

Dynamic Control Method and Application of Highway Toll Station Lanes

N Yan^{1*}, J Tan², W An³, J Gao¹

1. School of Civil Engineering, Chongqing Jiaotong University, Chongqing, China
2. Construction Department, CR Land, Shenzhen, China
3. School of Transportation and Municipal Engineering, Chongqing Jianzhu College, Chongqing, China

ABSTRACT

To address the issue of poor matching of lane allocation with traffic demand and composition, such as exit and entrance of highway toll stations, Electronic Toll Collection (ETC) and Manual Toll Collection (MTC), a dynamic lane control method for highway toll stations is proposed. Based on the lane control method, a mixed integer linear programming optimization model for the optimal opening scheme of toll lanes is established to achieve rational allocation of toll lanes. The results show that when the traffic demand at the entrance is significantly greater than that at the exit, dynamic control will allocate more toll lanes at the entrance. Compared with conventional control, the traffic capacity can be increased by 12.5%, 50% and 33.33%, respectively, under the three traffic schemes. When the traffic demand at the exit is greater than that at the entrance, dynamic control will allocate more toll lanes at the exit. The traffic capacity can be increased by 6.67%, 25%, and 33.33%, respectively, under the three traffic schemes. When the traffic composition at the entrance and exit matches the toll lanes, the traffic capacity and lane allocation of these two control methods are the same. In various traffic scenarios, dynamic control can adjust the lanes at the entrance and exit according to the traffic composition at the entrance of the toll station, so as to accurately match the traffic demand and capacity in all directions of the toll station, and improve the traffic efficiency of the toll station to realize intelligent expansion.

1. INTRODUCTION

With the rise of highway network encryption and expansion, the highways near urban areas are increasing. In addition to serving as a transit transportation service, suburban highways also carry the daily traffic short-distance travel needs of urban residents, with the latter traffic demand being the main type of transportation demand. Considering the obvious "tidal phenomenon" in daily urban travel, especially the roads near the urban area, the demand for inbound traffic is mainly in the morning rush hour and outbound traffic is mainly in the evening rush hour, leading to the "tidal phenomenon" in toll stations of suburban highway. However, due to the limited land area of urban roads, it is difficult for toll stations to physically expand, resulting in severe traffic congestion at some toll stations. Even more, the queuing of toll stations often overflows to the main line of the highway or intersection, causing potential traffic safety hazards. In this case, achieving smart expansion of toll stations through intelligent operation management is the focus of future research.

*Corresponding Author: yanfupi@126.com

Based on the Electronic Toll Collection (ETC) technology, some scholars have conducted related researches on free-flow toll collection [1], ETC vehicle behavior and rapid guidance at toll stations [2], and analysis on the impact of traffic signs and driver lane changes [3], to improve the efficiency of toll stations. However, since the utilization rate of ETC is difficult to reach 100% in a short period of time, the mixed configuration of ETC lanes and Manual Toll Collection (MTC) lanes is the conventional design of the current lane layout of toll stations. The current research hotspot is to optimize the optimal opening scheme of toll lanes under different traffic flow scenarios [4-7].

Ahmad et al. [8] evaluated the waiting time of toll station traffic queues under different toll lane configurations based on simulation, so as to achieve the optimal lane opening of toll stations. Wang et al. [9] adjusted the lane allocation scheme from the perspective of vehicle queuing. Ramandanis et al. [10] developed 39 alternative toll lane alternatives to analyze the operation performance of toll stations under different ETC penetration rates, thereby selecting the best opening scheme of toll lanes. Liu et al. [11] established a two-level programming model at the road network level to optimize the layout of ETC lanes in the highway network. To further improve the traffic capacity and service level of toll stations and reduce the mutual interference between the upstream and downstream of toll stations, some scholars studied from the coordinated management and control between entrance and exit ramps of toll stations and connecting intersections [12-14], coordinated control between toll stations and service areas [15], and variable speed limit control of toll stations [16], to ensure the orderly operation of traffic in toll plazas.

The above studies have limited effect on the alleviation of traffic pressure of tidal traffic at toll stations. This is because the maximum toll lanes that can be provided at the entrance and exit of toll stations cannot be adjusted, resulting in the difficulty of accurately matching the traffic capacity at the entrance and exit of toll stations with traffic demands. For this reason, some domestic highway toll stations have piloted "tidal lanes" to increase the lanes available at the exit or entrance of the toll station by adjusting the toll lane directions, thereby enhancing corresponding directional traffic capacity and alleviating the problem of tidal traffic congestion [17-18]. Taking the Xiong'an North Toll Station on the Jingxiong Expressway as the research object, Qiao et al. [19] set some ETC lanes in the middle of the toll plaza as tidal lanes, so as to analyze the setting schemes of tidal lanes by simulation. It was found that adjusting the number of tidal lane openings can help improve the traffic efficiency of toll stations. However, due to the limitations of research objects and methods, it is difficult to generalize the results to other toll stations.

Based on the concept of dynamic lane control for urban intersections [20-21], considering the adjustable toll directions (entrance, exit) and toll functions (ETC, MTC), a universal optimization model for the optimal opening scheme of toll lanes is established to adapt to different design schemes and traffic flow scenarios of toll stations. At the same time, with the launch of new generation intelligent transportation products such as intelligent remote-controlled command of mobile guardrails, has made the changes in toll direction and function of the toll plaza lanes, and the position switch of the central isolation guardrails, fast, convenient and intelligent, providing a technical support for dynamic control of toll plaza lanes.

2. MODEL AND METHODS

2.1. Dynamic control method

The layout of a highway toll station is shown in Fig.1. The conventional control method is that the exit lane of the toll plaza can only be used for exit traffic, and the entrance direction lane can only be used for entrance traffic. Each toll lane can be adjusted to ETC lane or MTC lane as needed, that is, the toll direction cannot be adjusted, but the toll function can be adjusted. Conventional control methods cannot adjust the maximum number of toll lanes available at the exit and entrance of toll stations and are not suitable for toll stations with tidal characteristics in traffic flow.

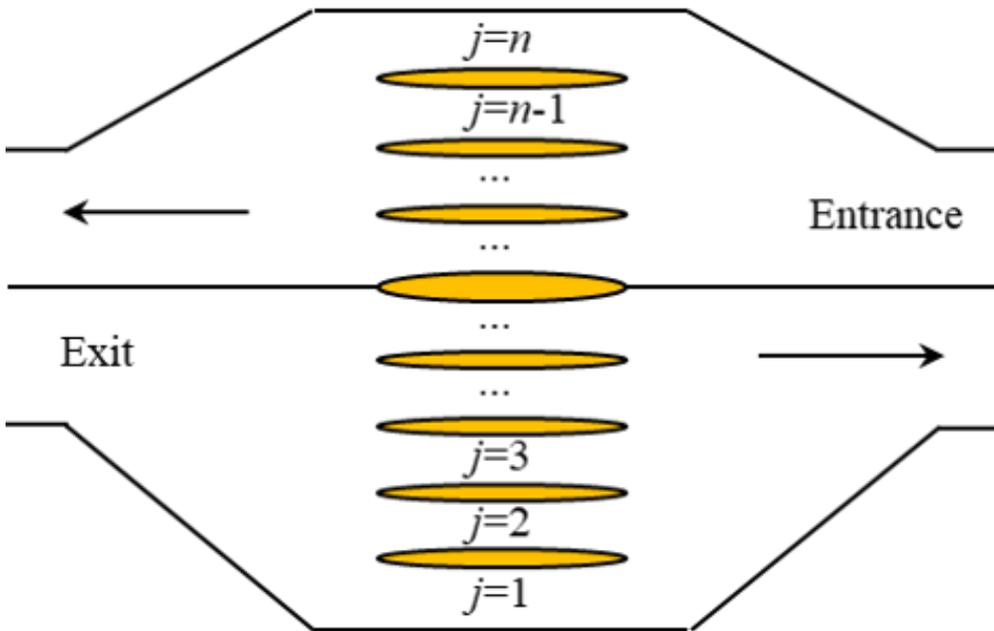


Fig.1 Plane layout of the toll station

The dynamic control method is that the toll lanes can adjust the toll function and direction, which can dynamically allocate the lanes according to the traffic demand in each direction and has high application flexibility. For example, when there is a high demand for exit traffic, the excess toll lanes at the entrance are allocated to the exit traffic by the conventional control method; when there is a high demand for entrance traffic, the excess toll lanes at the exit are allocated to the entrance traffic by the conventional control method. From this, it can be seen that the dynamic control method breaks through the limit of the maximum number of toll lanes that can be used at the entrance and exit of the conventional control method. It can adapt to various traffic scenarios, maximize the utilization of traffic resources of toll stations, and achieve accurate matching of traffic demand and capacity at the entrance and exit of toll stations.

2.2. Model optimization

2.2.1. Objective functions

To fully leverage the advantages of dynamic control methods in terms of traffic capacity and construct a linear objective function, maximizing the shared traffic coefficient of toll stations is taken as the first level optimization objective. The shared traffic coefficient represents the ratio of the reserved capacity of toll stations to traffic demand, and maximizing the shared traffic coefficient means maximizing traffic capacity. The second-level optimization aims to set location preferences for different types of toll lanes, such as setting the exit MTC lanes at the outermost direction of the exit, and the entrance MTC lanes at the outermost side of the entrance. The specific preferences for setting different types of toll lanes can be determined according to the needs of toll station operation and management. The objective function of this document is constructed as follows:

$$\max \quad A\varphi + B \sum_{i \in I} \sum_{j \in J} \left(\frac{x_{i,j}}{|j-r_i|+1} \right), A \gg B \quad (1)$$

where, A and B are the weight coefficients of the objective function, and A is much greater than B ; φ is the traffic coefficient shared by the toll stations; $x_{i,j}$ is whether i chooses lane j , and $x_{i,j} \in \{0,1\}$ respectively indicates No and Yes; $i \in I = \{1,2,3,4\}$ represents the traffic flow that selects exit MEC, exit ETC, entrance MTC and entrance ETC, respectively; j is the number of lanes, $j \in J = \{1,2, \dots, n\}$, as shown in Fig.1; r_i indicates that the toll station tends to prioritize the traffic flow i to use the lanes $j = r_i$ and $r_i \in J$, and r_i is the total number of lanes in the toll plaza.

2.2.2. Constraint conditions

This article does not consider the combination of ETC/MTC, assuming that each toll lane is allowed at most one type of traffic, as shown in Eq. (2). Based on the traffic direction at the exit of the toll station, to avoid conflicting traffic of various flows, the exit MTC lanes should be located on the right side of the exit ETC lanes; the exit ETC lanes should be located on the right side of the entrance ETC lanes, and the entrance ETC lanes should be located on the right side of the entrance MTC lanes, as shown in Eqs. (3)-(5). Each traffic flow has at least 1 toll lane for use, as shown in Eq. (6). Each toll lane is only possible for use if it is designed with supporting facilities for traffic i , as shown in Eq. (7). The value constraint for the use of markings $x_{i,j}$ in toll lanes is Eq. (8).

$$\sum_{i \in I} x_{i,j} \leq 1, \forall j \in J \quad (2)$$

$$x_{i,j} \leq 1 - x_{i',j'}, \forall i \in \{1\}, i' \in \{2\}, j \in \{2, \dots, n\}, j' \in \{1, \dots, j-1\} \quad (3)$$

$$x_{i,j} \leq 1 - x_{i',j'}, \forall i \in \{2\}, i' \in \{4\}, j \in \{2, \dots, n\}, j' \in \{1, \dots, j-1\} \quad (4)$$

$$x_{i,j} \leq 1 - x_{i',j'}, \forall i \in \{4\}, i' \in \{3\}, j \in \{2, \dots, n\}, j' \in \{1, \dots, j-1\} \quad (5)$$

$$\sum_{j \in J} x_{i,j} \geq 1, \forall i \in I \quad (6)$$

$$x_{i,j} \leq X_{i,j}, \forall i \in I, j \in J \tag{7}$$

$$x_{i,j} \in \{0,1\}, \forall i \in I, j \in J \tag{8}$$

where, $X_{i,j}$ indicates whether lane j has designed as supporting facility for traffic i , and $X_{i,j} \in \{0,1\}$ respectively indicates No and Yes. $X_{i,j}$ is a constant, determined by the traffic design of each lane of the toll station.

The shared traffic coefficient of the toll station shall not be greater than the traffic coefficient of each traffic flow, as shown in Eq. (9). The traffic of flow i is equal to the sum of the traffic flows of each toll lane selected, as shown in Eq. (10). Each toll lane can only be greater than 0 if it allows the traffic flow i to use its corresponding lane, as shown in Eq. (11). The traffic flow i in the available toll lane is equal, as shown in Eq. (12). Based on the queuing theory, it can be seen that the service intensity of each toll lane is less than 1, as shown in Eq. (13).

$$\varphi \leq \varphi_i, \forall i \in I \tag{9}$$

$$\varphi_i Q_i = \sum_{j \in J} q_{i,j}, \forall i \in I \tag{10}$$

$$0 \leq q_{i,j} \leq M x_{i,j}, \forall i \in I, j \in J \tag{11}$$

$$-M(2 - x_{i,j} - x_{i,j'}) \leq q_{i,j} - q_{i,j'} \leq M(2 - x_{i,j} - x_{i,j'}), \forall i \in I, j \in J, j' \in J - \{j\} \tag{12}$$

$$\frac{q_{i,j}}{\mu_i} < 1, \forall i \in I, j \in J \tag{13}$$

where, φ_i is the traffic coefficient of traffic flow i ; Q_i is the traffic demand/(pcu·h-1) of traffic flow i ; $q_{i,j}$ is the flow/(pcu·h-1) that traffic flow i chooses the lane j ; M is a large integer to determine the linear constraint construction, and μ_i is the average service rate /(pcu·h-1) of toll lanes in traffic flow i .

2.2.3. Model discussion

The decision variables of the optimization model in this paper include the shared traffic coefficient φ , the lane markings $x_{i,j}$, and a series of intermediate variables φ_i and $q_{i,j}$ introduced for the establishment of linear constraints. The objective function is Eq. (1), and the constraints are Eqs. (2)-(13), which is a mixed integer linear programming model that can be solved by standard branch delimitation method or majority solver. The above optimization model is applicable to the optimal opening scheme of lanes when all toll lanes are activated in traffic congestion at the toll station. In non-congested hours, the objective of toll station operation and control is to ensure that the toll lanes opened under a certain level of service should be as small as possible. Consequently, the decision variables of the optimization model for the optimal opening scheme of toll station lanes in non-congestion hours include the lane markings $x_{i,j}$ and a series of intermediate variables φ_i and $q_{i,j}$

introduced for the establishment of linear constraints. The objective function is Eq. (14), and the constraint conditions are Eqs. (2)-(9), (9)-(13), and (15), which is also a mixed integer linear programming model. According to limiting $X_{i,j}$, the optimization model in this paper can be used for conventional design, i.e., the directions of toll lanes cannot be adjusted.

$$\min \quad A \sum_{i \in I} \sum_{j \in J} x_{i,j} - B \sum_{i \in I} \sum_{j \in J} \left(\frac{x_{i,j}}{|j-r_i|+1} \right), A \gg B \quad (14)$$

$$\frac{q_{i,j}}{\mu_i} \leq \gamma, \forall i \in I, j \in J \quad (15)$$

where, γ is the traffic intensity threshold, $0 < \gamma < 1$.

3. RESULTS AND ANALYSIS

3.1. Case analysis

The toll station shown in Fig. 1 is selected as the research object. Under conventional control methods, there are 7 lanes in the exit direction and 5 lanes in the entrance direction, 12 toll lanes in total. The average service time of each type of toll lanes is taken as 16 s for the exit MEC lanes, 4 s for the exit ETC lanes, 8 s for the entrance MTC lanes and 4 s for the entrance ETC lanes. 9 traffic input schemes for toll stations are designed to compare the advantages and disadvantages of conventional control methods and dynamic control methods, as shown in Table 1. The optimization model established in this paper is used to solve, and the results are shown in Table 2.

Table 1. Traffic input schemes for the toll station

Exit traffic flow/(pcu·h-1)	Entrance traffic flow/(pcu·h-1)	Schemes	ETC Usage/%	Schemes	ETC Usage/%	Schemes	ETC Usage/%
2000	4000	1	60	4	70	7	80
3000	3000	2	60	5	70	8	80
4000	2000	3	60	6	70	9	80

As shown in Table 2, when the ratio of traffic demand at the exit and entrance of the toll station matches the number of toll lanes at the exit and entrance, the capacity and lane allocation of the toll station under conventional control and dynamic control are the same, such as Scheme 2, Scheme 5 and Scheme 8. At this time, the dynamic control does not change the direction of toll lanes. When the traffic demand at the entrance of the toll station is significantly greater than that at the exit, dynamic control will allocate the excess toll lanes at the exit to the entrance. Compared with conventional control, the traffic capacity can be increased by 12.5%, 50%, and 33.33%, respectively, under Scheme 1, Scheme 4, and Scheme 7. Conversely, when the traffic demand at the exit of the toll station is significantly greater than the entrance, dynamic control will allocate the excess toll lanes of the entrance to the exit. Compared with conventional control, the traffic capacity can be increased by 6.67%, 25%, and 33.33% respectively under Scheme 3, Scheme 6 and Scheme 9. Form this, it can be seen that dynamic control can flexibly adjust the allocation of lanes at the entrance and exit of toll stations to best match the traffic demand, maximize the traffic capacity of toll stations, and effectively alleviate the problem of tidal congestion at the toll station. At the same time,

as the ETC usage increases, the traffic capacity of the toll stations has also been improved to a certain extent, indicating that the promotion and utilization of ETC can improve the efficiency of toll stations.

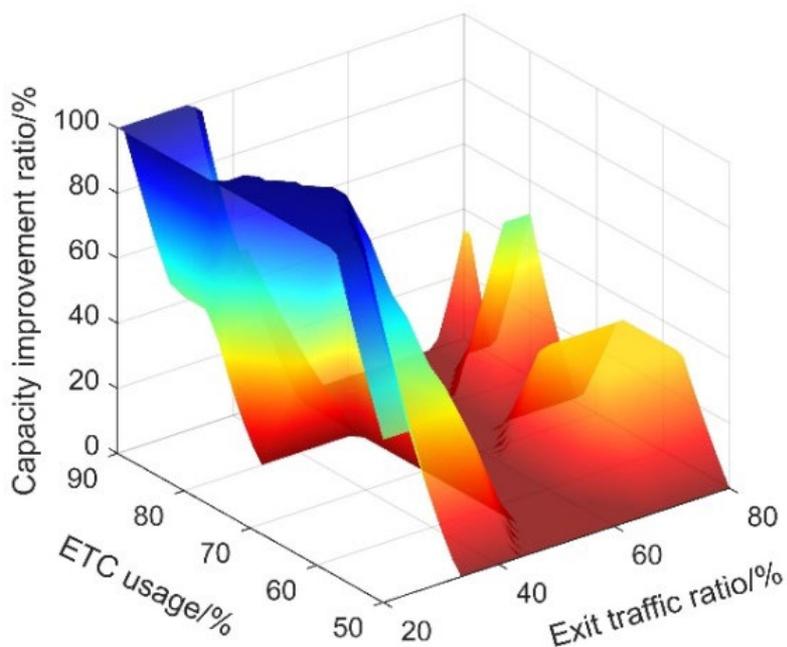
Table 2. Comparison of results

Scheme	Control methods	Number of lanes				Common traffic coefficient	Relative proportion of common traffic coefficient /%
		Exit MEC	Exit ETC	Entrance MTC	Entrance ETC		
1	Conventional control	6	1	3	2	0.75	12.50
	Dynamic control	4	2	3	3	0.84	
2	Conventional control	5	2	3	2	0.94	0.00
	Dynamic control	5	2	3	2	0.94	
3	Conventional control	5	2	2	3	0.70	6.67
	Dynamic control	6	2	2	2	0.75	
4	Conventional control	4	3	2	3	0.75	50.00
	Dynamic control	3	2	3	4	1.13	
5	Conventional control	4	3	2	3	1.00	0.00
	Dynamic control	4	3	2	3	1.00	
6	Conventional control	4	3	1	4	0.75	25.00
	Dynamic control	5	3	2	2	0.94	
7	Conventional control	4	3	2	3	0.84	33.33
	Dynamic control	2	4	2	4	1.13	
8	Conventional control	3	4	2	3	1.13	0.00
	Dynamic control	3	4	2	3	1.13	
9	Conventional control	3	4	1	4	0.84	33.33
	Dynamic control	4	4	1	3	1.13	

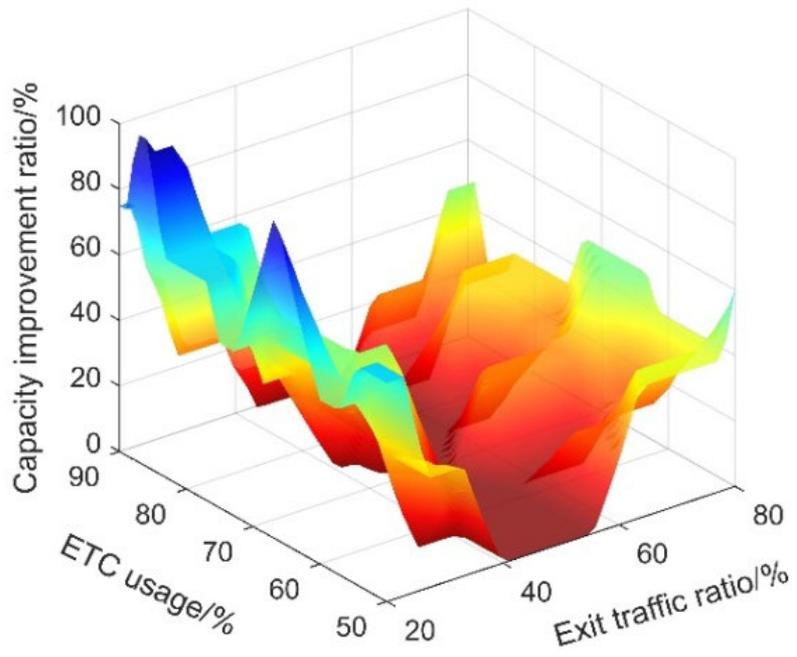
3.2. Sensitivity analysis

The trend of the traffic capacity improvement ratio of dynamic control method compared with conventional control method changing with ETC usage, and the ratio of exit traffic to total traffic at the toll station is further analyzed. Three lane layout schemes are designed: 8 lanes in the toll plaza (5 exit lanes and 3 entrance lanes under conventional control), 12 lanes (7 exit lanes and 5 entry lanes under conventional control), and 16 lanes (9 exit lanes and 7 entrance lanes under conventional control). The ETC usage is set at 50% -90%, and the proportion of exit traffic is taken as 20% -80%.

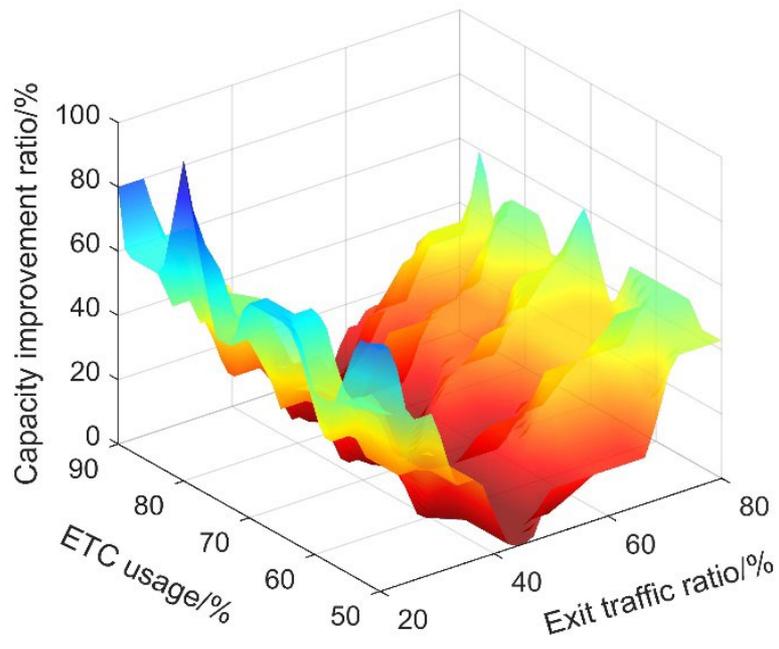
As shown in Fig.2, when the ratio of exit traffic matches the lanes at the entrance and exit of the toll station, these two control methods have the same capacity, and the ratio of dynamic control to capacity improvement is 0. When the exit traffic is greater than the matching value of lanes, as the ratio of exit traffic increases, dynamic control will allocate more toll lanes in the exit direction, thereby improving the capacity of the toll station. When the ratio of exit traffic is less than the matching value of lanes, as the ratio of exit traffic decreases, the traffic increases in the entrance direction, making dynamic control allocate more toll lanes in the entrance direction. To this end, the improvement ratio of traffic capacity in dynamic control is rapidly increased compared with conventional control. In various traffic scenarios, the traffic capacity of dynamic control is not less than that of conventional control, indicating that dynamic control has obvious advantages of traffic capacity. The ratio of traffic capacity improvement is affected by the ETC usage rate, and there are significant fluctuations. The reason is that toll stations have optimal capacity when the ETC usage at the toll station matches the ETC and MTC lanes.



(a) 8 lanes

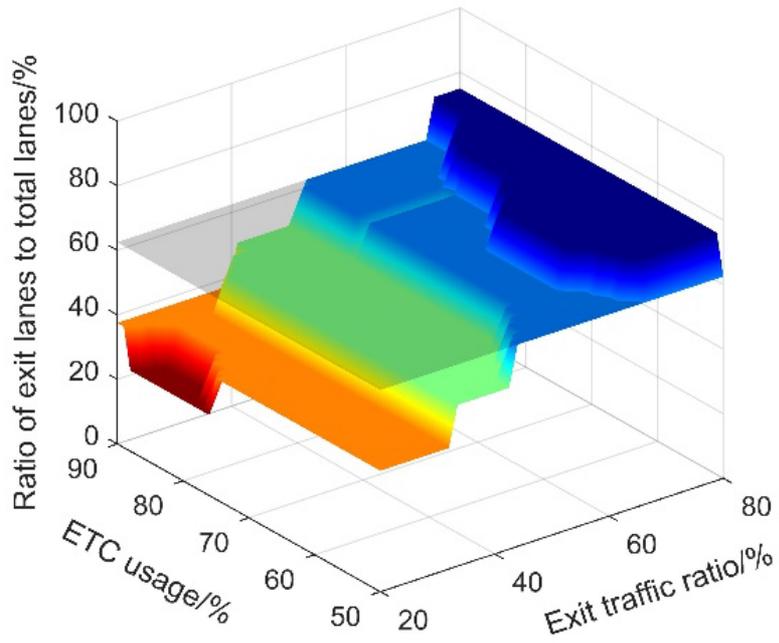


(b) 12 lanes

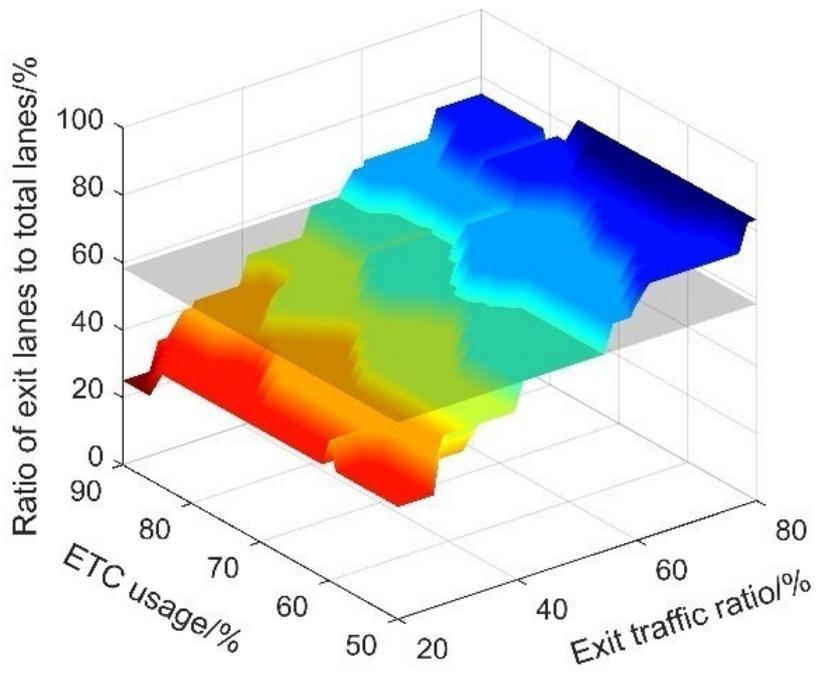


(c) 16 lanes

Fig. 2. The trend of traffic capacity improvement ratio of the toll station



(a) 8 lanes



(b) 12 lanes

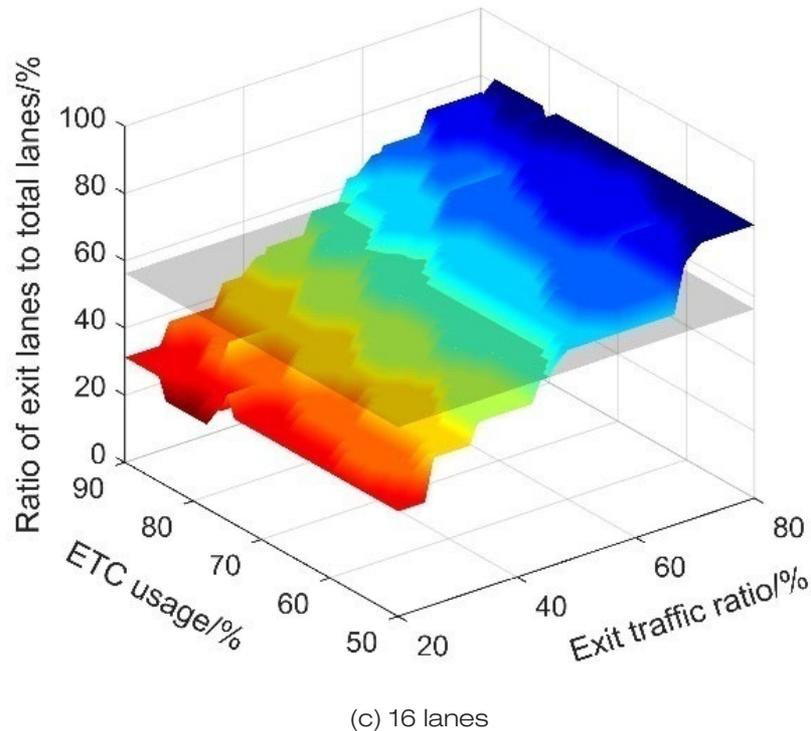


Fig.3. Entrance and exit lane allocation under dynamic control method

Lane allocation at the entrance and exit of the toll plaza is shown in Fig. 3. The surface is the dynamic control method, and the plane is the conventional control method. The conventional control method has fixed entrance and exit lanes, without changing in various traffic scenarios. The ratio of exit lanes to the total lanes is 62.5%, 58.33% and 56.25% respectively when the total lanes are 8, 12, and 16. Compared Fig. 2 with Fig. 3, it can be seen that when the ratio of exit lanes under these two control methods is the same (i.e. the part where the overlap of the surface and plane overlap in the figure), the traffic capacity improvement ratio of dynamic control is 0. When the ratio of exit lanes under dynamic control exceeds that under conventional control (that is, the part of the surface on the plane in the figure), dynamic control allocates more toll lanes for exit to improve the exit capacity. When the ratio of exit lanes under dynamic control is smaller than that under conventional control (i.e. the part where the curve is located below the plane in the figure), dynamic control would allocate more toll lanes for entrance to improve the entrance capacity. From Fig. 3, it can be clearly seen that dynamic control can dynamically adjust the lanes at entrance and exit of the toll station according to traffic demand, so as to accurately match the traffic demand and traffic capacity of the toll station.

4. CONCLUSIONS

- 1) Referring to the concept of dynamic lane control of intersections, allowing toll plaza lanes to adjust their toll directions (entrance, exit) and toll functions (ETC, MTC), this article proposes a dynamic control method for highway toll lanes. The toll lanes are adjusted to the exit MEC lanes, exit ETC lanes, entrance MTC lanes and entrance ETC lanes as needed to alleviate the problem of tidal congestion at toll stations.
- 2) Based on a case study, it is found that dynamic control can flexibly adjust the allocation of lanes at the entrance and exit of toll stations to optimally match the traffic demand. When the demand for entrance traffic is significantly greater than the exit, more toll lanes are allocated for entrance; when the demand for exit traffic is significantly greater than the entrance, more toll lanes are allocated for exit. Dynamic control can overcome the disadvantages of conventional control not being able to adjust the maximum number of lanes available at the exit and entrance of toll stations, making the application more flexible.
- 3) Through sensitivity analysis, it is found that when the traffic composition at entrance and exit matches the lanes at entrance and exit of the toll station under conventional control, the traffic capacity of these two control methods is the same. Otherwise, as the ratio of exit traffic increases or decreases, it will aggravate the uneven traffic distribution of toll stations. Dynamic control can ensure the capacity of toll stations when adjusting the lanes at the entrance and exit. With the aggravation of uneven traffic distribution, dynamic control can improve the traffic capacity of toll stations more significantly than conventional control.

REFERENCES

- [1] Andurkar, G. K., & Ramteke, V. R., "Smart highway electronic toll collection system", *International Journal of Innovative Research in Computer and Communication Engineering*, 2015, 3(5): p. 4087-4094.
- [2] Hua, W., Yue, Y., Wei, Z., Chen, J., & Wang, W., "A cellular automata traffic flow model with spatial variation in the cell width", *Physica A: Statistical Mechanics and Its Applications*, 2020, 556: p. 124777.
- [3] Zhi, S., Qunjie, P., Chaolun, W., Wang, X., & Chen, L., "The Influence of Figures in Warning Signs at the Manual Toll Station on the Lane Change Timing of Drivers in the Context of Virtual Reality of High-Proportion ETC Vehicles", *Complexity*, 2022, 2022: p. 7121952.
- [4] Bari, C. S., Chopde, R., & Dhamaniya, A., "Lane Choice Behavior at Toll Plaza under Mixed Traffic Conditions Using TODIM Method: A Case Study", *Transportation in Developing Economies*, 2022, 8(2): p. 26.
- [5] Kim, S., "The toll plaza optimization problem: Design, operations, and strategies", *Transportation Research Part E: Logistics and Transportation Review*, 2009, 45(1): p. 125-137.
- [6] Neuhold, R., Garolla, F., Sidla, O., & Fellendorf, M., "Predicting and Optimizing Traffic Flow at Toll Plazas", *Transportation Research Procedia*, 2019, 37: p. 330-337.
- [7] Yu, B., & Mwaba, D., "Toll Plaza Lane Choice and Lane Configuration Strategy for Autonomous Vehicles in Mixed Traffic", *Journal of Transportation Engineering, Part A: Systems*, 2020, 146(12): p. 04020133.

- [8] Ahmad, S., Ali, N., Ali, S., & Javid, M. A., “A Simulation-Based Study for the Optimization of Toll Plaza with Different Lane Configuration: A Case Study of Ravi Toll Plaza Lahore, Pakistan”, *Journal of Applied and Emerging Sciences*, 2021, 11(2): p. 157-165.
- [9] Wang, P., Zhao, J., Gao, Y., Sotelo, M. A., & Li, Z., “Lane Work-Schedule of Toll Station Based on Queuing Theory and PSO-LSTM Model”, *IEEE Access*, 2020,8: p. 84434-84443.
- [10] Ramandanis, I. D., Politis, I., & Basbas, S., “Assessing the environmental and economic footprint of electronic toll collection lanes: A simulation study”, *Sustainability*, 2020, 12(22): p. 9578.
- [11] Liu, C.L., Tao, S., Zhao, C., Ji, Y.X., Du, Y.C., “Optimal Deployment of Electronic Toll Collection Lanes for Highway Network”, *China Journal of Highway and Transport*, 2022, 35(5): p. 179-188.
- [12] Kan, X., Lu, X. Y., & Skabardonis, A., “Increasing Highway Capacity by Efficiently Timing its Nearby Arterial Traffic Signals”, *Transportation Research Record: Journal of the Transportation Research Board*, 2018,2672(18): p. 27-34.
- [13] Zhao, J., Ma, W., & Xu, H., “Increasing the capacity of the intersection downstream of the highway off-ramp using presignals”, *Computer-Aided Civil and Infrastructure Engineering*, 2017, 32(8): p. 674-690.
- [14] Chen, Y. Y., Chen, Y. H., & Chang, G. L., “Optimizing the integrated off-ramp signal control to prevent queue spillback to the highway mainline”, *Transportation Research Part C: Emerging Technologies*, 2021, 128: p. 103220.
- [15] Pérez-Acebo, H., Romo-Martín, A., & Findley, D. J., “Spatial distribution and the facility evaluation of the service and rest areas in the toll motorway network of the European Union”, *Applied Spatial Analysis and Policy*, 2022, 15(3): p. 821-845.
- [16] Wang, P., Zhang, Y., Wang, S., Li, L., & Li, X., “Forecasting travel speed in the rainfall days to develop suitable variable speed limits control strategy for less driving risk”, *Journal of Advanced Transportation*, 2021, 2021: p. 6639559.
- [17] Kumar, A., Thakare, A., & Tawalare, A., “Strategy to Reduce Queuing Time at Toll Plaza”, In: Mathew, T., Joshi, G., Velaga, N., Arkatkar, S. (eds) *Transportation Research. Lecture Notes in Civil Engineering*, Springer, Singapore, 2020,45: p. 453-464.
- [18] An, W., Gao, J., Song, L., & Li, Y., “Intelligent Opening Optimization Algorithm of Expressway Toll Channel Based on Lane Control”, *International Journal of Robotics and Automation*, 2022, 37(6): p. 1-7.
- [19] Qiao, J.G., Li, E.Y., Liu, Y.M., Peng, R., Tao, R., “Study on the number of tidal lanes open at smart highway toll stations”, *Journal of Hebei University of Science and Technology*, 2023, 44(1): p. 83-92.
- [20] Wu, W., Liu, Y., Liu, W., Zhang, F., Dixit, V., & Waller, S. T., “Autonomous intersection management for connected and automated vehicles: A lane-based method”, *IEEE Transactions on Intelligent Transportation Systems*, 2021, 23(9): p. 15091-15106.
- [21] Tang, Q., & Sohr, A., “Exit lane allocation with the lane-based signal optimization method at isolated intersections”, *Canadian Journal of Civil Engineering*, 2020, 47(12): p. 1345-1358. <https://doi.org/10.1139/cjce-2019-0306>

