{v_a}} - \frac{{c_0}^2}{2{\rho_0}^2}\left[\frac{\rho_0}{{c_0}^2}\left(\frac{\partial^2 p}{\partial \rho^2}\right)_{\rho=\rho_0} - 1\right]\boldsymbol{\nabla}\overline{{\rho_a}^2} - \frac{1}{{\rho_0}^2}\left(\zeta+\frac{\mu}{3}\right)\boldsymbol{\nabla\nabla}\cdot\overline{(\rho_a\boldsymbol{v_a})} - \frac{\mu}{{\rho_0}^2}\nabla^2\overline{(\rho_a\boldsymbol{v_a})} - \frac{1}{{\rho_0}^2}\left(\zeta+\frac{4\mu}{3}\right)\overline{\rho_a\nabla^2\boldsymbol{v_a}} \quad (1)$$

with $\boldsymbol{v_a}$ being the particle velocity and p the oscillating pressure due to the ultrasound, c_0 denotes the speed of sound, ρ_0 the density of the fluid (assumed constant on the low frequency time scale), ρ_a the (quickly oscillating) density perturbation. ζ denotes the bulk viscosity and μ the dynamic viscosity. Overbar denotes the time-averaging mentionned above.

It is emphasized that the expression (1) may not be curl-free, if it is to cause a fluid motion. Otherwise, i.e. if the force (1) can be interpreted as the gradient of some potential, it solely would modify the pressure distribution in the flow field, without triggering any fluid motion; this conclusion becomes more evident when the transport of vorticity is analysed directly. It can be seen in (1) that those terms not related to viscosity are obviously gradients of suitably defined quantities, which expresses the fact that viscosity is indeed needed to describe the triggering of fluid motion. This in turn can be explained by the requirement that momentum has to be transferred somehow from the ultrasound to the fluid.

The acting force described by equation (1) is derived from the solution of the governing equation for acoustics, including an attenuation term, which writes [3]

$$-\frac{\omega^2}{a^2}p + \text{h.o.t.} = \nabla^2 p + \frac{4}{3}\frac{i\omega\mu}{a^2\rho_0}\nabla^2 p \quad (2)$$

where ω denotes the angular frequency and h.o.t. summarize higher order terms.

3. DIMENSIONLESS ESTIMATION

The driving force of the phenomenon emerges from the attenuation of the acoustic sound waves in the fluid. Depending on its intensity, the force term (1) may induce a steady flow. A prediction of the behaviour can be made by writing the momentum balance of the fluid [4,5] in dimensionless form:

$$\frac{D\vec{w}}{Dt} = -\frac{[p_{st}]}{\rho[w]^2}\nabla p_{st} + \frac{1}{Re}\nabla^2\vec{w} - \frac{[w_v]^2}{[x_D]}\frac{[x]}{[w]^2}\frac{\partial}{\partial x_l}\overline{(w_{vl}\,w_{vm})} \quad (3)$$

where the quantities and lengths are now dimensionless: p_{st} denotes the (slowly varying or steady) pressure with $[p_{st}]$ being the characteristic scale, $\vec{w}$ is the (slowly varying or steady) fluid velocity with the scale $[w]$; $[x]$ denotes the characteristic length and $[x_D]$ the characteristic attenuation length. The characteristic acoustic velocity can be expressed as follows

$$[w_v] = \left(\frac{k}{\rho\omega}\right)_s [p]. \tag{4}$$

with the wave number k being the proportionality constant between angular frequency and speed of propagation, ρ being the fluid density and p being the oscillating pressure as in (1).

The coefficient

$$\Pi = \frac{[w_v]^2}{[x_D]}\frac{[x]}{[w]^2} = \frac{k^2[p]^2}{\rho^2\omega^2[x_D]}\frac{[x]}{[w]^2} \tag{5}$$

denoting the scale for the force term, proves to be useful, as the numerical complexity increases with increasing Π, and non-linear solvers tend to have convergence problems then. In the present work the case investigated is shown in figure 1.

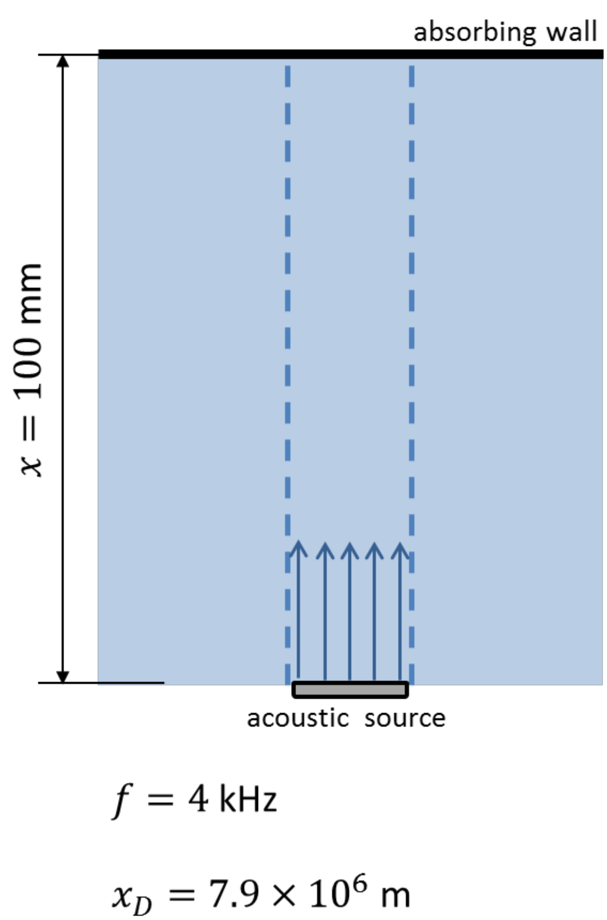

Figure 1: Illustration of the investigated setup for the numerical model with an acoustic source and a sample fluid

The test-case is chosen for validation purposes with Eckart streaming [6]. Given the parameters in figure 1, the force term in equation (1) scales with a factor Π of 0.002. Compared to the other terms of the momentum balance, the force term is therefore significantly smaller.

4. NUMERICAL MODEL

4.1. Modelling essentials

In this study the commercial tool COMSOL Multiphysics™ is used which conveniently allows the division of the simulation setup into three steps

1) Acoustics simulation
2) Force term evaluation
3) Fluid dynamics simulation (CFD)

Acoustics and fluid dynamics are coupled as per a time-averaged force stemming from acoustics and triggering the fluid motion. To simplify time averaging, quickly oscillating acoustic quantities are expressed by means of complex quantities. This allows a separation of frequency domains by splitting the analysis into high-frequency acoustics and low-frequency fluid dynamics. Density and pressure perturbations are coupled in the acoustics part, which requires a compressible fluid description. In the stationary flow evaluation compressibility has no relevance.

The body force source term (1) can create significant non-linearities in the momentum balance, leading to severe convergence issues. As foreseeable in (3), the model is simplified assuming isothermal behavior, neglecting cavitation and turbulent effects. The latter is delicate in terms of physical accuracy due to widely spread Reynolds numbers. In proximity of the acoustic source the Reynolds number can be up to 600'000, whereas in the vast outer region the Reynolds number is typically of the order of 1000. Nonetheless it improves numerical stability significantly. Further stabilization is achieved by ramping up the force term stepwise from a vanishingly small factor (e.g. 1E–8) to 1. For practical use a number of 500 intermediate iteration steps is recommended.

4.2. Setup and Boundary Conditions

The numerical model for the test-case is set up with a 2D-axismmetric geometry according to figure 2. The boundary conditions for the test-case are listed in table 1.

Table 1: Boundary conditions for the acoustics and fluid flow (CFD) part of the simulation for the experimental case

Boundary	Part	Acoustic BC	CFD BC
1	acoustic source	acceleration	free slip wall
2	continuity	sound hard	free slip wall
3	continuity	sound hard	free slip wall
4	rigid wall	plane wave radiation	no slip wall
5	rigid wall	sound hard	no slip wall
6	interior wall	sound hard	not applicable

The particularity of this case is that for the acoustics the represented domain is split by an interior wall at boundary 6. The interior wall limits the propagation of sound waves to the domain formed by boundaries 1, 5 and 6. The acoustic source emits waves towards the absorbing wall, where no reflection occurs. For the fluid flow simulation the interior wall is ignored.

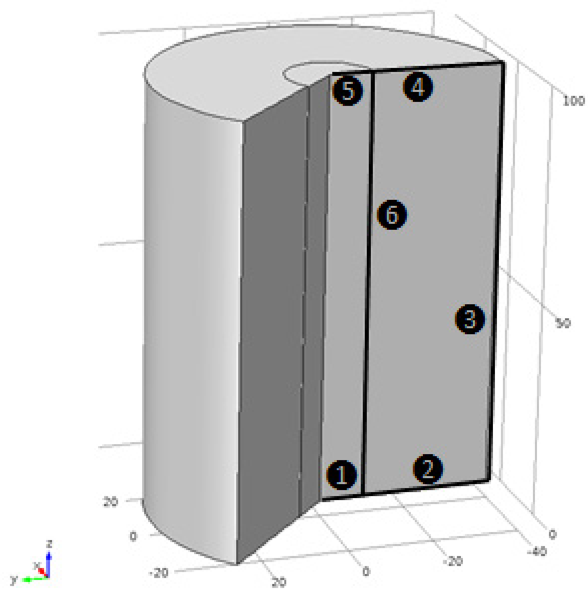

Figure 2: Revolved representation of the 2D-axisymmetric geometry of the test-case, units in mm

4.3 Mesh

The mesh consists of hexa-elements with refined resolution on boundaries as shown in figure 3. In the present case the total number of elements is 16'000 distributed over a surface of 4000 mm^2.

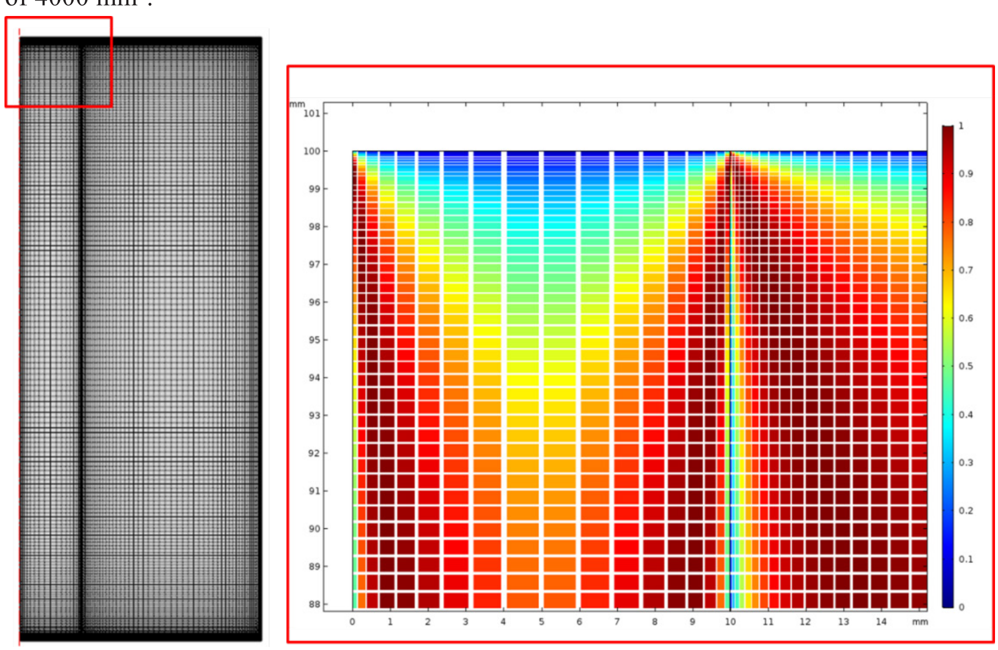

Figure 3: Mapped test case mesh. The color legend displays the quality of the mesh elements, which is at the lowest in proximity of the boundaries due to large aspect ratios of the boundary layers.

5. TEST CASE VALIDATION

5.1 Simulation results

Figure 4 shows the RMS velocity of the sound field for the acoustic simulation. As expected given the boundary conditions from table 1, the sound waves are propagating uniformly from the source to the absorbing boundary. Outside the range of the sound beam the vibrational velocity equals zero.

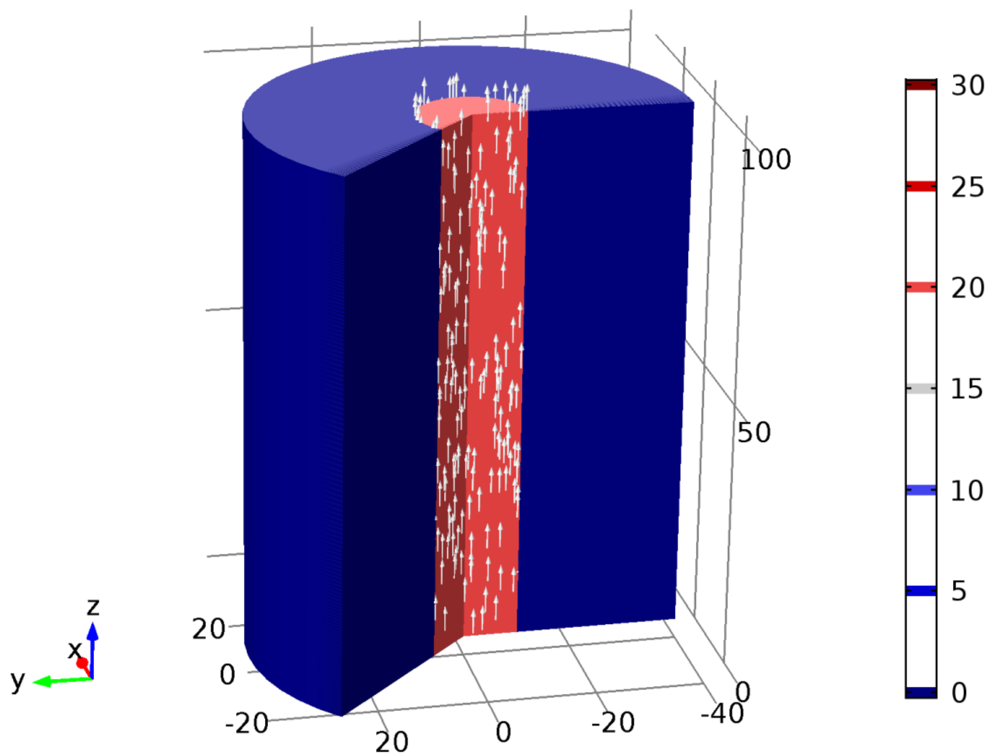

Figure 4: Acoustic RMS velocity of 21 m/s with anisotropic distributed velocity vectors, grid units in mm. As expected, the velocity magnitude is uniform.

Figure 5 shows the resulting velocity field from the test-case. The characteristic jet emerging from the acoustic source and the corresponding streamlines can be seen. Due to numerical stabilization methods and the assumptions taken, the jet is not uniformly distributed over the rotational axis.

5.2 Comparison with analytical solution

For this test case known as Eckart streaming, a solution was found for the typical streaming pattern. The characteristic radial velocity profile is given by [1]

$$[v_z] = \begin{cases} \frac{1}{2}\left(1-\frac{\psi^2}{\chi^2}\right)\left(1-\frac{1}{2}\chi^2\right)(1-\psi^2) - \ln\chi\,, & 0 \le \psi \le \chi \\ -\left(1-\frac{1}{2}\chi^2\right)(1-\psi^2) - \ln\psi\,, & \chi \le \psi \le 1 \end{cases} \tag{6}$$

where $\psi = r/r_0$ and $\chi = r_1/r_0$ with r_1 being the beam radius and r_0 being the tube radius.

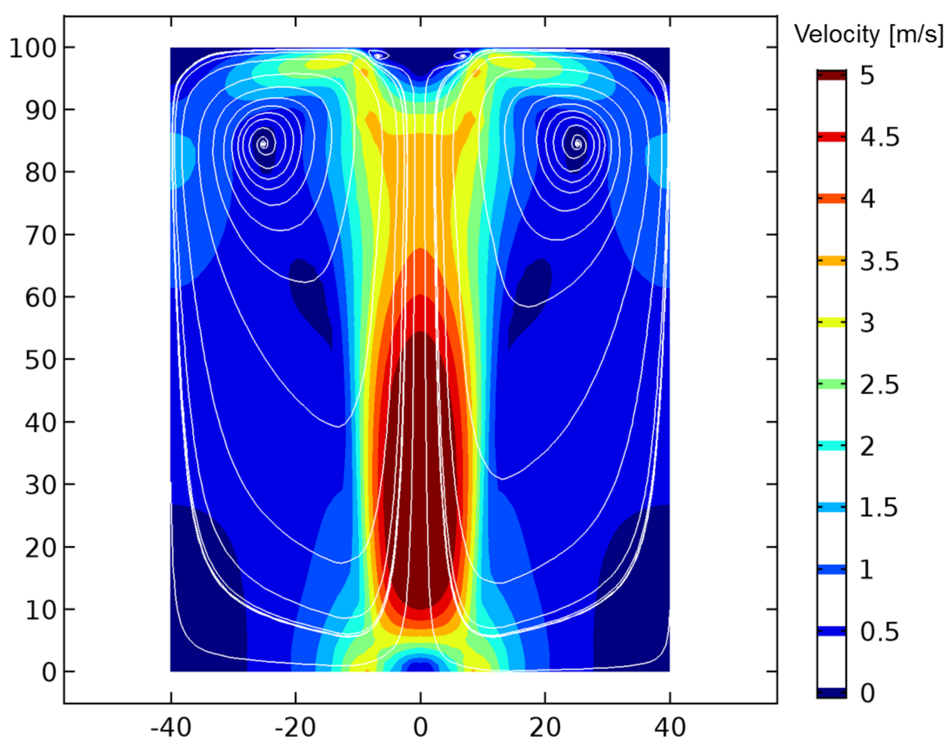

Figure 5: Resulting fluid flow velocity field and streamlines, grid units in mm

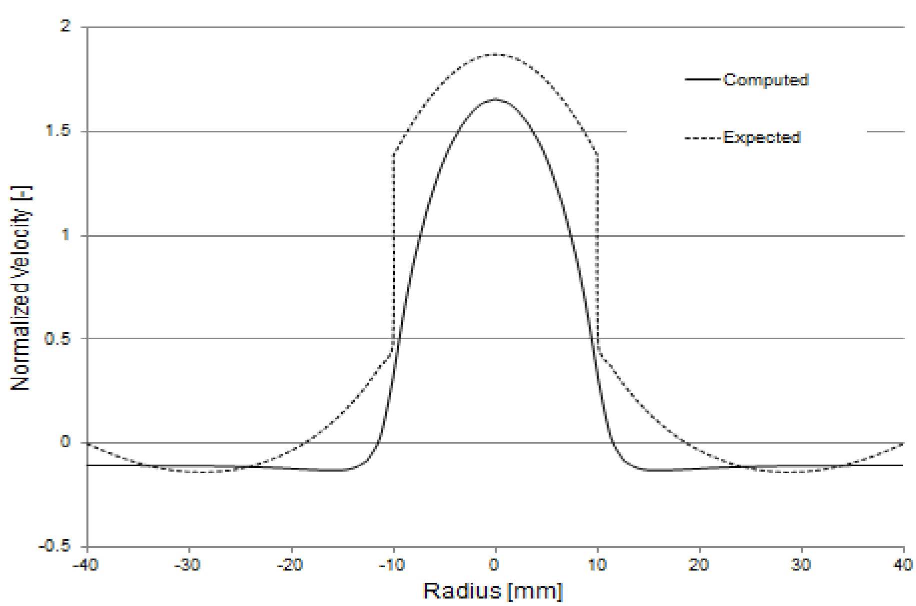

Figure 6: Comparison of analytical solution and computed solution for the normalized velocity profile at z=20mm

From figure 6 it can be seen that the velocity profiles in dimensionless notation are in good agreement. Both experience their peak velocity at the center of the beam axis. The computed solution has a smooth transition from positive to negative velocity, whereas the analytical solution follows a sharp decline at the beam borders.

6. CONCLUSIONS

The proposed approach to model ultra-sound driven flows is physically consistent and appropiate. The challenging coupling of high-frequency acoustics and low-frequency fluid flow requires numerical stabilization methods and simplifications. Thus, dimensionless analysis and analytical verification for test-cases prove to be meaningful to confirm the model's functionality. The methodology presented is particularly useful for testing parameter variations, geometrical modifications and material adjustments. On an industrial scale it can conveniently be used to optimize operating conditions and to spot areas of improvement.

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