

Designing An Integrated Scheduling Model for Production and Maintenance and Repairs in an Open Workshop Production System in Stable Conditions

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Abstract

Manufacturing success depends on flexibility to adapt to demand and design changes. Open workshop production systems offer greater adaptability than traditional methods. While existing research connects production planning and maintenance, there is a gap in studying integrated scheduling for production, maintenance, and repair in open workshops. This study aims to develop a mathematical model for integrated scheduling to enhance scheduling accuracy, equipment reliability, and production efficiency. The study compares the efficiency of the MOKA and MOSA algorithms to solve 12 generated problems, evaluating them based on criteria such as NPS, CPU time, MID, MS, and SNS. The mathematical model validation covered three stages of production: injection and mold making, assembly, and testing, involving three devices and seven personnel at each stage. The analysis emphasized the importance of accurate scheduling and maintenance planning to optimize production and reduce downtime. Heuristic optimization techniques were used to assess dependencies between key objectives. The ϵ -constraint method, sensitivity analysis, and Taguchi's method were applied to optimize the model.

Results highlighted the critical role of preparation time, revealing that longer preparation times lead to a 10% cost increase, while shorter preparation times reduce production costs by 28%. The optimization of algorithms like MOSA and MOKA was key to improving performance. The study found that MOKA is more effective for smaller to medium-sized problems, while MOSA performs better for larger problems. Future work may focus on developing hybrid models that combine the strengths of both algorithms or dynamic parameter tuning to improve performance across different problem scales.

Keywords: Integrated Scheduling, Open Workshop Production, Maintenance Optimization, Meta-Heuristic Algorithms, MINLP Model

1. Introduction

Adaptability to changes in demand and design of products with little cost and time is considered a key factor in the success of manufacturing industries(1, 2). Traditional production systems, such as factory production and flow production, are not able to respond quickly and simultaneously to such changes. In contrast to traditional systems, the use of new systems such as cellular production and open workshop can be a suitable solution to achieve such an ability. Open workshop production system is an effective approach to implement production technology principles (3, 4). The open workshop production system is actually a combined approach of workshop production and flow production; and it is used to produce products of medium size and variety (5, 6). This system is similar

to a workshop system, except that the sequence of steps for each product is not predetermined, meaning no priority or delay is defined for the processing operations of a product, and usually the goal in this production system is to minimize the completion time of all tasks.

Integrated scheduling models in open workshop systems significantly impact production and maintenance efficiency by aligning maintenance with production needs, preventing disruptions, and optimizing costs. Studies emphasize the interdependence of production planning and preventive maintenance, showing that increased stochastic dependence affects production and maintenance costs, as well as available production capacity (6, 7). These models consider long-term maintenance policies alongside short-term conditions, aiming to minimize preventive and corrective maintenance costs while optimizing various production costs like setup, tardiness, and safety stock penalties (6, 8, 9). By incorporating changing machine failure rates and predictive maintenance into job-shop production scheduling, these models establish multi-objective optimization to minimize processing costs and product processing time, enhancing decision-making for machine activities and production planning (6, 8, 9).

Most research in production planning and scheduling has focused on workshop environments with parallel machines, emphasizing scheduling. However, a research gap exists in non-scheduling approaches within open workshop environments. This research develops a multi-objective integrated optimization model for production, maintenance, and repair scheduling in an open workshop system. It considers MAKESPAN conditions (early and late completion) and analyzes device failure curves to optimize scheduling decisions. Given the complexity of open workshop scheduling, where operators handle both production and quality inspection while managing storage and Kanban processes, the study also addresses the challenge of learning levels in non-repetitive tasks to mitigate time-dependent deterioration effects. Due to the NP-HARD nature of the MAKESPAN problem, the research introduces two meta-heuristic algorithms, MOKA and MOSA, to optimize the model under robust uncertainty conditions using a penalty function.

The MOKA meta-heuristic algorithm is a memory-based optimization method with a predefined number of iterations. It refines the selection process by prioritizing elite prey, similar to the NSGA-II algorithm, but with the added advantage of memory retention. While MOKA has been applied to supply chain optimization, its use in scheduling open workshop systems remains unexplored. This research evaluates and analyzes MOKA's effectiveness in this context. Unlike supply chain mathematical modeling, where the objective function is cost-based, open workshop scheduling follows a MAKESPAN-based optimization approach, a novel application that has not been previously studied. Also, given that the model in question is NP-HARD, a basic MOSA algorithm will be used to evaluate the MOKA algorithm to evaluate the performance of the algorithm in large dimensions and high limits.

2. Methods

The model examined in this research includes “ n ” tasks to be processed in a maximum of m machines. The proposed mathematical model is a two-objective model of batch production in an open workshop environment with stable parameters. In this research, the problem of designing a simultaneous production scheduling model in an open workshop environment is discussed, taking into account the capacity limitation and preparation dependent on the sequence and package delivery approach. This model is based on mixed linear integer programming (MILP) (10).

2.1. Mathematical modelling

It should be mentioned that because in the definition of the problem, the model of this research is considered a general and general state, then it can be used in different industries such as component manufacturing industries. To describe our above model, the following indices, parameters and variables are used in the model:

A) Collections and indexes:

I : Collection of all production parts

i, i' : Piece index

J : The set of all the steps it goes through

j و j' : Index of steps to be taken

K : Collection of all equipment in each stage

k و k' : equipment index

P : Collection of all production and maintenance personnel

p : Index of production and maintenance personnel

A_i : The first stage of part production i

Z_i : The last stage of producing the part i

B) parameters:

ST_{ijk} : The duration of the preparation of part i in step j on equipment k

SST_{ij} : Entry time of part i to stage j (when the part enters the system for production)

PT_{ijk} : The duration of production of part i in stage j on equipment k

PPT_{pjk} : The duration of preventive maintenance by personnel p in stage j on equipment k

PET_{pjk} : The duration of maintenance of corrective repairs by personnel p in stage j on equipment k

TC_{ijk} : Production cost of part i in step j on equipment k

TD_{pjk} : The cost of preventive maintenance and repairs of manpower p at stage j on equipment k

TED_{pjk} : The cost of maintenance and corrective repairs of manpower p in stage j on equipment k

TEC_{ijk} : The cost of energy consumption (production overhead) of producing part i in stage j on equipment k

$r(k)$: density function of failure probability of each equipment k

The probability density function of operator learning on each device means that it is its cumulative distribution function. The learning rate of each device in each period is obtained from the following equation:

$$r(k) = \frac{f(k)}{1 - F(k)} \quad \forall \quad k$$

According to the set goals of minimizing the loss caused by operator learning and preparation of equipment for production, therefore, the failure of production equipment and operator learning is evaluated from a possible process in the interval $(0-t)$ which is calculated from the following equation:

$$\int_0^t r(k)_t dt$$

2.2. Decision variables

C_{ijk} : The end time of production activities of part i in stage j in equipment k

CT_{jk} : End time of preventive maintenance activities in stage j in equipment k

CET_{jk} : End time of corrective maintenance activities in stage j in equipment k

Y_{ijk} : If part i is serviced by device k in step j , 1 otherwise zero value

YY_{pjk} : If manpower p performs preventive maintenance operation on equipment k in stage j , 1 otherwise zero value

Y_{pjk} : If manpower p performs corrective maintenance operations on equipment k in step j , 1 otherwise zero value

X_{ijk} : If the part after the part is produced in the step of equipping, one otherwise zero

2.3. Limitations

- 1- Constraint 1 guarantees that the production part is done on one machine in each stage of its production process

$$\sum_{k \in K_{ij}} y_{ijk} = 1 \quad \forall i, j$$

- 2- Constraint 2 guarantees that on every production device in the jam stage, either corrective maintenance or preventive maintenance is performed by manpower.

$$Y_{pjk} + Y_{EY_{pjk}} = 1 \quad \forall p, j, k$$

- 3- Constraint 3 guarantees that if the part production is done on machine k in the j th stage, then a production machine is selected for this production.

$$C_{ijk} \leq M * y_{ijk} \quad \forall i, j, k \in K_{ij}$$

- 4- Constraint 4 states that if a preventive net operation occurs on the production device k in stage j , then operational personnel will be assigned to the process to perform the operation.

$$CT_{jk} \leq M * \sum_p Y_{pjk} \quad \forall j, k$$

- 5- Constraint 5 states that if a corrective net operation occurs on production device k in stage j , then operational personnel will be assigned to the process to perform the operation.

$$CET_{jk} \leq M * \sum_p Y_{EY_{pjk}} \quad \forall j, k$$

- 6- Constraint 6 guarantees that the production completion time of part i in stage j (the second stage onwards) is the result of the production times in the previous stages and the preparation time of the part in the current stage and the time of the part entering the production process and the duration of production The piece is in the current stage and the duration of the preventive or corrective note is on the production device.

$$C_{ijk} \geq \sum_{k'} C_{ij'k'} + ST_{ijk} + SST_{ij} + PT_{ijk} + CT_{jk} + CET_{jk} - M(1 - Y_{ijk})$$

$$\forall i \in I, j, j' \in J, j \neq A_i, j' = j - 1, k \in K_{ij}$$

- 7- Limitation 7 guarantees that the production schedule in the first stage will include the duration of the production of the part and the preparation of the work and preventive and corrective maintenance and repairs.

$$C_{ijk} \geq Y_{ijk} * (ST_{ijk} + SST_{ij} + PT_{ijk} + CT_{jk} + CET_{jk})$$

$$\forall i \in I, \forall j \in A_i, \forall k \in K_{ij}$$

- 8- Constraint 8 guarantees that maintenance operations and preventive repairs in each stage (except for the first stage of production) are the results of the activity times of operational personnel in the same period and the previous period.

$$CT_{jk} \geq \sum_{k'} CT_{j'k'} + \sum_p PPT_{pjk} + M * \left(1 - \sum_p YY_{pjk}\right) \quad \forall j, k, j' \neq A_i, j' = j - 1$$

- 9- Constraint 9 guarantees that preventive maintenance operations in the first stage include the duration of human effort for preventive repairs.

$$CT_{jk} \geq \sum_p PPT_{pjk} * \sum_p YY_{pjk} \quad \forall j, k, j' = A_i$$

- 10- Constraint 10 guarantees that maintenance operations and corrective repairs at each stage (except for the first stage of production) are the results of the activity times of operational personnel in the same period and the previous period.

$$CET_{jk} \geq \sum_{k'} CET_{j'k'} + \sum_p PET_{pjk} + M * \left(1 - \sum_p YEY_{pjk}\right) \quad \forall j, k, j' \neq A_i, j' = j - 1$$

- 11- Constraint 11 ensures that corrective maintenance operations in the first stage include the duration of manpower efforts for preventive maintenance.

$$CT_{jk} \geq \sum_p PPT_{pjk} * \sum_p YY_{pjk} \quad \forall j, k, j' = A_i$$

- 12- Limitations 12-14 ensures that the sequence of production operations in the shop floor system is respected.

$$13- X_{ii'jk} + X_{i'ijk} \leq 1 \quad \forall i, i' < i', j, k \quad (12)$$

$$2X_{ii'jk} \leq Y_{ijk} + Y_{i'jk} \quad \forall i, i' < i', j, k \quad (13)$$

$$Y_{ijk} + Y_{i'jk} \leq X_{ii'jk} + X_{i'ijk} + 1 \quad \forall i, i' < i', j, k \quad (14)$$

- 14- Limits of non-interference of activity 15 and 16: These restrictions guarantee that if piece i' is produced earlier than piece i, then piece i is in the waiting queue for the production of i', then the production time of i will be longer than piece i', and constraint 16 is the opposite of constraint 15.

$$C_{ijk} \geq C_{i'jk} + ST_{ijk} + SST_{ij} + PT_{ijk} + CT_{jk} + CET_{jk} - M * X_{ii'jk} - 2M + MY_{ijk} + MY_{i'jk} \quad \forall k, j, i, i', i < i' \quad (15)$$

$$C_{i'jk} \geq C_{ijk} + ST_{i'jk} + SST_{ij} + PT_{i'jk} + CT_{jk} + CET_{jk} - M * X_{ii'jk} - 2M + MY_{ijk} + MY_{i'jk} \quad \forall k, j, i, i', i < i' \quad (16)$$

2.4. Objective functions

According to the presented limitations, the objective function of the mathematical model is as follows:

$$\begin{aligned} MIN C1 = & \sum_i \sum_j \sum_k TC_{ijk} * \left(MAX \sum_j \sum_k C_{ijk} - SST_{ij} \right) + \sum_p \sum_j \sum_k TD_{pjk} * \left(MAX \sum_j \sum_k CT_{jk} \right) \\ & + \sum_p \sum_j \sum_k TED_{pjk} * \left(MAX \sum_j \sum_k CET_{jk} \right) + \sum_i \sum_j \sum_k y_{ijk} * TEC_{ijk} \end{aligned}$$

The first objective is to minimize production costs, preventive and corrective maintenance and repairs, and production overhead costs.

$$MIN C2 = \sum_p \sum_j \sum_k (1 - r(k)) * (MAX \sum_j \sum_k CT_{jk}) + \sum_p \sum_j \sum_k r(k) * (MAX \sum_j \sum_k CET_{jk})$$

The second objective function is to minimize the possibility of disruption in production machines

Considering that the presented mathematical model is based on MINLP, it will be converted to MIP model by changing the variable in the objective function. Therefore, we need to change the variable:

$$MAX \sum_j \sum_k C_{ijk} = t_i$$

and instead, in the objective function, it becomes t_i , which is itself a limitation:

$$t_i \geq MAX \sum_j \sum_k C_{ijk} - SST_{ij} \quad \forall i \in I$$

2.5. Correctness measurement based on the enhanced constraint epsilon approach

In this method, we always optimize one of the objectives, provided that we define the highest acceptable limit for the other objectives in the majority of the constraints, and for a two-objective problem, we will have the following mathematical representation:

$$\text{Min } f_1(x)$$

$$\text{Subject to } f_2(x) \leq \varepsilon_2, f_3(x) \leq \varepsilon_3, \dots, f_p(x) \leq \varepsilon_p, \quad x \in S$$

First get the maximum and half of each objective function without considering other objective functions in the space. Then, with the help of the values obtained from the previous step, they calculate the interval associated with each of the target functions. If we call the maximum and minimum values of the objective functions by and respectively, then the interval of each of them is calculated as follows:

$$r_i = f_i^{\max} - f_i^{\min}$$

The interval r_i is divided into q_i intervals. Then, in the ε_i following relationship, it is possible to obtain $q_i + 1$ as many different values as can be calculated from the following formula.

$$k = 0, 1, \dots, q_i \quad \varepsilon_i^k = f_i^{\max} - \frac{r_i}{q_i} \times k$$

2.6. Multi-objective Keshtel algorithm (MOKA)

In this research, a multi-purpose version of it has been presented, and its pseudo-code is given below:

- 1 .Land the (N) Keshtels and calculate them fitness
- 2 .Do non-dominate sorting and calculate crowding distance
- 3 .Sort Keshtels respect to the crowding distance
- 4 .Find the Lucky Keshtels (LK).
- 5 .Find the best lucky Keshtel.
- 6 . For each LK (N1)
 - 6.1 .Swirl the Nearest Keshtel (NK) around the LK.
 - 6.2 .If NK finds better food than LK, replace NK with LK, find new NK, go to step 6.1

- 6.3 .If the food still exists, attract the NK, go to step, 61. if not, go to step 8.
- 7 .Let the LKs remain in the lake.
- 8 .Startle the Keshtels which have found less food and land new ones. (N3)
- 9 .Move the remained Keshtels in the lake between other Keshtels. (N2)

2.7. Mult objective optimization algorithm (MOSA)

In this research, a multi-purpose version of it has been presented, and its pseudo-code is given below:

1. Parameter setting
2. Initialize and evaluation fitness functions (x , $f_j(x)$)
3. Best solution = (x , $f_j(x)$)
4. For 1 to max-iteration
 - 4.1. Do mutation operator (x')
 - 4.2. Calculate the fitness function and (Δf_j)
 - 4.3.1. If $\Delta f_1 \leq 0$ && $\Delta f_2 \geq 0$
Update the Best solution =(x' , $f_j(x')$)
Update the solution $x=x'$
 - 4.3.2. Else if $\Delta f_1 \geq 0$ && $\Delta f_2 \geq 0$ || $\Delta f_1 \leq 0$ && $\Delta f_2 \leq 0$
Put this solution in Pareto set
 - 4.3.3. Else $\Delta f_1 \geq 0$ && $\Delta f_2 \leq 0$

2.8. A hybrid AHP-VIKOR

First, the largest eigenvalue of the matrix of pairwise comparisons (λ_{max}) should be calculated. Then the inconsistency index is calculated with the following equation:

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

where n in the above equation represents the number of rows or columns of the comparison matrix (number of criteria). In the next step, the inconsistency rate is calculated using following formulas:

$$RI = \frac{1.98(n - 2)}{n}$$

$$CR = \frac{CI}{RI}$$

It should be noted that RI (random inconsistency index) is extracted from the relevant table or formula above and if the inconsistency rate is less than or equal to 0.1 ($CR \leq 0.1$). Then we conclude that there is compatibility in paired comparisons, and if not, it is necessary for the decision maker to reconsider the paired comparisons.

2.9. Multi-criteria optimization and compromise solution (VIKOR)

In the first step, the weight and importance of each of the criteria must first be obtained through the AHP value determination model (criteria weighting models).

In the second step, a decision-making matrix is first formed, in which the preference of each option compared to the criterion is given. Then you normalize it using the following formula:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$$

In the third step, the normalized matrix of the previous step is weighted. For weighting, the values of the normal matrix of each option are multiplied by the weight of the criteria (previously obtained from AHP methods).

In the fourth step, in order to determine the highest and lowest value of the weighted normal matrix, f_i^+ and f_i^- of largest and smallest number of each column is determined. Here, the biggest number means the number that has the most positive value and the smallest one means the most negative value. So, if the criteria are negative, the largest number becomes the lowest value and the smallest becomes the largest value and vice versa.

$$f_i^+ = \max_j f_{ij} \quad ; \quad f_i^- = \min_j f_{ij}$$

In the fifth step, the desirability index (S) and the dissatisfaction index (R) are determined, which are calculated using following formulas:

$$S_j = \sum_{i=1}^n W_i \cdot \frac{f_i^* - f_{ij}}{f_i^* - f_i^-}$$

$$R_j = \max_i \left[W_i \cdot \frac{f_i^* - f_{ij}}{f_i^* - f_i^-} \right]$$

In the last step, to rank the options, the value of Q is calculated, which is calculated using this formula:

$$Q_j = v \cdot \frac{S_j - S^-}{S^* - S^-} + (1 - v) \cdot \frac{R_j - R^-}{R^* - R^-}$$

where V is a constant number equal to 0.5, S_j is the total value of S for each option, S^* is the largest index number of S for each option, the smallest index number of S is for each option, R_j is the total value of R for each option and the smallest and largest index number, respectively R is for each option; And finally, the lowest value of Q is selected as the best option.

3. Numerical analyses and parameter setting

This section analyzes the performance of the proposed algorithms in solving sample problems as their parameters are independently changed, aiming to find the best algorithm parameter values. In addition, the performance of the provided algorithms is evaluated and compared in depth.

3.1. Estimation of modeling parameters using discrete event simulation

Based on the production process, the simulation approach using ARENA software evaluates parameters such as the duration of transportation between workstations across different routes and the queuing time at each station. This allows for the assessment of waiting times within the production process. In this study, a specific workstation was selected for simulation and optimization of the production schedule. Additionally, there are three routes for movement between each station. The simulation model is illustrated in Figure 1.

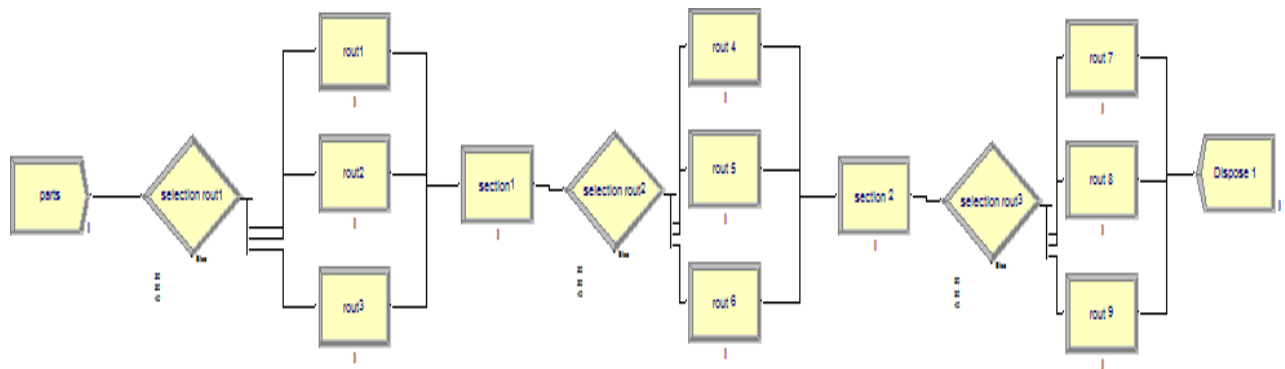


Figure 1. Simulation of production lines

3.2 Mathematical model validation

In this section, we validate the mathematical model of a workshop environment in a manufacturing company. The production process comprises three stages: injection and mold making, assembly of parts, and testing. In each of these stages, three production and executive devices are directly involved in the production and maintenance process, with a total of seven personnel participating. Based on the specific conditions of the problem under study, the arrangement of devices and production processes is planned as Figure 2.

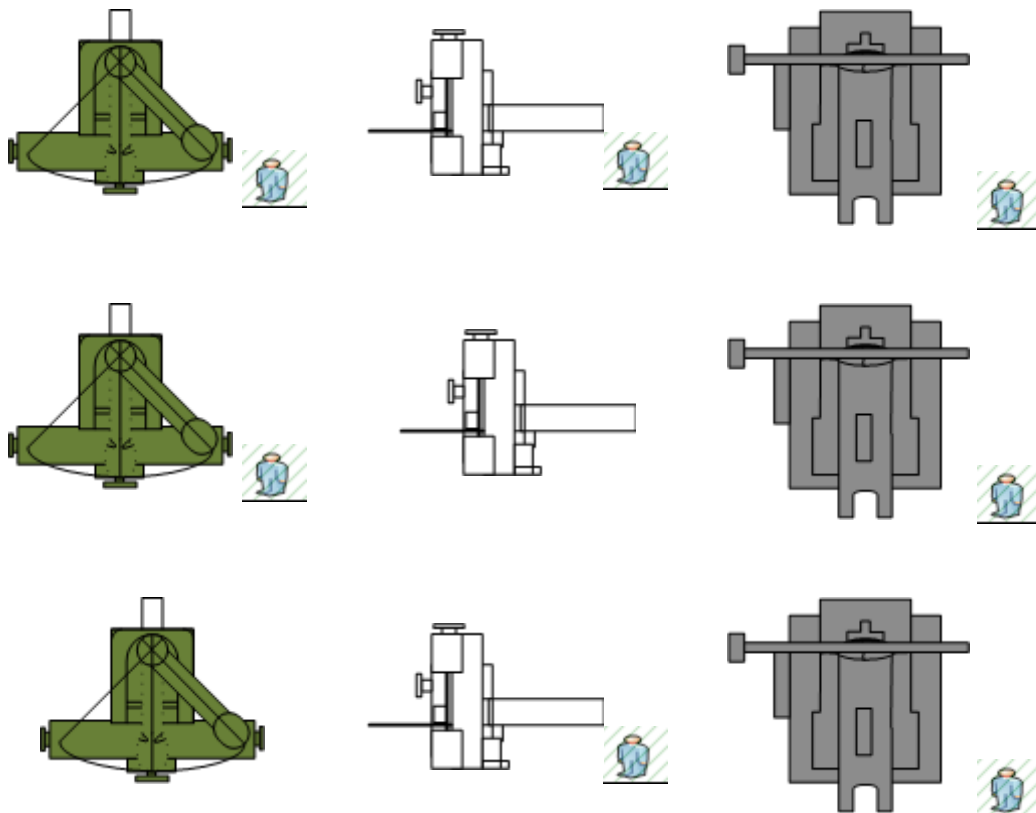


Figure 2. Arrangement of production lines and operational and executive personnel

According to the layout of the production space in question, the parameters of the problem studied in this company, the following is collected:

A. The duration of preparation of part i in step j on equipment k (Figure 3)

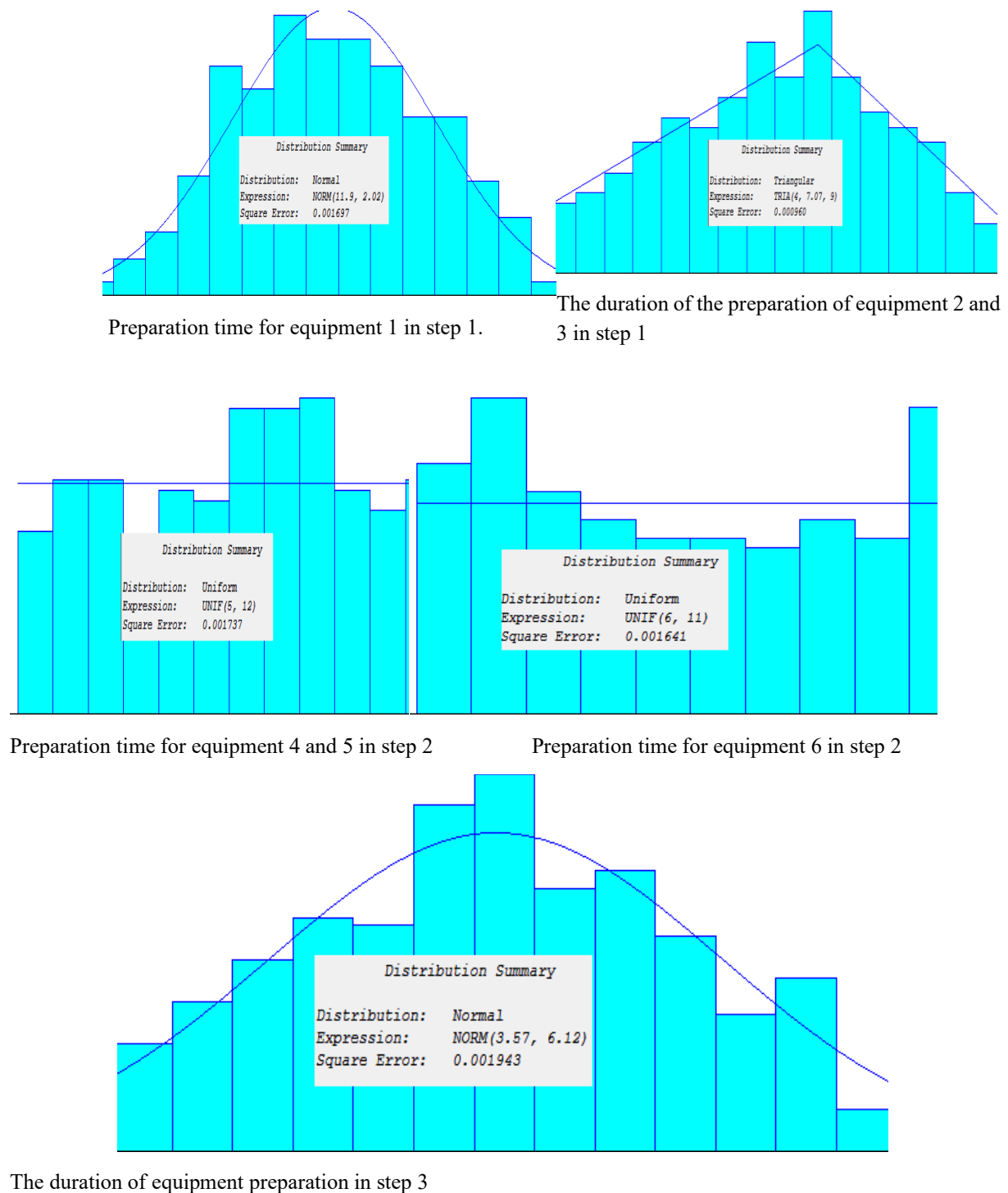


Figure 3. Duration of preparation of part i in stage j on equipment k

B. Entry time of part i to stage j (when the part enters the system for production) (Figure 4)

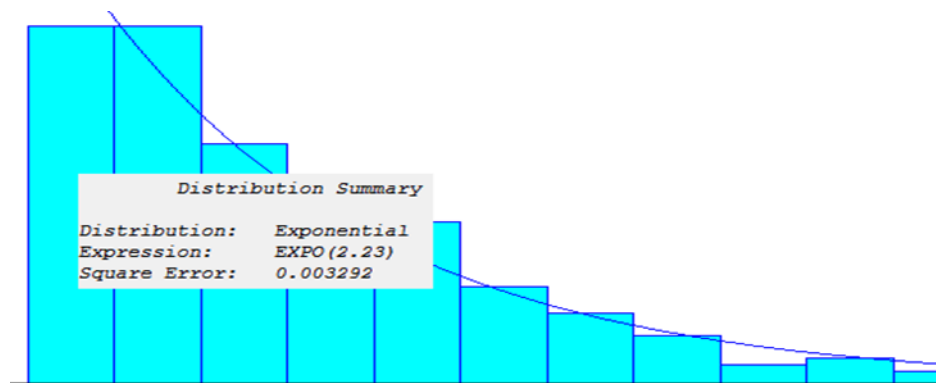
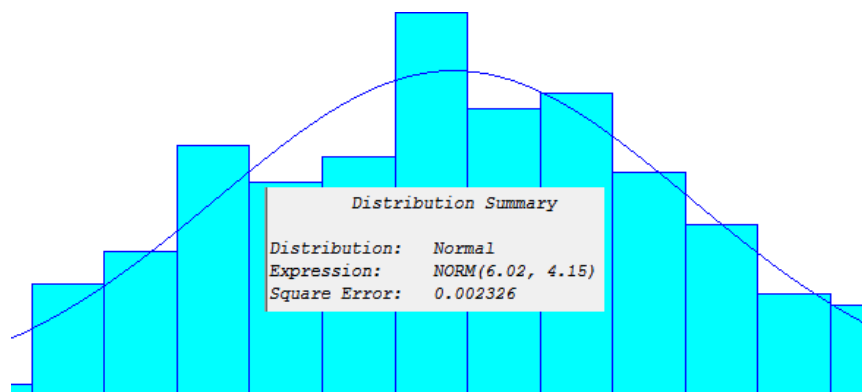


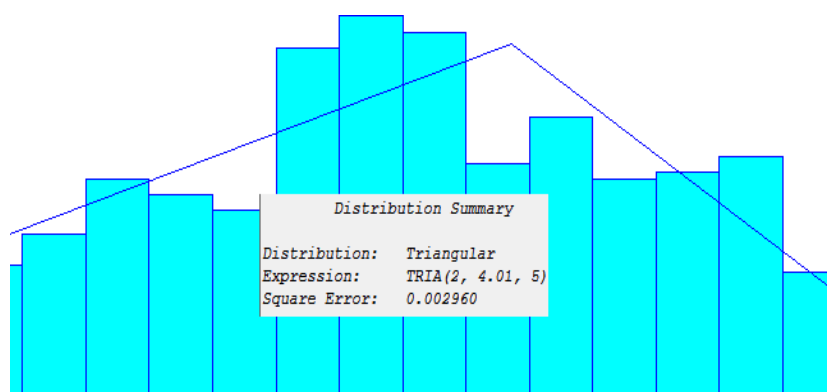
Figure 4. The input rate of parts to production departments

C. The duration of production of part i in stage j on equipment k

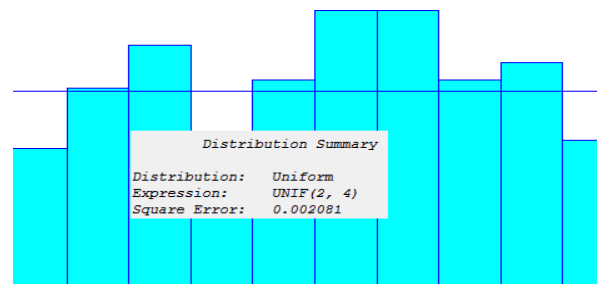
It has been done according to the time measurement performed on the production equipment, the duration of the production of parts for different production devices and timed separately from each other and analysis of the information (Figure 5: 5a- 5e)



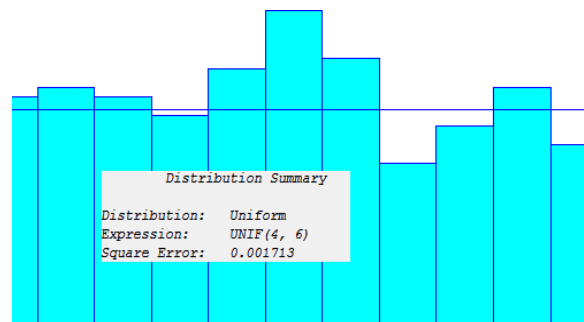
5a. The duration of production of equipment 1 in step 1



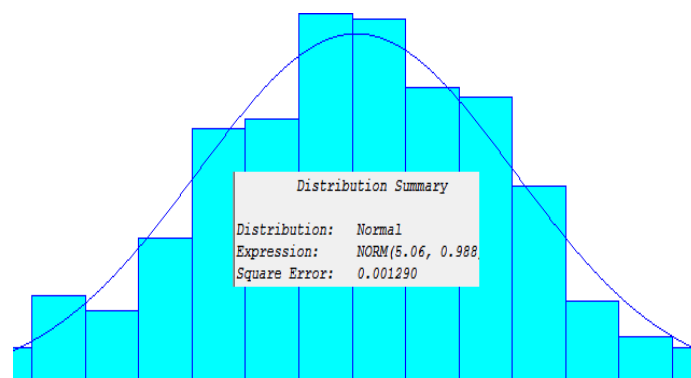
5b. The duration of production of equipment 2 & 3 in step 1



5c. The duration of production of equipment 4 and 5 in stage 2



5d. The duration of production of equipment 6 in stage 2



5e. The duration of equipment production in step 3

Figure 5- Production time of part i in stage j on equipment k

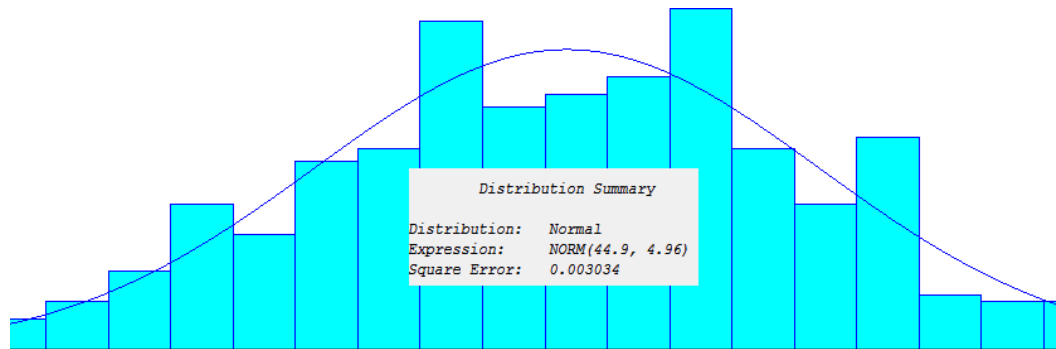
D. The duration of the maintenance of preventive maintenance by personnel p in stage j on equipment k

According to the assessment done and the time measurement done in preventive maintenance and repairs, each stage is individually timed, so the duration of the preventive note for different stages is as follows:

Step 1. The duration of preventive maintenance and repairs in the first stage of production according to the evaluation, it has been shown that in the first stage of the preventive net process. It follows a normal distribution with a mean of 45 minutes and a standard deviation of 5 minutes

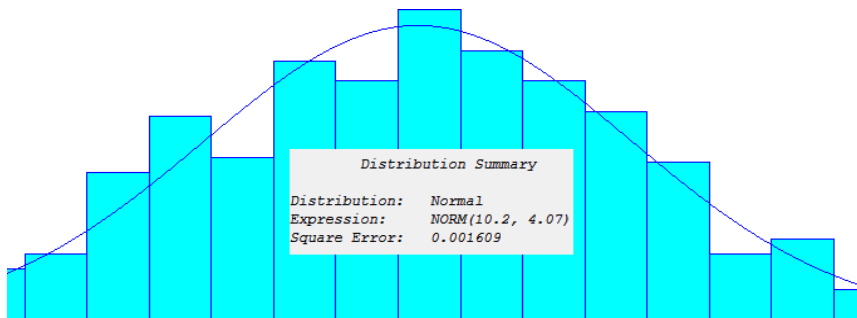
(Figure 6a)

6a. The duration of preventive maintenance and repairs of the first stage of production



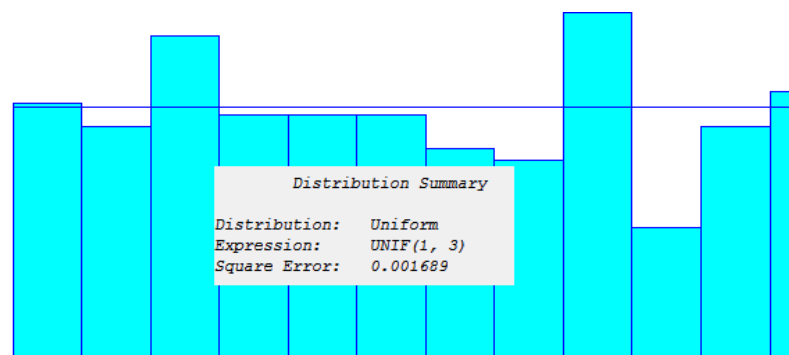
Step 2. The duration of preventive maintenance and repairs of the second stage of production according to the evaluation, it has been shown that in the second stage of the preventive net process

It follows a normal distribution with a mean of 10 minutes and a standard deviation of 4 minutes (Figure 6b).



6b. The duration of preventive maintenance and repairs of the second stage of production

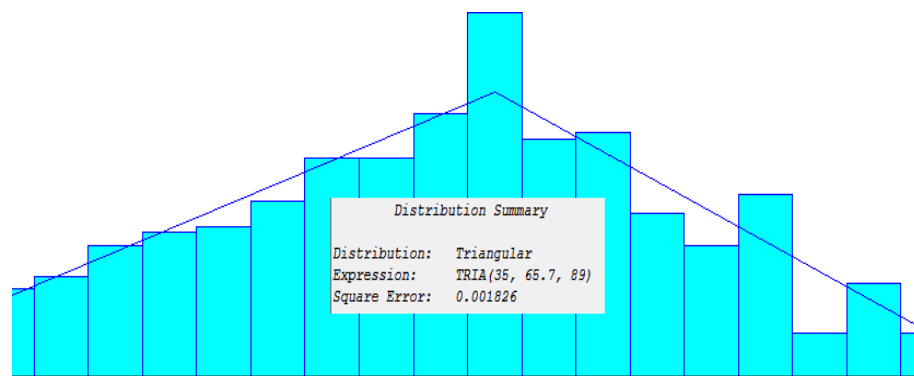
Step 3. The duration of preventive maintenance and repairs of the third stage of production according to the evaluation, it has been shown that in the second stage of the preventive maintenance process, the uniform distribution is done with an average of 2 minutes (Figure 6c).



6c. The duration of preventive maintenance and repairs of the third stage of production

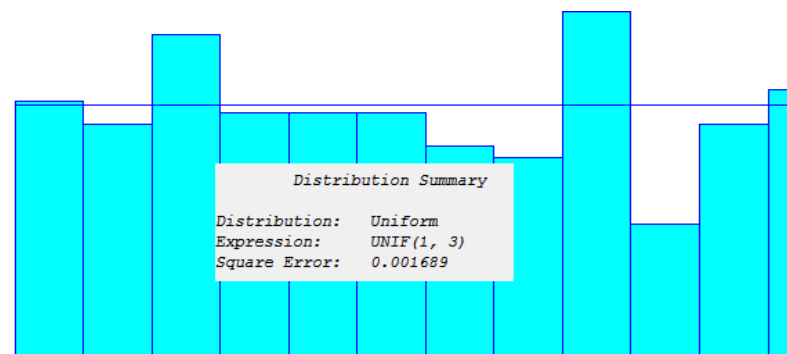
E. The duration of the maintenance of corrective repairs by personnel p in stage j on equipment k

Step 1. According to the evaluation, it has been shown that in the first stage of the reformation process. This process is done from the triangular distribution with the lower limit of 35 minutes, the middle limit of 65 minutes, and the upper limit of 90 minutes (Figure 7a).



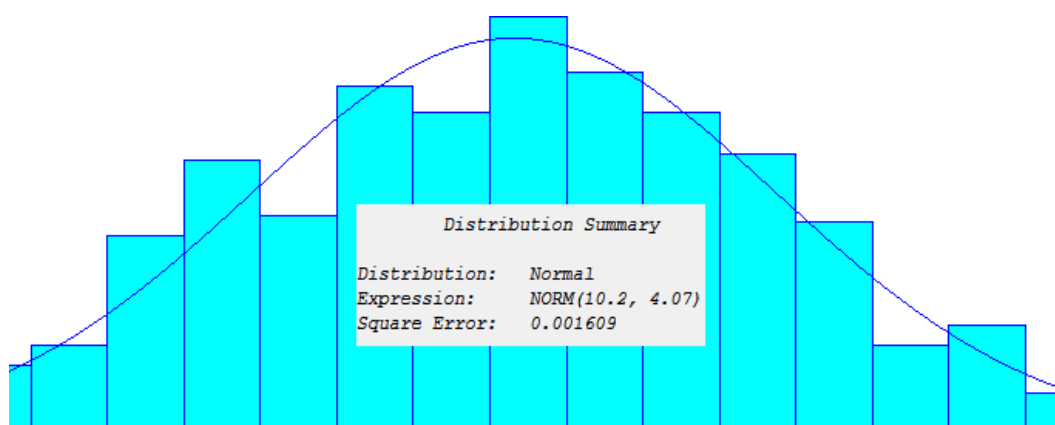
7a. The duration of maintenance and corrective repairs of the first stage of production

Step 2. According to the evaluation, it has been shown that in the second stage of the process it is carried out from a uniform distribution with a mean of 2 minutes (Figure 7b).



7b. The duration of maintenance and corrective repairs of the second stage of production

Step 3. According to the evaluation, it has been shown that in the third stage of process it follows a normal distribution with a mean of 10 minutes and a standard deviation of 4 minutes. Also, the production costs based on the production schedule are 150,000 tomans per hour of activity. Therefore, according to the input parameters of the problem under study, the evaluation of the dependencies of the objectives has been done using the heuristic method of epsilon constraint (Figure 7c).



7c. The duration of maintenance and corrective repairs of the third stage of production

4. Validation of the model using the ϵ -constraint method

The ϵ -constraint method is probably the most widely used approach to solve multi-objective optimizations (MOOs). This technique relies on solving a series of single-objective problems in which one objective is kept in the objective function while the others are transferred to auxiliary constraints that bound them within some allowable levels.

$$\begin{aligned} & \text{Min/Max}(f(x) + \theta * (s_2 + s_3 + \dots + s_i \dots + s_n)) \\ & \quad 1 \quad \quad \quad r_2 \quad \quad \quad r_3 \quad \quad r_i \quad \quad r_n \quad | \\ & \text{St:} \\ & f_2(x) - s_2 = \epsilon_2 \\ & f_3(x) - s_3 = \epsilon_3 \\ & \dots \\ & i \in [2, n] \\ & s_i \in \mathbb{R}^+ \end{aligned}$$

According to the above relationship, Pareto optimal solutions are obtained, where r_i is the domain of the i -th objective function, θ is a small number between .001 to .000001, and s_i is a non-negative additional variable. First, the value of $NISf_i$ (the worst value) and $PISf_i$ (the best value) are obtained for each objective function, then the domain value of the i -th objective function is calculated according to the following equation:

$$r_i = PISf_i - NISf_i$$

After that, r_i is divided into intervals equal to l_i . Then l_{i+1}

The relationship below the epsilon value was obtained based on these points (Grid). In this method

For all obtained epsilons, the model must be solved according to the relation, η number of points (Grid point) has been achieved.

Finally, the following values were obtained for each of the variables (Table 1):

Table 1. values for each of the variables

r_2	18848
l_i	10
NIS_2	52
$PISF_2$	18900
θ	0.0001

The, the number of epsilons were calculated:

ϵ : 1936, 3820, 5704, 7588, 9472, 9472, 11356, 13240, 15124, 17008, 18900

Finally, we solved the enhanced epsilon model using Games software for each of the obtained epsilons. The set of Pareto optimal solutions obtained as presented in Table 2:

Table 2. The rate of objective function

ε	the rate of the first objective function	the rate of the second objective function
1936	24821	1958
3820	45376	3850
5704	65626	5705
7588	87181	7597
9472	13091	9478
11356	17587	11363
13240	24741	13253
15124	31895	15144
17008	39518	17009
18900	48205	18900

5. Evaluation and sensitivity analysis of key modeling parameters

According to the evaluation made in the real production environment, in the previous part, the mathematical model was developed and it was shown how the improvements of the production situation are implemented along with the maintenance and repair services. Therefore, in this section, the sensitivity analysis of the mathematical model in real space is discussed.

A. The sensitivity of the preparation time of parts at each stage

According to the sensitivity analysis, it was shown that the reduction and increase of the preparation time has a direct effect on the system costs, and because of this, the longer the preparation time, the more the costs will increase up to 10%, and as the preparation time decreases, it is found that the cost of production is reduced by 28%.

Therefore, it has been shown in the sensitivity analysis that the duration of preparation plays a key role in the overall costs of production and the scheduling of maintenance and repairs, and it is necessary to pay special attention to the performance of the organization. be created. The executive policies of the organization should be implemented in such a way that the topics of training and learning of operators in 15 workshop systems improve, because the issue of the preparation time of the device and parts is completely dependent on the skill of the operator, and with the training and learning of the operator, the system costs are reduced by 28% (Data not shown).

As shown in the above sensitivity analysis, the longer the preparation time is, the more the probability of disruption decreases, and the disruption is minimized up to 4%, and on the other hand, as the preparation time decreases, the probability of disruption in the production system is reduced to 29%. increase. As a result, the preparation time has an inverse effect on the probability of malfunction, therefore, it is suggested to pay special attention to the problem of operator learning in the production system, because as the preparation time increases, the production errors and malfunctions will decrease, and a balance should be maintained regarding disruption and system costs must be created (Data not shown).

6. Experiment design by Taguchi method

Taguchi method reduces the time of parameter setting by reducing the number of tests. First, specify the parameters that are set in each algorithm, and then, using the Minitab software, present the levels of parameters and orthogonal arrays for the tests, and after determining the number of tests for each algorithm, test the algorithms

with the same specified level. We performed ten times and from the results; obtained from these ten tests, we took the average, then we weighted them and obtained the S/N graphs and selected the best parameters.

At first, it is necessary to obtain and mention the levels of each algorithm. For this work, related articles were studied and candidate levels were identified from among them, which is explained in Table 3.

Table 3. Different levels for parameters of each algorithm

Algorithm	Algorithm parameters	Parameter level		
		Level 1	Level 2	Level 3
MOSA	T0	40	50	60
	α	0.91	0.95	0.98
	Max-iteration	$8*(i+j+t)$	$12*(i+j+t)$	$14*(i+j+t)$
MOKA	M1	15%	20%	25%
	M2	25%	30%	40%
	Smax	15	25	30
	N-Keshtel	100	150	250
	Max-iteration	$4*(i+j+t)$	$6*(i+j+t)$	$8*(i+j+t)$

Using Minitab 16 software, the designers of the experiments were successfully performed and the L9 orthogonal arrays were selected for the MOSA algorithm; But for the MOKA algorithm, L27 orthogonal arrays were considered. After running the algorithms for each of the mentioned tests, the response values for the Taguchi method were obtained. These values and orthogonal arrays are presented in the Table 4. Finally, after drawing the signal-noise graphs of the algorithm, the best values were obtained (Table 5).

Table 4. L27 orthogonal array and computational results for the MOKA algorithm

Experiment	M1	M2	Smax	N-Keshtel	Max-iteration	MOKA Response
1	1	1	1	1	1	0.0000114085
2	1	1	1	1	2	0.0000080784
3	1	1	1	1	3	0.0000122764
4	1	2	2	2	1	0.0000081444
5	1	2	2	2	2	0.0000082496
6	1	2	2	2	3	0.0000139157
7	1	3	3	3	1	0.0000054548
8	1	3	3	3	2	0.0000116998
9	1	3	3	3	3	0.0000046843
10	2	1	2	3	1	0.0000064355
11	2	1	2	3	2	0.0000192760
12	2	1	2	3	3	0.0000075266
13	2	2	3	1	1	0.0000100477

14	2	2	3	1	2	0.0000349183
15	2	2	3	1	3	0.0000097563
16	2	3	1	2	1	0.0000073975
17	2	3	1	2	2	0.0000112974
18	2	3	1	2	3	0.0000049005
19	3	1	3	2	1	0.0000065089
20	3	1	3	2	2	0.0000215562
21	3	1	3	2	3	0.0000047569
22	3	2	1	3	1	0.0000205043
23	3	2	1	3	2	0.0000036033
24	3	2	1	3	3	0.0000211287
25	3	3	2	1	1	0.0000319259
26	3	3	2	1	2	0.0000083761
27	3	3	2	1	3	0.0000154806

Table 5. L9 orthogonal array and computational results for MOSA algorithm

Experiment	$T0$	α	Max-iteration	MOSA Response
1	1	1	1	0.00001506
2	1	2	2	0.00001824
3	1	3	3	0.00002464
4	2	1	2	0.00001255
5	2	2	3	0.00003098
6	2	3	1	0.00002042
7	3	1	3	0.00000649
8	3	2	1	0.00001432
9	3	3	2	0.00001662

7. Computational results

After designing the experiment and adjusting the parameters, the appropriate parameters have been specified in the algorithm and the algorithms were implemented for the generated problems and compared with each other. As a result, 12 problems were executed with 2 algorithms. After solving the proposed mathematical model using the mentioned methods, Tables 6 & 7 show the result for the problem.

Table 6. Computational results of the algorithms for 12 problems, part 1

Problem	NPS		CPU Time	
	MOSA	MOKA	MOSA	MOKA

1	8	17	1540.9660	5358.5110
2	1	15	4219.9140	28312.2200
3	11	12	9678.5920	90243.9700
4	8	17	16464.7200	215506.5000
5	13	15	36780.8600	1417215.0000
6	13	16	38512.4500	2026231.0000
7	10	17	62413.2700	2856213.0000
8	8	15	164712.7000	10239191.0000
9	10	16	783896.8000	36226808.0000
10	16	14	703949.2000	47964842.0000
11	15	17	1230565.0000	137000000.0000
12	13	17	2874044.0000	247000000.0000
Problem	MS			
	MOSA		MOKA	
1	125000000000		132000000000	
2	338000000000		415000000000	
3	775000000000		516000000000	
4	647000000000		479000000000	
5	947000000000		945000000000	
6	1180000000000		2690000000000	
7	1080000000000		1190000000000	
8	1130000000000		1300000000000	
9	2940000000000		1650000000000	
10	4890000000000		3340000000000	
11	1990000000000		4580000000000	
12	4530000000000		2070000000000	

Table 7. Computational results of the algorithms for 12 problems, part 2

Problem	NPS		MID	
	MOSA	MOKA	MOSA	MOKA
1	8	17	8990.1490	17817.7000
2	11	15	10990.3800	39197.5800

[illegible]

8. Identifying the best algorithm

Considering that in this issue we have the standard index including: MID, SNS, MS, NPS and CPU time, and there are also two options of algorithms including: MOKA and MOSA, identifying the best method is a difficult task. Therefore, by using multi-criteria methods, we choose the best method in different dimensions; which is used to obtain the weights of the criteria (indices) from the AHP method and to sort the options (algorithms) from the VIKOR method. To obtain the weights of the criteria, at first, pairwise comparisons are performed according to Tables (8 & 9).

Next the VIKOR method was applied to choose the best option. For this purpose, we consider problems 1 to 4 as small dimension problems, problems 5 to 8 as medium dimension problems, and problems 9 to 12 as large dimension problems. Then we take the average of each dimension for each option relative to the criterion (Table 10).

Table 8. The matrix of pairwise comparisons of criteria relative to each other

	NPS	CPU time	MID	MS	SNS
NPS	1	3	5.0	5.0	2
CPU time	33.0	1	2.0	33.0	2.0
MID	2	5	1	2	2
MS	2	3	5.0	1	2
SNS	5.0	5	5.0	5.0	1

Table 9. Normalized pairwise comparison matrix with final weight and inconsistency rate (CR=50.0)

	NPS	CPU time	MID	MS	SNS	وزنها
NPS	0.213269	0.1765	0.224697	0.135886	0.340221	0.16754
CPU time	0.87497	0.0588	0.080584	0.080681	0.03004	0.051864
MID	0.537633	0.2941	0.569539	0.597745	0.436018	0.374083
MS	0.622043	0.1765	0.300402	0.271654	0.361252	0.266056
SNS	0.099837	0.2941	0.20369	0.149469	0.167193	0.140458

Table 10. Score matrix of options relative to criteria

problem average	گزين هها	NPS	CPU time	MID	MS	SNS
Problems 1-4	MOSA	75.6	487325.23	467725.3	2.429727	4.964632
	MOKA	75.11	1305.335	95765.1	6.581085	5.764987
Problems 5-8	MOSA	5.7	417025.46	510575.4	893257	3065857
	MOKA	25.12	846425.2622	1518.4	1139881	2744901
Problems 9-12	MOSA	75.9	354125.108	808775.6	1383045	6263086
	MOKA	75.11	46958.13515	87805.6	1444900	5538413

Finally, our results showed that, in small dimensions, MOKA was chosen as the best option, followed by MOSA algorithm (Table 11). In medium dimensions, MOKA is chosen as the best option, followed by MOSA algorithm (Table 12). In large dimensions, MOSA is chosen as the best option, followed by MOKA algorithm (Table 13).

Table 11. Results of the VIKOR method for problems with small dimensions

	Normal decision matrix					Balanced normal decision matrix					Selection indicators			
	NPS	CPU MS	SNS	MID	time	MID	CPU time	NPS	MS	SNS	Q	S	R	
MO SA	0.3359 0816	0.0578 1178	0.7336 6148	0.3751 0434	0.5924 9318	0.0622 5884	0.0034 1286	0.2563 7818	0.0910 096	0.0971 1138	2	1.034 2	0.8039 8708	0.3613 4948
MO KA	0.5847 3668	0.5072 751	0.4141 971	0.8253 9502	0.4699 4048	0.1083 8416	0.0484 0056	0.1446 8458	0.1230 698	0.0770 479	0.1695	0.3463	0.2191	0.0538

Table 12. Results of the VIKOR method for problems with medium dimensions

	Normal decision matrix					Balanced normal decision matrix					Selection indicators			
	NPS ^{CPU} time MID MS SNS					NPS ^{CPU} time MID MS SNS					Q	S	R	
M	0.55	0.429	0.54	0.01	0.380	0.09	0.070	0.000	0.189	0.104	0.362	0.798	0.991	1531
OS	6393	4219	5132	2089	3955	3685	2719	1212	9328	482	4159 5159 ²			
A														
M OK A	9 5 5 2					5 5					5			
	0.621	0.85	0.49	0.547	0.49	0.115	0.050		0.132	0.08	0.153	0.342	0.259	
	3817	3454	8971	9975	8141	1551	0.174		9829	1676	2983	5632	2286 1	
	5	1	1	5	9	5	0629 3393		5	2	5	5	5	
						5								

Table 13. Results of the VIKOR method for problems with large dimensions

	Normal decision matrix					Balanced normal decision matrix					Selection indicators		
	NPS	CPU time	MID	MS	SNS	NPS	CPU time	MID	MS	SNS	S	R	Q
MOSA	0.363	0.006	0.588	0.427	0.541	0.067	0.000	0.205	0.103	0.088	0.344	0.779	0.895
	7568	5399	7945	4039	5	2971	8421	7846	5	2923	4066	7396	0282
											6555		1
													5

MOKA	0.594	0.437	0.0780.046				0.943		
	0.4466501	7541 58065 0.7951063	5754 50.081	6573 0.2077726	0.1083589		0.355 2103	1.016 2	5
		0.4791781		5	1167		5	1651	
		5							

9. Conclusion

The open workshop production system is characterized by its complexity and diversity, producing a wide variety of products in high quantities but low volumes. Common in industries such as automotive, aerospace, electronics, and SMEs, this system offers high flexibility, allowing quick changes in production operations to respond to market demands and customer needs. It features diverse production paths, optimizing resource use and minimizing idle times, thus enhancing productivity and reducing costs. Accurate and optimal production planning and scheduling are crucial due to the system's complexity, requiring advanced methods and algorithms to manage production and maintenance processes efficiently. An integrated scheduling model combining production and maintenance processes is vital for optimizing performance in these systems. This model enhances resource coordination, reduces equipment downtime, and uses advanced optimization algorithms to solve complex scheduling problems. It also includes robust methods to handle instabilities and changes in production conditions, ensuring flexibility and consistent performance.

Achieving stable start-up times in an open workshop production system requires a comprehensive approach that integrates supply chain efficiency, proactive maintenance, and real-time monitoring. Advanced technologies and strategic planning help enhance productivity and minimize downtime. The model incorporates sequence-dependent preparation times to manage scheduling complexity, while the just-in-time approach ensures timely, high-quality production in a competitive market. Effective asset management is essential due to the high costs of equipment failures. To address these challenges, the research introduces a MINLP mathematical model for optimizing maintenance and scheduling, offering an improved strategy for production efficiency.

10. Limitations

Limited access to real data led the study to rely on hypothetical samples, affecting accuracy. The integrated scheduling model involves complex, time-consuming calculations, especially with meta-heuristic algorithms. The study relied on simulations rather than extensive field tests, which future research can address. Despite these limitations, the model has shown promise in improving productivity, reducing costs, and minimizing equipment downtime.

11. Future research priorities

To enhance accuracy and adaptability, more complex models should integrate real data and account for production and maintenance details. Hybrid optimization methods, such as combining genetic algorithms with other techniques, can improve scheduling efficiency. Industries should adopt advanced information systems, IoT, and Big Data for better data management and optimization. Extensive field tests across industries will help validate and refine models. Collaboration with industry partners can facilitate implementation. Additionally, incorporating human and organizational factors into scheduling models can improve overall productivity and system efficiency.

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