Research on Intelligent Formation Operation Performance of Straddle-Type Rapid Transit Vehicles based on Model Predictive Control

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ABSTRACT
The high construction and operation costs of urban rail transit have prompted scholars to explore new urban rail transit systems and operating modes. Based on the new straddle-type rapid transit system proposed by our team, the collaborative control of intelligent formation vehicles is discussed in this paper. The mathematical formulation of straddle-type rapid transit vehicle (SRTV) operating in intelligent formation operation mode (IFOM) was first constructed, and then the formation vehicle safety protection strategy based on limited speed difference (LSD) was analyzed. Subsequently, based on the Model Predictive Control (MPC) algorithm, a formation collaborative operation controller was established for IFOM operation. Based on the typical operating scenarios of formation vehicles exiting, cruising, and entering the station, formation performance evaluation indicators were adopted, with a focus on optimizing the time for formation vehicles to exit and enter the station. The simulation results show that the formation operates well and the vehicles in the formation can safely and stably travel in formation, proving that the use of LSD speed limitation strategy and MPC formation controller can ensure the safety and stability of SRTV vehicle formation.

1. INTRODUCTION

As an important solution to alleviate urban road traffic congestion, the urban rail transit system still faces many problems in the rapid development process. For example, there are significant investments in construction and operation, as well as issues such as mismatched transportation capacity and passenger flow during the operation process. The former is limited by expensive land demolition and high labor maintenance cost. However, for the latter, in recent years, there are many ways to solve the mismatch between transport capacity and passenger flow, including variable grouping, flexible grouping, virtual coupling (VC), etc.

As a more advanced and flexible solution, VC has become a hot spot in relevant research fields. Many projects have conducted some research on the key technologies of VC. Some of the most prominent projects are the X2Rail series project under the Shift2Rail program and MovingRail projects [1-3]. They conducted research on some key technologies and economy of VC, and the research objects were also extended to freight railway, high-speed railway, intercity railway and urban rail transit. In addition, some companies also conducted engineering experiments, such as CAF in Spain and AVP in Russia, which conducted line

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experiments on trams and heavy-loaded trains respectively [4]. They plan to use VC technology to enhance precise and flexible capacity adjustment capabilities and solve the problem of uneven passenger flow in urban rail transit [5].

But does VC really apply to a wide variety of track operating lines? Our team is also actively exploring the application of virtual coupling in urban rail transit [6-7]. We have found that the application of VC in urban rail transit requires specific consideration for different line traffic volumes. For metro lines with large traffic volume, virtual coupled train sets (VCTS) and smaller formation operation can be used to reduce operation cost during peak passenger flow period. However, when large formation operation is adopted in peak passenger flow period, the operation time of VCTS will be increased compared with fixed formation, because the whole formation can only start to carry passengers after the last vehicle stops. Moreover, large formation will result in an increase in VCTS's length compared to the same number of fixed formation trains, which will reduce line utilization during peak periods. Meanwhile, VCTS will increase turn-back time and reduce operation efficiency.

It can be seen that from the perspective of line operation, the use of VC can bring benefits during low peak periods, but it inevitably exposes drawbacks such as reduced operational efficiency and complex operational management during high passenger flow. On the other hand, from the perspective of system construction costs, the length of VCTS with the same number of formations is greater than fixed train formations, which will lead to larger station scale and increased construction cost. In addition, if VC enables each coach to operate independently, it will increase the on-board signal system and cost for each metro coach.

Therefore, we think that VC application in metro has conditions and limitations, and the future of this technology should be used in urban rail transit lines with small and medium-sized traffic volume. The latter will not face significant passenger flow pressure, but can also combine with the passenger flow prediction system to 'transport as required', better meeting the actual demand of the line. Therefore, we put forward the scheme of using VC on the new urban rail transit system, which adopts the new SRTV proposed by our team [6-7]. The intelligent formation cooperative operation scheme is shown in Fig. 1.

Compared to Chongqing Straddle Monorail Line 3, the smaller coach of SRTV result in lower system and maintenance costs. The rubber tire running system of the vehicle can also achieve better traction and braking capabilities, improving the longitudinal safety of vehicle operation. In addition, the pantograph-catenary system of the vehicle is canceled and the vehicle is driven by its own power battery, thus reducing the construction and maintenance costs. By using VC technology, this system is very suitable for existing urban rail transit lines with small and medium-sized traffic volume.

In order to realize the application of VC in the new urban rail transit system, the first thing to be solved is the vehicle collaborative control problem, which is studied in many papers. Some scholars have designed the MPC method to verify the VCTS operation method [8-9]. Some have adopted the method of optimal control to simulate the cruise process of VCTS [10-13]. In addition, for the multi-objective control problem of VCTS, some scholars combined dynamic programming algorithm and sequential Quadratic programming algorithm to achieve online control of VCTS [14-16]. Besides, some studies have proposed corresponding control methods for specific line conditions, which can ensure the safe operation of VCTS [17-20].

The VCTS collaborative control method has been studied in the above paper, but for the rubber wheel system vehicle, the related research is still blank. Hence, we first need to study the security of SRTV when using VC. On this basis, it is also necessary to analyze whether VC can meet the index requirements of operation efficiency. Therefore, based on the MPC algorithm, this article conducts research on the collaborative control problem of intelligent formation SRTV, mainly studying their safety protection strategies, control algorithms, and operational efficiency of formation entry and exit stations.

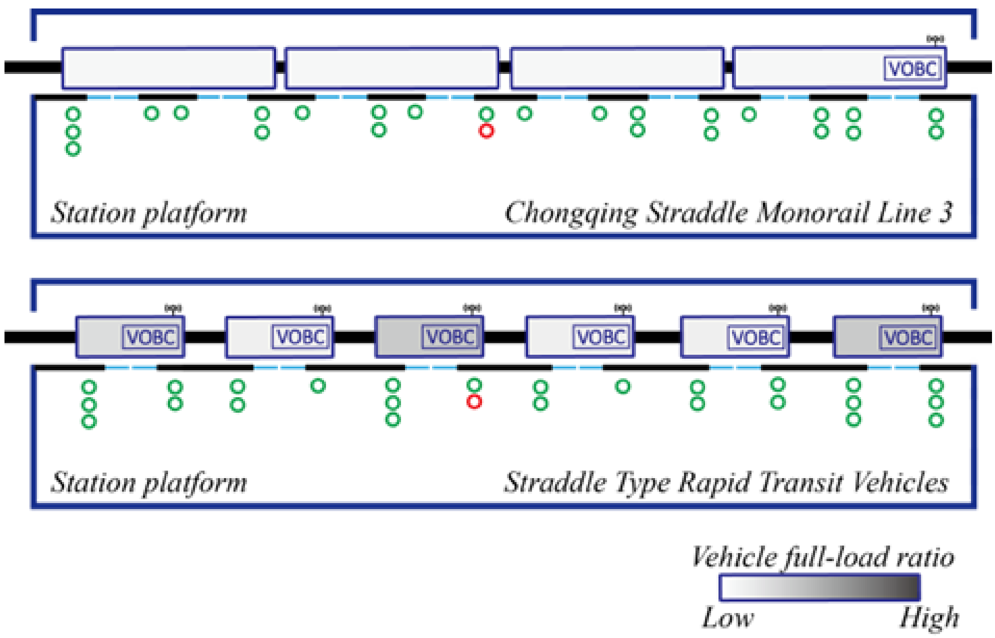

Fig. 1. Intelligent formation operation scheme of SRTV

2. MATHEMATICAL FORMULATION FOR INTELLIGENT FORMATION SRTV OPERATION CONTROL

SRTV has two operation modes: single vehicle operation mode (SVOM) in low peak period and IFOM in peak period. When the vehicle is operating in SVOM, a single vehicle is an independent formation, which automatically calculates the running speed curve based on the command from the control center. When vehicles operate in IFOM, each vehicle transmits its own position, speed, acceleration and other information through vehicle-vehicle communication. The leader receives commands from the control center to drive autonomously, while the follower follows the preceding vehicle in sequence. The schematic diagram of SRTV operating in IFOM is shown in Fig. 2.

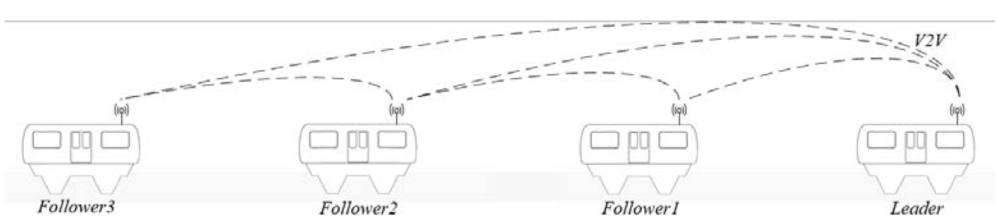

Fig. 2. Schematic diagram of SRTV operating in IFOM

The dynamic formula of SRTV operating in IFOM can be expressed as:

$$\begin{cases} \dot{s}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) - f_i(t) \end{cases} \tag{1}$$

where $s_i(t)$ is the position of the i-th vehicle at time t, $v_i(t)$ is the speed of the i-th vehicle at time t; $u_i(t)$ is the actual acceleration of the i-th vehicle at time t; $f_i(t)$ is the acceleration generated by all running resistance of the i-th vehicle at time t.

Since the vehicle will start to respond after the delay time when receiving the instruction, the dynamic formula of SRTV operating in IFOM can be written as:

$$\begin{cases} s_i(t) = v_i(t) \\ v_i(t) = u_i(t) - f_i(t) \\ u_i(t) = \left(u_{i,des}(t) - u_i\right)\left(\frac{1}{\tau_i}\right) \end{cases} \tag{2}$$

where τ is the delay in the powertrain of the i-th vehicle, $u_{i,des}(t)$ is the expected acceleration of the i-th vehicle at time t.

3. SAFETY PROTECTION STRATEGY FOR SRTV OPERATING IN IFOM

The main methods for calculating the limit speed curve of trains include absolute braking distance (ABD) and relative braking distance (RBD). For SRTV operating in IFOM, the distance between vehicles is very small, and ABD and RBD are no longer suitable for this article due to the long protection distance. Therefore, this article adopts a LSD based on relative coordinates as a safety protection strategy [21]. The speed and position relationship between adjacent vehicles is shown in Fig. 3.

In Fig. 3, at time t, $v_{ija}(t)$ represents the actual safe operating speed of the vehicle, $v_{ijb}(t)$ represents the safe operating speed of the vehicle with speed measurement error, $v_{ijc}(t)$ represents the speed at which the braking is established and $v_{ijd}(t)$ represents the speed at which the maximum braking force is reached.

According to the relative position of vehicles at point d in Fig. 3, it can be concluded that:

$$s_{\mathrm{ij}de}(t) = s_{ij}(t) - s_{ijad}(t) \tag{3}$$

where $s_{ij}(t)$, $s_{ijad}(t)$ and $s_{ijde}(t)$ are the relative distance of the vehicle, the distance from a to point d, and the distance from point d to point e, respectively.

$$v_{ijd}(t) = \sqrt{2 \cdot \Delta aijde_{ijmax}} \tag{4}$$

where $\Delta a_{ijmax}(t)$ is the maximum relative braking acceleration of adjacent vehicles at time t.

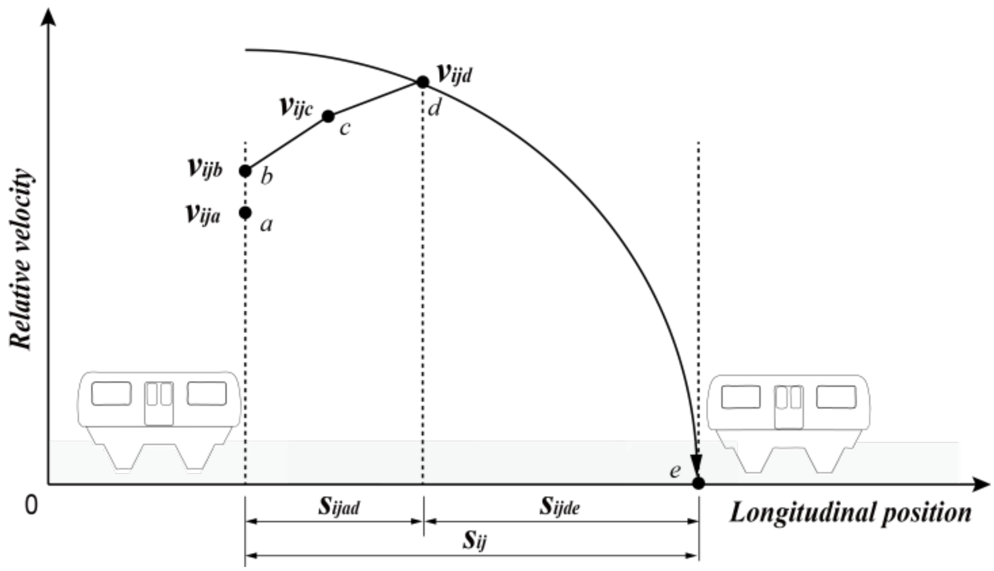

Fig. 3. The speed and position relationship between adjacent vehicles

$$v_{ijb}(t) = v_{ijd}(t) - \Delta a_{ijrea}(t) \cdot t_{ijrea} - \Delta a_{ijcoa}(t) \cdot t_{ijcoa} \tag{5}$$

where $\Delta a_{ijrea}(t)$ and $\Delta a_{ijcoa}(t)$ are the relative accelerations of adjacent vehicles before establishing braking, and the relative accelerations of adjacent vehicles during braking, respectively; t_{ijrea} and t_{ijcoa} are the delay required for establishing braking for the rear vehicle, and the delay required for establishing braking to the maximum braking force, respectively.

Therefore, it can be concluded that:

$$s_{ijad}(t) = v_{ija}(t) \cdot t_{ijrea} + \frac{1}{2}\Delta a_{ijrea}(t) \cdot t_{ijrea}^2 + v_{ijc}(t) \cdot t_{ijcoa} + \frac{1}{2}\Delta a_{ijcoa}(t) \cdot t_{ijcoa}^2 \tag{6}$$

$$v_{ijc}(t) = v_{ijb}(t) + \Delta a_{ijrea}(t) \cdot t_{ijrea} \tag{7}$$

$$v_{ija}(t) = v_{ijb}(t) - 2v_e \tag{8}$$

where v_e is the speed measurement error of the train control equipment. Based on the data indicators of mainstream on-board speed measurement equipment in the current market, this article sets v_e to 1km/h.

Therefore, the speed limit for safe operation of vehicles based on LSD can be written as:

$$v_{j\,lim\,it}(t) = v_{ija}(t) + v_i(t) \tag{9}$$

4. DESIGN OF MODEL PREDICTIVE CONTROLLER

4.1. Control objectives of SRTV operating in IFOM

The SRTV operating in IFOM operates with a fixed time interval strategy, and the ideal relative position relationship of the vehicle can be expressed as follows:

$$s_{ijdes}(t) = s_{stop} + v_j(t) \cdot t_s \tag{10}$$

where $s_{ijdes}(t)$ is the expected distance between the front and rear vehicles, sstop is the stopping distance between the formation vehicles, and ts is the operating time distance between the formation vehicles. In this article, s_{stop} and t_s are set to 3m and 0.3s, respectively.

The control objective of SRTV operating in IFOM is to achieve consistency in relative distance and speed between adjacent vehicles, which can be expressed as:

$$\begin{cases} \lim\limits_{t \to t_k} (s_i - s_j) = s_{ijdes} \\ \lim\limits_{t \to t_k} (v_i - v_j) = 0 \end{cases} \tag{11}$$

where s_i and s_j are the positions of adjacent front and rear vehicles, v_i and v_j are the speeds of adjacent front and rear vehicles, and t_k is the maximum convergence time for stable operation of the formation.

4.2. Prediction model

Based on the position, speed, and acceleration information transmitted by V2V between vehicles, the MPC controller calculates the expected acceleration of the vehicles. Then the state information and control instructions are obtained repeatedly in the next control cycle until the whole formation reaches a stable operating state. Therefore, the state space formula of SRTV operating in IFOM can be written as:

$$\dot{s}(t) = A(t)s(t) + B(t)u(t) + W(t) \tag{12}$$

where $s(t)$, $v(t)$ and $u(t)$ are the position, speed, and acceleration information of the vehicle, respectively. $A(t)$, $B(t)$ and $C(t)$ are as follows:

$$A(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \tag{13}$$

$$B(t) = \left[0,0,\frac{1}{\tau}\right]^T \tag{14}$$

$$w(t) = [0, -f(t), 0]^T \tag{15}$$

4.3. Constraint condition

Firstly, it is necessary to limit the safe distance between vehicles.

$$s_{ij}(t) - s >_{ijmin} \tag{16}$$

where $s_{ij}(t)$ and $s_{ijmin}(t)$ are the actual distance and minimum safety interval between adjacent vehicles at time t.

Secondly, the operating speed of the vehicle must be less than the safe operating speed limit of the vehicle.

$$v_j(t) < v_{j\,lim\,it}(t) \tag{17}$$

In addition, the vehicle needs to ensure that the speed cannot exceed the line speed limit.

$$0 \leq v_i(t) \leq v_{itrack}(s_i(t)) \tag{18}$$

where $v_{itrack}(s_i(t))$ is the speed limit at the line location of vehicle i at time t.

Due to the limited acceleration and braking capability of vehicle power and braking system, vehicle acceleration needs to meet the requirements of actual acceleration.

$$u_{i\,min}(t) \leq u_i(t) \leq u_{i\,max}(t) \tag{19}$$

where $u_{imin}(t)$ and $u_{imax}(t)$ are the maximum deceleration and maximum acceleration limited by the vehicle's power and braking systems respectively.

4.4. Controller design

According to the control objectives of SRTV operating in IFOM, the following state variables can be defined:

$$s_{ijdev}(t) = s_i(t) - s_j(t) - s_{ijdes}(t) \tag{20}$$

$$v_{ij}(t) = v_j(t) - v_i(t) \tag{21}$$

where $s_{ijdev}(t)$ and v_{ij} are the distance deviation and speed deviation of adjacent vehicles at time t.

The primary goal of vehicle operation in IFOM is to ensure the longitudinal safety of the vehicle and achieve the desired operating interval between adjacent vehicles. In the model predictive controller, the control objective can be expressed as the sum of distance deviations within the minimum prediction step.

$$J_1(k) = \sum_{k=1}^{M} s_{ijdev}(k)^2 \tag{22}$$

where M is the prediction step size.

Secondly, it is also necessary to minimize the sum of velocity deviations of adjacent vehicles within the predicted step size.

$$J_2(k) = \sum_{k=1}^{M} v_{ij}(k)^2 \tag{23}$$

In addition, it is necessary to minimize traction and braking control to make the vehicle run as smoothly as possible. Therefore, it is necessary to set the control law deviation as the control target.

$$J_3(k) = \sum_{k=1}^{M} (u_i(k) - u_i(k-1))^2 \tag{24}$$

where $u_i(k)$ and $u_i(k-1)$ are the control law at time k and $k-1$ of vehicle i respectively.

In order to balance the relationship between the three control objectives, optimization weights η_1, η_2, and η_3 are added to allocate the weights between the above three optimization objectives. They represent the weights occupied by distance deviation, speed deviation, and control law deviation, respectively. Thus, the ultimate controller optimization objective can be expressed as:

$$\min_{U_i} J(k) = \sum_{k=1}^{M} \eta_1 J_1(k) + \eta_2 J_2(k) + \eta_3 J_3(k) \tag{25}$$

Finally, the vehicle control objective operating in IFOM can be expressed as:

$$\min_{U_i} J(k) = \sum_{k=1}^{M} \eta_1 J_1(k+j|k) + \eta_2 J_2(k+j|k) + \eta_3 J_3(k+j|k) \tag{25}$$

s.t.

1) $s_{ij}(t) - s_{ij\,min}(t) > 0$
2) $v_j(t) < v_{j\,limit}(t) > 0$
3) $0 \le v_i(k+j|k) \le v_{itrack}(s_i(k))$
4) $u_{i\,min}(t) \le u_i(t) \le u_{i\,max}(t)$
5) $j = 0,1,\ldots,M-1$

After linearization, the above optimization problem could be converted to Quadratic Programming for solving, and the optimal solution control sequence could be expressed as:

$$U(k) = [u_i(k|k), u_i(k+1|k),\ldots,u_i(k+M-1|k)] \tag{27}$$

At the moment k, the control sequence of the first column is used to control the vehicle running. At the moment $k+1$, the control process is repeated to complete the rolling optimization control over time.

5. SIMULATION

5.1. Simulation evaluation index

Compared with fixed formation vehicles, when SRTV drive in IFOM, the enter and exit times of each vehicle are inconsistent, which has become a major factor limiting the operation efficiency. The process of SRTV operating in IFOM entering and exiting the station is shown in the Fig. 4.

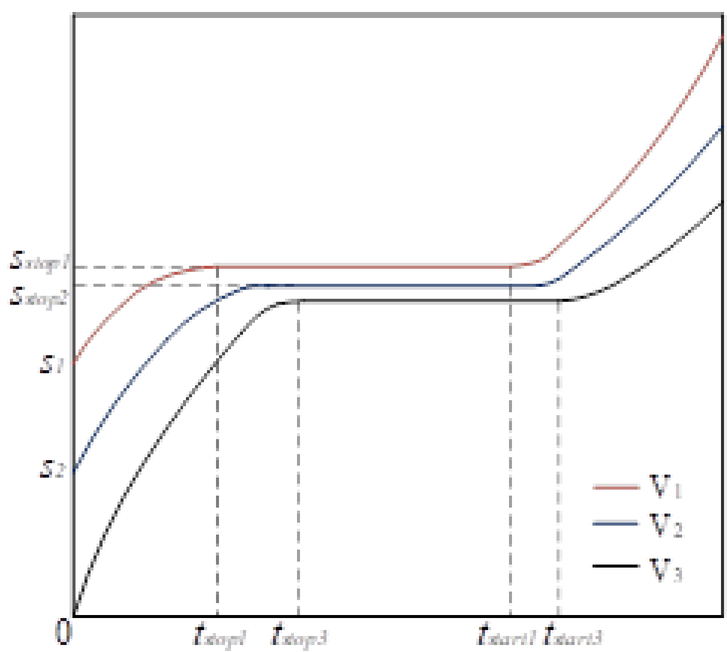

Fig. 4. The process of SRTV operating in IFOM entering and exiting the station

As shown in Fig. 4, if the vehicle is driving in SVOM, it stops at tstop1 and starts at tstart1. The time between the two times is the passenger service time. But if the vehicle operates in IFOM, the following vehicle is still in motion when the leader stops. In this case, it is necessary to wait for each vehicle to stop in sequence before opening the doors at the same time. Therefore, the more the number of formations, the longer the dwell time of intelligent formation vehicles in the platform will be. Similarly, the service efficiency of intelligent formation vehicles will also decrease when each vehicle starts, as the rear vehicle is still stationary when the leader vehicle starts.

For the above reasons, in order to increase the operational efficiency of the line, the time to enter and leave the station must be shortened. Based on the indicators proposed in [4], we define the following evaluation indicators for the operational performance of intelligent formation vehicles:

$$\begin{cases}\Delta t_{startij} < 1s \\ \Delta t_{stopij} < 2s\end{cases} \tag{28}$$

where $\Delta t_{startij}$ is the start time difference between adjacent vehicles after the leader starts, Δt_{stopij} is the stop time difference between adjacent vehicles after the leader stops. Especially, when the speed is greater than 0.01m/s, the vehicle starts, and when the speed is less than 0.01m/s, the vehicle stops.

In addition, as parking must be aligned with the platform screen door, the position deviation between each vehicle and the platform screen door during parking needs to be less than a certain value.

$$d_{stopi} < 0.3m \tag{29}$$

where d_{stopi} is the deviation of the parking position.

5.2. Simulation result

By simulating typical operating scenarios of vehicles, simulate the operation process of intelligent formation vehicles from leaving the station, cruising, and entering the station. For simulation, the line is set as a straight line. The speed curve of SRTV operating in IFOM is shown in Fig. 5.

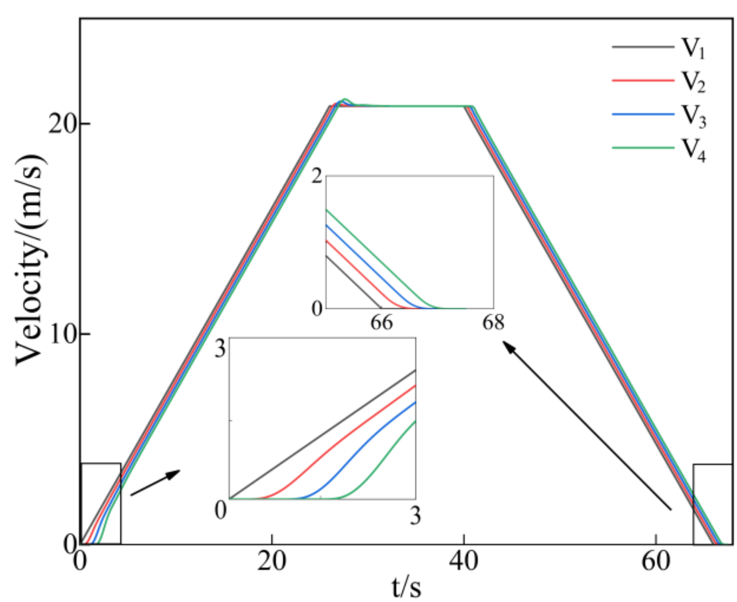

Fig. 5. The speed curve of SRTV operating in IFOM

It can be seen that after the leader vehicle starts, each following vehicle starts and runs in sequence. When the formation starts to stop, the leader vehicle stops, and each following vehicle stops in sequence. During each stage of formation operation, each vehicle can operate safely and stably.

The key time node statistics of intelligent formation vehicles are shown in Table 1, and the operating performance indexes of intelligent formation vehicles are shown in Table 2.

Table 1. The key time node statistics of intelligent formation vehicles

	V_1	V_2	V_3	V_4
The starting time of the vehicle (s)	0	0.34	0.86	1.38
The stopping time of the vehicle (s)	66	66.53	66.8	67.17

Table 2. The running performance indicators

	V_{12}	V_{23}	V_{34}
The starting time deviations of adjacent vehicles (s)	0.34	0.51	0.52
The stopping time deviations of adjacent vehicles (s)	0.53	0.27	0.37

From Tables 1 and 2, it can be seen that the entire formation start-up time is 1.38 seconds. The starting time deviations of adjacent vehicles are 0.34s,0.51s and 0.52s, respectively, which are all less than 1s. When the formation enters the station, the leader vehicle stops at 66s, and the stopping time deviations of adjacent vehicles are 0.53s, 0.27s and 0.37s, respectively, which are all less than 2s. The stopping time of the whole formation is 1.17s. It can be seen that the formation takes a relatively short time to complete the entry and exit of the station, which has little impact on the operation efficiency for SRTV.

The speed deviation of adjacent vehicles is shown in Fig. 6.

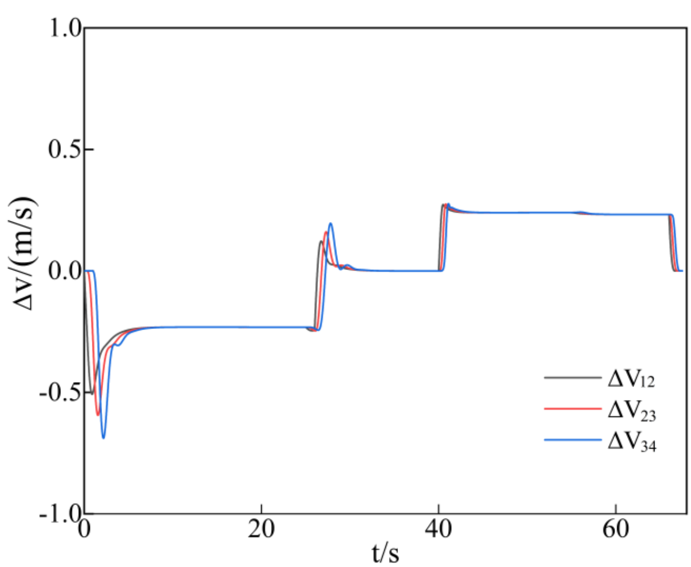

Fig. 6. The speed deviation of adjacent vehicles

It can be seen from Fig. 6 that the speed deviation of adjacent vehicles in formation is the largest at the start-up stage, exceeding 0.5m/s, while the speed difference in other periods does not exceed 0.5m/s, indicating that the formation has good control effect.

The curves of vehicle speed and LSD limit speed are shown in Fig. 7.

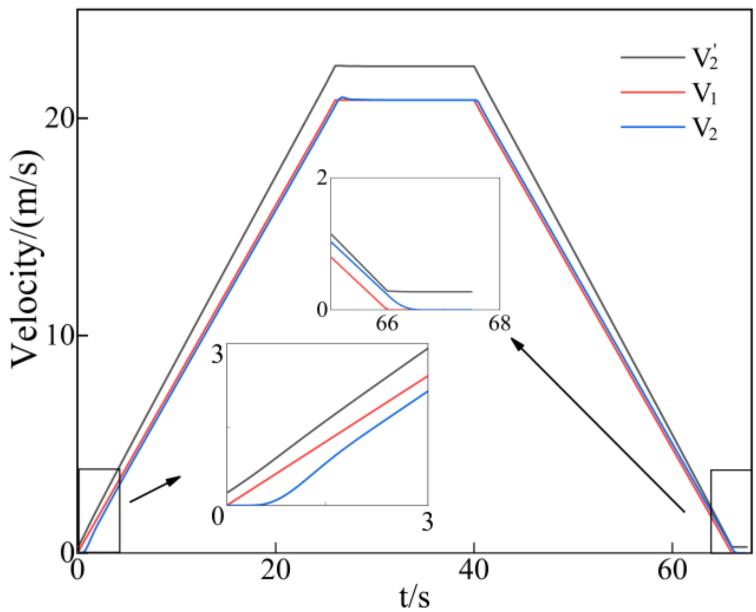

Fig. 7. The curves of vehicle speed and LSD limit speed

In Fig. 7, V_2' represents the LSD limiting speed curve of V_2. It can be seen that during each time period of vehicle operation, the speed of V_2 is always lower than the LSD speed limit curve. Especially, after the leader vehicle stops, the speed limit curve of V2 suddenly becomes a fixed value, and at this time, the speed limit curve of V_2 changes to ABD. It has been proven that LSD speed limit can ensure safe operation of vehicles at small intervals, meeting the safety protection requirements for vehicle starting and stopping scenarios.

The longitudinal and relative position curves of formation vehicles are shown in Fig. 8 and 9, respectively. From Fig. 8 and 9, it can be seen that the distance between adjacent vehicles is always greater than 3m from starting to stopping. Due to the use of a fixed time distance control strategy, the distance between adjacent vehicles is an invariant value when the vehicle is moving at a constant speed. From the simulation results, it can also be seen that the distance between adjacent vehicles varies steadily with vehicle speed.

The parking position deviation of formation vehicles is shown in Table 3

Table 3. The key time node statistics of intelligent formation

	V_1	V_2	V_3	V_4
The deviation of the parking position(m)	0	0.071	0.151	0.223

As can be seen from Table 3, the deviation of the parking position gradually increases, and the parking position deviation of the rear vehicle reaches 0.223m. The deviation of each vehicle's parking position is less than 0.3m, which meets the indicator requirements of accurate parking.

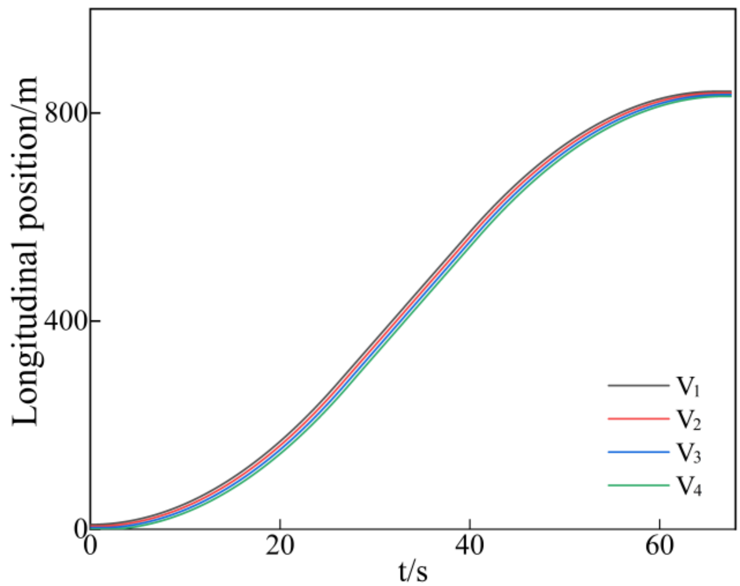

Fig. 8. Longitudinal position curve of formation vehicles

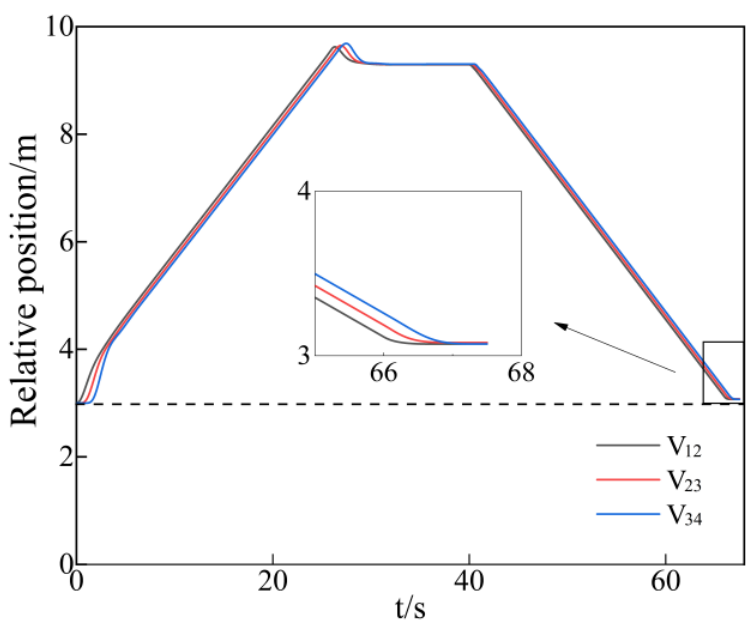

Fig. 9. Relative position curve of formation vehicles

6. CONCLUSION

The mathematical formulation of SRTV operating in IFOM was first constructed, and then the formation vehicle safety protection strategy based on LSD speed limitation was analyzed. Subsequently, based on the MPC control algorithm, a formation collaborative operation controller was established for IFOM operation. Based on the typical operating scenarios of formation vehicles exiting, cruising, and entering the station, formation performance evaluation indicators were adopted, with a focus on optimizing the time for formation vehicles to exit and enter the station. The simulation results show that the starting and stopping time differences between adjacent vehicles within the formation can be controlled within 1s and 2s respectively, and the parking position deviation of each vehicle can be controlled within 0.3m. It is proved that the LSD speed limiting strategy and MPC formation controller can ensure the safety and stability of SRTV driving in formation.

REFERENCES

[1] Shift2Rail. CONNECTA [EB/OL]. [2022-12-01].

[2] Shift2Rail. X2RAIL1 [EB/OL]. [2022-12-01].

[3] Shift2Rail. MOVINGRAIL [EB/OL]. [2022-12-01].

[4] AVP Technology. Virtual coupling technology [EB/OL].[2022-12-01].

[5] Liu, H. J., Tang, T., Zhang, Y. B., Chai, M., Luo, X. L., Discussion on the Key Performance Indicators and Technologies of Virtual Coupling in Metros. Urban Rapid Rail Transit, 2023.36(1): p. 28-35.

[6] Wu, H., Du, Z., Yang, Z., Wen, X., Xu, Z., Xin, L., Coordination and matching of tire system of new straddle-type rapid transit vehicle. Vehicle System Dynamics, 2023. 61(2): p. 573-588.

[7] Du, Z., Wu, H., Yang, Z., Wen, X., Research on intelligent formation operation performance of straddle-type rapid transit vehicles in heterogeneous operating environment. Mechanika, 2023. 29(1): p. 59-66.

[8] Felez, J., Kim, Y., Borrelli,F., A Model Predictive Control Approach for Virtual Coupling in Railways. IEEE Transactions on Intelligent Transportation Systems, 2019. 20(7): p. 2728-2739.

[9] Wang, Q., Chai, M., Liu, H., Tang, T., Optimized control of virtual coupling at junctions: A coopera-tive game-based approach. Actuators, 2021.10(9): p. 207.

[10] Liu, Y., Zhou, Y., Su, S., Xun, J., Tang, T., An analytical optimal control approach for virtually coupled high-speed trains with local and string stability. Transportation Research Part C: Emerging Technologies, 2021. 125:p. 102886.

[11] Di Meo, C., Di Vaio, M., Flammini, F., Nardone, R., Santini, S., Vittorini, V., ERTMS/ETCS virtual coupling: Proof of concept and numerical analysis. IEEE Transactions on Intelligent Transportation Systems, 2020.21(6): p. 2545-2556.

[12] Zhang, Z., Song, H., Wang, H., Wang, X., Dong, H., Cooperative multi-scenario departure control for virtual coupling trains: A fixed-time approach. IEEE Transactions on Vehicular Technology, 2021.70(6): p. 8545-8555.

[13] Quaglietta, E., Wang, M., Goverde, R. M. P., A multi-state train-following model for the analysis of virtual coupling railway operations. Journal of Rail Transport Planning & Management, 2020.15: p. 100195.

[14] Li, J. X., Wang, H. F., Zhang,Q. H., Chai, M., Train Dynamic Coupling Optimization Control Based on Dynamic Programming-Sequential Quadratic Programming Method. Railway Standard Design, 2022. 66(4): p. 168-174.

[15] Liu, Y., Liu, R., Wei, C., Xun, J., Tang, T., Distributed model predictive control strategy for constrained high-speed virtually coupled train set. IEEE Transactions on Vehicular Technology, 2022.71(1): p. 171-183.

[16] Luo, X., Liu, H., Zhang, L., Xun, J., A model predictive control based inter-station driving strategy for virtual coupling trains in railway system.2021 IEEE International Intelligent Transportation Systems Conference (ITSC),Indianapolis, IN, USA, 2021, p. 3927-3932.

[17] Su,S., She, J.,Li,K., Wang, X., Zhou, Y., A nonlinear safety equilibrium spacing-based model predictive control for virtually coupled train set over gradient terrains. IEEE Transactions on transportation electrification, 2022. 8(2): p. 2810-2824.

[18] Zhou, Q., Zhang, C., Bao, F., Zhang, L., Xiao, X., The safety braking protection model of virtually coupled train platoon in subway. 2020 10th Institute of Electrical and Electronics Engineers International Conference on Cyber Technology in Automation, Control, and Intelligent Systems (CYBER). Xi'an, China, 2020, p. 401-406.

[19] Xun, J., Chen, M., Liu, Y., Liu, F., An overspeed protection mechanism for virtual coupling in railway. IEEE Access, 2020. 8:p. 187400-187410.

[20] Su, S., Liu, W., Zhu, Q., Li, R., Tang, T., Lv, J., A cooperative collision avoidance control methodology for virtual coupling trains. Accident Analysis & Prevention, 2022.173: 106703.

[21] Chen, M. L., Safety Protection Control Method Based on Cooperative Control for Train Formation Operation. Master thesis, Beijing Jiaotong University, 2021.