International Journal of Multiphysics
Volume 18, No. 3, 2024
ISSN: 1750-9548

Optimal Algorithms for Two Agent Parallel Batch Scheduling with Rejection

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Abstract

Efficient scheduling can make full use of resource, and achieve many goals, such as maximizing efficiency and saving energy. The scheduling problems involving two agents and rejection to perform their respective jobs on a parallel batch machine are considered. The manager need to choose the jobs for processing and arrange its schedule. The objective is to minimizes the objective of first agent, and keep the other agent's objective below a given threshold. Three objective functions in scheduling theory are studied, and we analyze the problem complexity and give optimal algorithms for three problems. The optimal algorithms can help the manager to reduce the production time and cost.

Keywords: Scheduling, two agents, rejection penalty, parallel batch.

1. Introduction

The scheduling theory is an important research area which contains a lot of scheduling models, such as agent scheduling, scheduling with rejection and batch scheduling. For agent scheduling, there are some agents which has its own performance measure and compete for some resource. In last decade, agent scheduling are studied with many applications, such as production scheduling, project scheduling, and computer network. In the real world, due to limited resources, the manager often rejects some jobs which have long processing times but with little profit, and some rejection penalty should be paid . A parallel-batching machine can handing jobs in batches, and the same batch have same completion time. These three scheduling models are modern scheduling ,and have great theory price and practical value.

Agent scheduling problems were studied by Agnetis et al. [1]. They consider many different models, such as single machine and parallel machines, and different scheduling objectives, and provide optimal algorithms to solve various problems. Single machine agent scheduling were studied by Agnetis et al. [2] and Perez-Gonzalez et al. [3] .They provide optimal algorithms to solve various problems .The agent scheduling with total late work was studied by Zhang et al. [4] , and give optimal algorithm for the objection function. Freud et al.[5] extended the model with job rejection, and proposed optimal algorithm for the objection function. Li et al. [6] study agent scheduling on a parallel batch machine. They consider many different models, such as different scheduling objectives, and provide optimal algorithms to solve various problems. Fan et al. [7] study agent scheduling on a parallel batch machine, and the objective to minimize total completion time, and give optimal algorithm for the objection function. Oron et al. [8] study two agent scheduling where the jobs have equal processing times, and proposed optimal algorithm for the objection function. Wang et al. [9] studied scheduling with different job sizes to minimize the makespan, and give optimal algorithm for the objection function. Tang et al. [10] analyzed the agent scheduling problem with deterioration, and proposed optimal algorithm for the objection function. Shabtay et al. [11] give a survey paper for scheduling with rejection .They consider many different models, such as single machine and parallel machines, and different scheduling objectives, and provide optimal algorithms to solve various problems. Sterna, M. [12] give a survey for early and late work scheduling, they provide optimal algorithms to solve various problems .Feng Qi et al. [13] study agent scheduling with rejection, and give optimal algorithm for the objection function. Agnetis et al. [14] study the price of fairness of two agents, and provide optimal algorithms to solve various problems. Gao et al. [15] studied agent scheduling with deteriorating, and give optimal algorithm for the objection function.

There are a lot of papers study on agent scheduling, parallel batch machine and job rejection separately, but there is no paper study about these three models together. We study the scheduling problems involving two agents and rejection on a parallel batch machine, analyze the problem complexity and give optimal algorithms for three problems.

2. Materials and Methods

There are two agents, agent A and B, and the set of jobs of each agent denoted as $J^A = \left\{ J_1^A, J_2^A, \cdots, J_{n_A}^A \right\}$ and $J^B = \left\{ J_1^B, J_2^B, \cdots, J_{n_B}^B \right\}$.There are only one parallel batch machine for production. For each job $J_j^A(J_j^B)$, it's processing time is $p_j^A(p_j^B)$, due date is $d_j^A(d_j^B)$, weight is $\omega_j^A(\omega_j^B)$ and rejection penalty is $e_j^A(e_j^B)$.

We define time-related objective functions in table 1.

Table 1 The time-related objective functions

C_j^G	completion time of J_j^G, $G \in \{A, B\}$
$L_j^G = C_j^G - d_j^G$	lateness of J_j^G. $L_{\max}^G = \max\{L_j^G\}$
U_j^G	If $L_j^G > 0$, then $U_j^G = 1$; otherwise $U_j^G = 0$
$\sum C_j^G$	total processing completion time of J_j^G, $G \in \{A, B\}$
$\sum \omega_j^G U_j^G$	total weighted number of J_j^G, $G \in \{A, B\}$

We denote three problems as follows according to the three-field notation:

P1: $1 \left| agent, p-batch, b \geq n, rej \right| \sum C_j^A + \sum e_j^A : \sum C_j^B + \sum e_j^B$

P2: $1 \left| agent, p-batch, b \geq n, rej \right| \sum C_j^A + \sum e_j^A : L_{\max}^B + \sum e_j^B$

P3: $1 \left| agent, p-batch, b \geq n, rej \right| \sum \omega_j^A U_j^A + \sum e_j^A : \sum \omega_j^B U_j^B + \sum e_j^B$

3. Results and Discussion

3.1 Problem P1

Lemma 1. There exists an optimal schedule that accepted jobs are arranged in SPT sequence for both agents .

We arrange the jobs in SPT order, $p_1^A \leq p_2^A \leq \cdots \leq p_{n_A}^A$, $p_1^B \leq p_2^B \leq \cdots \leq p_{n_B}^B$. Let $F_A(i, j, k, g)$, $F_B(i, j, l, h)$ as the optimal objective solution for agent A and B ,and satisfying two cases:

(1)The index of jobs which is arranging of agent A and B is i and j, and the number for processing is k and l.

(2) The job which has longest processing time in first batch of A and B is J_g^A, J_h^B.

Algorithm DP1

The boundary condition:

$$F_A(n_A+1, n_B+1, k, g) = \begin{cases} 0, & k=g=0 \\ +\infty, & else \end{cases}$$
$$F_B(n_A+1, n_B+1, l, h) = \begin{cases} 0, & l=h=0 \\ +\infty, & else \end{cases} \quad (1)$$

Recurrence relations:

(1) The processing job is i from agent A, and $g > i$.

$F_A(i, j, k, g) = F_A(i+1, j, k-1, g) + p_g^A$, and $F_B(i, j, l, h) = F_B(i+1, j, l, h) \le Q$.

(2) The processing job is i from agent A, and $g = i$

$F_A(i, j, k, g) = \min_{i+1 \le g' \le n_A+1} \{F_A(i+1, j, k-1, g') + kp_i^A\}$, and $F_B(i, j, l, h) = lp_i^A + F_B(i+1, j, l, h) \le Q$.

(3)The processing job is i from agent A, and the job is reject.

$F_A(i, j, k, g) = F_A(i+1, j, k, g) + e_i^A$, and

$F_B(i, j, l, h) = F_B(i+1, j, l, h) \le Q$.

(4) The processing job is j from agent B, and $h > j$.

$F_A(i, j, k, g) = F_A(i, j+1, k, g)$, and

$F_B(i, j, l, h) = F_B(i, j+1, l-1, h) + p_h^B \le Q$.

(5) The processing job is j from agent B, and $h = j$

$F_A(i, j, k, g) = kp_j^B + F_A(i, j+1, k, g)$, and

$F_B(i, j, l, h) = \min_{j+1 \le h' \le n_B+1} \{F_B(i, j+1, l-1, h') + lp_j^B\} \le Q$.

(6) The processing job is j from agent B, and the job is reject.

$F_A(i, j, k, g) = F_A(i, j+1, k, g)$, and

$F_B(i, j, l, h) = F_B(i, j+1, l, h) + e_j^B \le Q$.

Optimal value: $\min\{F_A(1,1,k,g) \mid F_B(1,1,l,h) \le Q\}$. (2)

Algorithm DP1 provide an optimal schedule in $O(n_A{}^3 n_B + n_A n_B{}^3)$ time for problem P1, which is pseudo-polynomial time optimal algorithm, so problem P1 is weakly NP-hard.

3.2 Problem P2

Lemma 2. There exists an optimal schedule that accepted jobs are arranged in SPT sequence for both agents.

We arrange the jobs in SPT order, $p_1^A \le p_2^A \le \cdots \le p_{n_A}^A$, $p_1^B \le p_2^B \le \cdots \le p_{n_B}^B$. Let $F_A(i, j, k, g)$ as the optimal objective solution of agent A, $L_B(i, j, l, h, E_B)$ is the smallest $L_{\max}$ of agent B, and satisfying two cases:

(1)The index of jobs which is arranging of agent A and B is i and j, and the number for processing is k and l.

(2) The job which has longest processing time in first batch of A and B is J_g^A, J_h^B.The rejection penalty of agent B is E_B.

Algorithm DP2

The boundary condition:

$$F_A(n_A+1, n_B+1, k, g) = \begin{cases} 0\,, & k = g = 0 \\ +\infty\,, & else \end{cases}$$
$$L_B(n_A+1, n_B+1, l, h, E_B) = \begin{cases} -\infty\,, & l = h = 0, E_B = 0 \\ +\infty\,, & else \end{cases} \quad (3)$$

Recurrence relations:

(1) The processing job is i from agent A, and $g > i$.

$F_A(i, j, k, g) = F_A(i+1, j, k-1, g) + p_g^A$, and $L_B(i, j, l, h, E_B) = L_B(i+1, j, l, h, E_B)$, $L_B(i, j, l, h, E_B) + E_B \le Q$

(2) The processing job is i from agent A, and $g = i$

$F_A(i, j, k, g) = \min_{i+1 \le g' \le n_A+1} \{F_A(i+1, j, k-1, g') + kp_i^A\}$, and $L_B(i, j, l, h, E_B) = L_B(i+1, j, l, h, E_B) + p_i^A$, $L_B(i, j, l, h, E_B) + E_B \le Q$.

(3)The processing job is i from agent A, and the job is reject.

$F_A(i, j, k, g) = F_A(i+1, j, k, g) + e_i^A$, and

$L_B(i, j, l, h, E_B) = L_B(i+1, j, l, h, E_B)$, $L_B(i, j, l, h, E_B) + E_B \le Q$.

(4) The processing job is j from agent B, and $h > j$.

$F_A(i, j, k, g) = F_A(i, j+1, k, g)$, and

$L_B(i, j, l, h, E_B) = \max\left\{p_h^B - d_j^B, L_B(i, j+1, l-1, h, E_B)\right\}$, $L_B(i, j, l, h, E_B) + E_B \le Q$.

(5) The processing job is j from agent B, and $h = j$

$F_A(i, j, k, g) = F_A(i, j+1, k, g) + kp_j^B$, and

$L_B(i, j, l, h, E_B) = \max\left\{p_j^B - d_j^B, L_B(i, j+1, l-1, h', E_B) + p_j^B\right\}$, $j+1 \le h' \le n_B+1$,

$L_B(i, j, l, h, E_B) + E_B \le Q$.

(6) The processing job is j from agent B, and the job is reject.

$F_A(i, j, k, g) = F_A(i, j+1, k, g)$, and

$L_B(i, j, l, h, E_B) = L_B(i, j+1, l, h, E_B - e_j^B)$, $L_B(i, j, l, h, E_B) + E_B \le Q$.

Optimal value: $\min\{F_A(1,1,k,g) \mid L_B(1,1,l,h,E_B)+E_B \le Q\}$ (4)

Algorithm DP2 provide an optimal schedule in $O(n_A{}^3 n_B + n_A n_B{}^3 E)$ time for problem P2, which is pseudo-polynomial time optimal algorithm, so problem P2 is weakly NP-hard.

3.3 Problem P3

For problem P3: $1 \mid agent,\ p-batch, b \ge n, rej \mid \sum \omega_j^A U_j^A + \sum e_j^A : \sum \omega_j^B U_j^B + \sum e_j^B$, we assume that all tardy jobs of the same agent are scheduled in a separate batch at the end.

Lemma 3. There exists an optimal schedule that accepted jobs are arranged in SPT sequence for both agents.

We arrange the jobs in SPT order, $p_1^A \le p_2^A \le \cdots \le p_{n_A}^A$, $p_1^B \le p_2^B \le \cdots \le p_{n_B}^B$. Let $F_A(i, j, g, h, W_B, E_B)$ as the optimal objective solution, and satisfying two cases:

(1)The index of jobs which is arranging of agent A and B is i and j , and the number for processing is k and l .

(2) The job which has longest processing time in first batch of A and B is J_g^A, J_h^B .The weighted number of tardy jobs and rejection penalty of agent B is W_B, E_B .

Algorithm DP3

The boundary condition:

$$F_A(n_A+1, n_B+1, g, h, W_B, E_B) = \begin{cases} 0, & g=h=0, W_B=0, E_B=0 \\ +\infty, & else \end{cases} \quad (5)$$

Recurrence relations:

(1) The processing job is i from agent A, and $g > i$.

$F_A(i, j, g, h, W_B, E_B) = F_A(i+1, j, g, h, W_B, E_B)$, and $p_g^A \le d_i^A, W_B + E_B \le Q$.

(2) The processing job is i from agent A, and $g = i$

$F_A(i, j, g, h, W_B, E_B) = F_A(i+1, j, g', h, W_B, E_B)$, and $p_i^A \le d_i^A, W_B + E_B \le Q$.

(3)The processing job is i from agent A, and the job is reject.

$F_A(i, j, g, h, W_B, E_B) = F_A(i+1, j, g, h, W_B, E_B) + e_i^A$, and

$W_B + E_B \le Q$.

(4) The processing job is i from agent A, and the job is tardy.

$F_A(i, j, g, h, W_B, E_B) = F_A(i+1, j, g, h, W_B, E_B) + \omega_i^A$, and

$p_g^A > d_i^A, W_B + E_B \le Q$.

(5) The processing job is j from agent B, and $h > j$.

$F_A(i, j, g, h, W_B, E_B) = F_A(i, j+1, g, h, W_B, E_B)$, and

$p_h^B \le d_j^B, W_B + E_B \le Q$.

(6) The processing job is j from agent B, and $h = j$

$F_A(i, j, g, h, W_B, E_B) = F_A(i, j+1, g, h', W_B, E_B)$, and

$p_j^B \le d_j^B, W_B + E_B \le Q$.

(7) The processing job is j from agent B, and the job is reject.

$F_A(i, j, g, h, W_B, E_B) = F_A(i, j+1, g, h, W_B, E_B - e_j^B)$, and $W_B + E_B \le Q$.

(8) The processing job is j from agent B, and the job is tardy.

$F_A(i, j, g, h, W_B, E_B) = F_A(i, j+1, g, h, W_B - \omega_j^B, E_B)$, and $p_h^B > d_j^B, W_B + E_B \le Q$.

Optimal value:

$$\min\{F_A(1,1,g,h,W_B,E_B) | W_B + E_B \le Q\} \quad (6)$$

Algorithm DP3 provide an optimal schedule in $O(n_A{}^2 n_B{}^2 EW)$ time for problem P3, which is pseudo-polynomial time optimal algorithm, so problem P3 is weakly NP-hard.

3.4 The superiority of our models

We consider two agent scheduling with job rejection on a common unbounded parallel batch machine, and consider three objective functions. There are some models related to our models, but our models are the generalization of the studied models before, and are more practical. Table 2 summarizes the complexity results of our paper and the studied models before. The superiority of our models can be find in table 2.

Table 2 Complexity results for agent scheduling problems

Scheduling problem	Complexity results
$1\|agent, p-batch, b \ge n\|\sum C_j^A : \sum C_j^B$	$O(nn_A n_B P)$, Li (2012)
$1\|agent, p-batch, b \ge n\|\sum C_j^A : L_{\max}^B$	$O(nn_A n_B P)$, Li (2012)
$1\|agent, p-batch, b \ge n\|\sum \omega_j^A U_j^A : \sum \omega_j^B U_j^B$	$O(n_A{}^2 n_B{}^2 W)$, Tang (2017)
$1\|agent, p-batch, b \ge n, rej\|\sum C_j^A + \sum e_j^A : \sum C_j^B + \sum e_j^B$	$O(n_A{}^3 n_B + n_A n_B{}^3)$, Algorithm DP1
$1\|agent, p-batch, b \ge n, rej\|\sum C_j^A + \sum e_j^A : L_{\max}^B + \sum e_j^B$	$O(n_A{}^3 n_B + n_A n_B{}^3 E)$, Algorithm DP2
$1\|agent, p-batch, b \ge n, rej\|\sum \omega_j^A U_j^A + \sum e_j^A : \sum \omega_j^B U_j^B + \sum e_j^B$	$O(n_A{}^2 n_B{}^2 EW)$, Algorithm DP3

4. Conclusion

We consider a single parallel batch machine scheduling with two agents and job rejection. We consider three objective functions which are not studied before, and analyze the problem complexity. Three pseudo-polynomial algorithms were introduced to solve the problems. The future research can study other scheduling criteria like maximum tardiness and total tardiness, and some settings with produce activities and machine types, such as parallel machines and flow shop.

Acknowledgment

The authors would like to acknowledge the project RZ2022ZR20 supported by Rizhao City Natural Science Foundation.

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