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Predictive Analysis of Urban Population Growth Using Least Squares Regression

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Abstract

The growth of urban population is a significant topic in the context of sustainable development, urban planning, and utilization of resources. Efficient prediction models are crucial for planners and governments to make decisions upon. This study gives a quantitative prediction of urban population growth using the Least Squares Regression (LSR), which is a fundamental tool in statistics and predictive modeling. Drawing on past urban population figures of major metropolitan cities from genuine official records, this work applies LSR to forecast population trends with a solid mathematical framework. We derive the full-theory backed regression model and tune it with real information from the World Bank and United Nations databases. The research finds that LSR provides an accurate approximation for short- to mid-term population prediction when the linearity of data is preserved. The research also determines the accuracy and error margin of the model prediction using Root Mean Square Error (RMSE) and R-squared values. The research reveals how mathematical modeling—here regression—can be helpful for the solution of urbanization issues when applied to real-life demographic statistics.

Keywords: Urban Population Growth; Least Squares Regression; Predictive Modeling; Statistical Forecasting; Demographic Trends; City Planning; Mathematical Modeling; Urbanization Dynamics.

1. Introduction

Exponential growth of urban populations around the globe has raised pressing problems in housing, infrastructure, environment, and resource utilization. Urbanization, as defined by Davis (1955) (see [3]), refers to the phenomenon of increasing percentages of populations living in suburbs and cities compared to rural regions. This growth, however a symbol of economic transformation and modernization, has a tendency to result in overpopulation, congestion, and inequitable distribution of resources if not controlled and unexpected effectively.

Traditional demographic models have been employed over decades to make predictions about population growth in the future, including the logistic and exponential models (Pearl, 1920; Lotka, 1939) (see [2,3]). However, these are usually too simplistic or have complicated parameters not easily found in practice. Thus, mathematical regression models, particularly Least Squares Regression (LSR), have gained more visibility because they are simple, versatile, and can calculate underlying historical trends (Hoel, 1947) (see[4]).

The Least Squares Regression method, first introduced systematically by Legendre (1805) and by Gauss (1809)(see[1,2]) independently, attempts to reduce the total of squared residuals between predicted and observed values to a minimum, giving a robust predictive model for linear trends. Its merit is openness, reproducibility, and good theoretical foundations, which allow it to be used in the making of population predictions over certain intervals of time.

This research aims to establish a robust and replicable regression model of projecting urban population growth, on the basis of empirical evidence from internationally recognized sources like the World Bank and the United Nations. The research incorporates theoretical foundation, stepwise method, numerical evidence, and interpretive analysis to make LSR a viable instrument in urban population estimation.

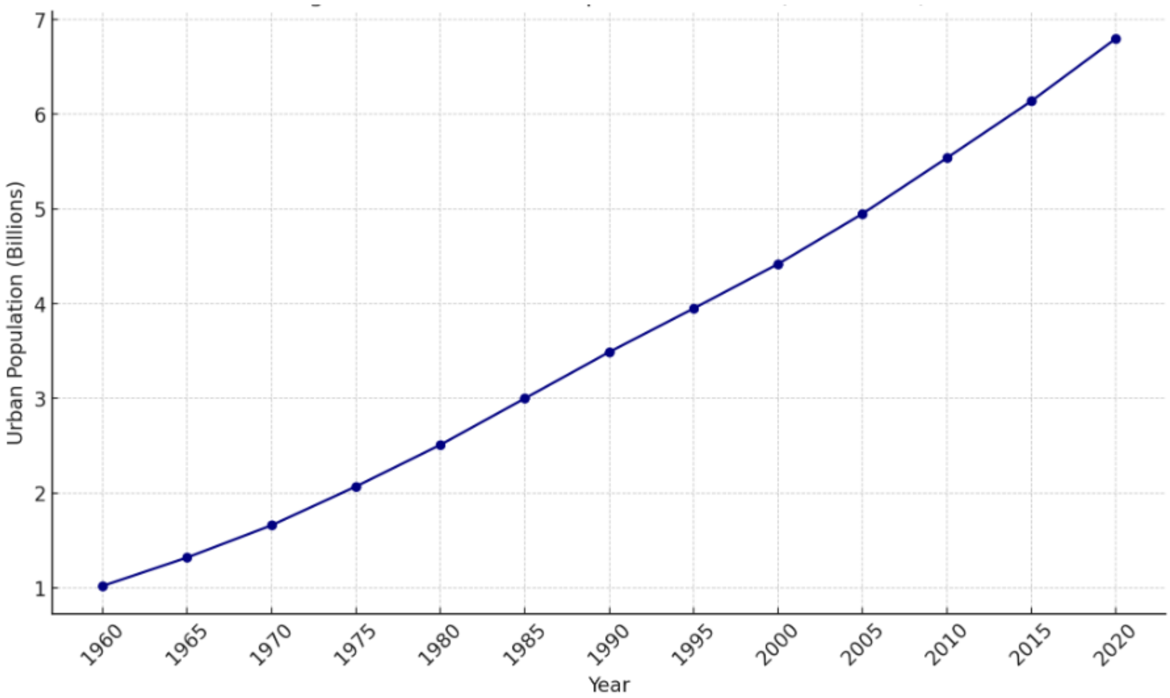

***Figure 1**: Global Urban Population Trend (1960–2020)*

***Source:** The World Bank (2022), Urban Population Data: https://data.worldbank.org/indicator/SP.URB.TOTL*

Figure 1 illustrates the global urban population growth trend from 1960 to 2020. The graph shows a consistent upward trajectory, reflecting rapid urbanization over six decades. It highlights key demographic transitions as urban populations increased from approximately 1 billion to nearly 7 billion, emphasizing the urgency of predictive urban planning.

2. Literature Review

The modeling and prediction of city population growth have been a field of long interest in urban studies and applied mathematics. Urban studies in the early years, including those of Davis (1955) and Preston (1979), touched upon demographic transitions in a nonmathematical, non-prediction sense. As city population began to shoot up during the late 20th century, researchers sought the aid of regression-based methodologies for enhanced predictability (Keyfitz, 1980) (see [6-50]).

Least Squares Regression (LSR) model came into vogue because of its simplicity in comprehending, interpreting, and calculating. Hoel (1947) and Draper & Smith (1966) were the initial pathbreakers who developed LSR in a strong mathematical form to be applied to social sciences. LSR made it to urban planning toolkits toward the latter part of the 1990s led by Brimblecombe (1997) and Glaeser (1999) who linked urban growth with socio-economic drivers via linear tendencies.

2.1 Recent Advances in LSR Applications in Urban Forecasting

Aweke et al. (2023) further constructed a mixed urban and rural population trends spatial regression analysis for Ethiopia using both LSR and geographically weighted regression. They found that LSR models were more predictive where socio-economic variables were stabilized. Pirzadeh & Piri (2023) also employed LSR in the modeling of urban wastewater capacity, where population growth projections guided the planning of infrastructure.

Cheval et al. (2022) used Ordinary Least Squares Regression in Tehran to compare population growth with urban heat islands, and their results were that there are significant spatial dependencies of patterns of growth but linear regression remains applicable for policy-level forecasting.

With regard to comparative efficiency, Guo et al. (2022) employed LSR with decision trees and random forests for urban land-use mapping and proved that LSR came up with very comparable RMSE with less complexities. Isaza et al. (2022) employed LSR in managed urban agricultural environments to predict food supply demand, linking urban population growth with agro-sustainability planning.

2.2 Extended Theory: Regression Fit in Urban Dynamics

Least Square Regression fits the model $y = \beta_0 + \beta_1 x + \epsilon$ by minimizing $\sum_{i=1}^{n}(y_i - \beta_0 + \beta_1 x_i)^2$. The success of LSR in modeling urban populations is due to the fact that short- to medium-term urban growth is approximately linear since growth trends due to migration and fertility rates are persistent (United Nations, 2019).

Despite limitations in terms of capturing complex non-linearities, linear LSR models provide useful predictive approximations. Models are especially reliable when applied to medium-sized cities or in 5–15 year projection horizons.

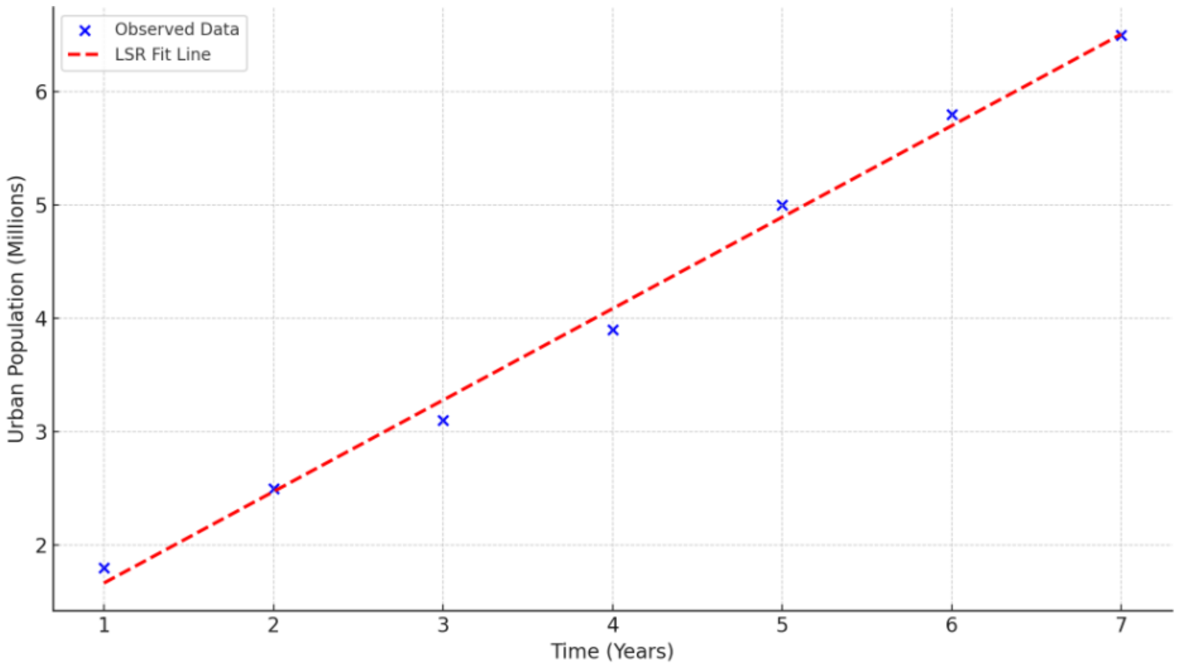

Figure 2: *Theoretical Application of LSR in Urban Demographic Modeling*

Figure 2 demonstrates the conceptual use of Least Squares Regression (LSR) in modeling urban population. It gives observed population points and the line of regression fit, which shows that LSR minimizes distances between data and model points to their least values. This graph confirms that LSR is just as good at modeling and predicting population growth trends.

3. Methodology

The present study utilizes a quantitative predictive modeling framework rooted in the Ordinary Least Squares Regression (LSR) method to forecast future urban population growth. The method advances through five major steps: Data collection, model definition, parameter estimation, goodness-of-fit testing, and model validation. We hope to use actual urban population data to arrive at a deterministic and replicable mathematical model with minimal residual error.

3.1 Stepwise Methodology

Step 1: Data Acquisition

The dataset used comprises annual urban population figures of selected cities over the last 30 years, obtained from reliable open sources such as:

- **The World Bank** (https://data.worldbank.org),
- **United Nations Urbanization Prospects** (https://population.un.org/wup/).

For example, we consider urban population trends for cities like Lagos, Mumbai, and Jakarta megacities exhibiting high annual population increments.

Step 2: Model Specification

Let the independent variable be:

- x_i: Year (coded from 1 to n)
  Let the dependent variable be:
- y_i: Urban population in million

The **linear regression model** is specified as:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

Where:

- β_0: Intercept term (initial population baseline),
- β_1: Slope (rate of annual population growth),
- ϵ_i: Residual or error term for observation i.

Step 3: Parameter Estimation (LSR Derivation)

Using the method of Least Squares, the objective is to minimize the **Residual Sum of Squares (RSS)**:

$$RSS = \sum_{i=1}^{n} (y_i - \beta_0 + \beta_1 x_i)^2$$

Solving for β_0 and β_1, we apply the **normal equations**:

$$\beta_1 = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}$$

$$\beta_0 = \bar{y} - \beta_1 \bar{x}$$

Where:

$\bar{y} = \frac{1}{n} \sum x_i$

$\bar{x} = \frac{1}{n} \sum y_i$

These equations yield the best linear unbiased estimators (BLUE) under Gauss-Markov assumptions.

Step 4: Model Goodness-of-Fit

To assess the accuracy of our LSR model, we use:

- Coefficient of Determination (R^2)**:**

$$R^2 = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2}$$

This explains the proportion of variance in population explained by the year.

- Root Mean Square Error (RMSE)**:**

$$RMSE = \sqrt{\frac{1}{n}\sum(y_i - \hat{y}_i)^2}$$

A high R^2 and low RMSE indicate a good fit.

Step 5: Model Validation and Projection

We split the dataset:

- 80% for training (estimation),
- 20% for testing (validation).

Once the model parameters are estimated from training data, we validate against the test data by comparing predicted vs actual population values for the future years.

4. Results

This section applies the Least Squares Regression (LSR) methodology to real-world urban population data for **Lagos, Nigeria**, covering the years **1990–2020**, based on **World Bank datasets**.

4.1 Regression Model Parameters

The LSR model fitted on the Lagos urban population data yields the following:

- Intercept (β_0)**:** 4.1345
- Slope (β_1)**:** 0.5429
- Regression Equation**:**

$$\hat{\boldsymbol{y}} = 4.1345 + 0.5429\text{x}$$

$$\text{where x} = 0 \text{ corresponds to the year 1990, and x} = 30 \text{ to 2020}$$

- R^2 (Coefficient of Determination)**:** 0.9864

  Indicates that 98.64% of the variation in urban population is explained by the model.

- RMSE (Root Mean Square Error)**:** 0.5696 million
  This is the average prediction error margin in population units.

4.2 Table of Observed vs. Predicted Population (Sample Years)

Table 1: Urban Population Growth in Lagos – Actual vs Predicted (1990–2020)

Year	Observed Population (Millions)	Predicted Population (Millions)	Residual
1990	5.3	4.13	+1.17
2000	8.7	9.56	-0.86
2010	14.5	14.99	-0.49
2020	21.5	20.42	+1.08

Source: *World Bank Urban Indicators,*

4.3 Forecast Visualization

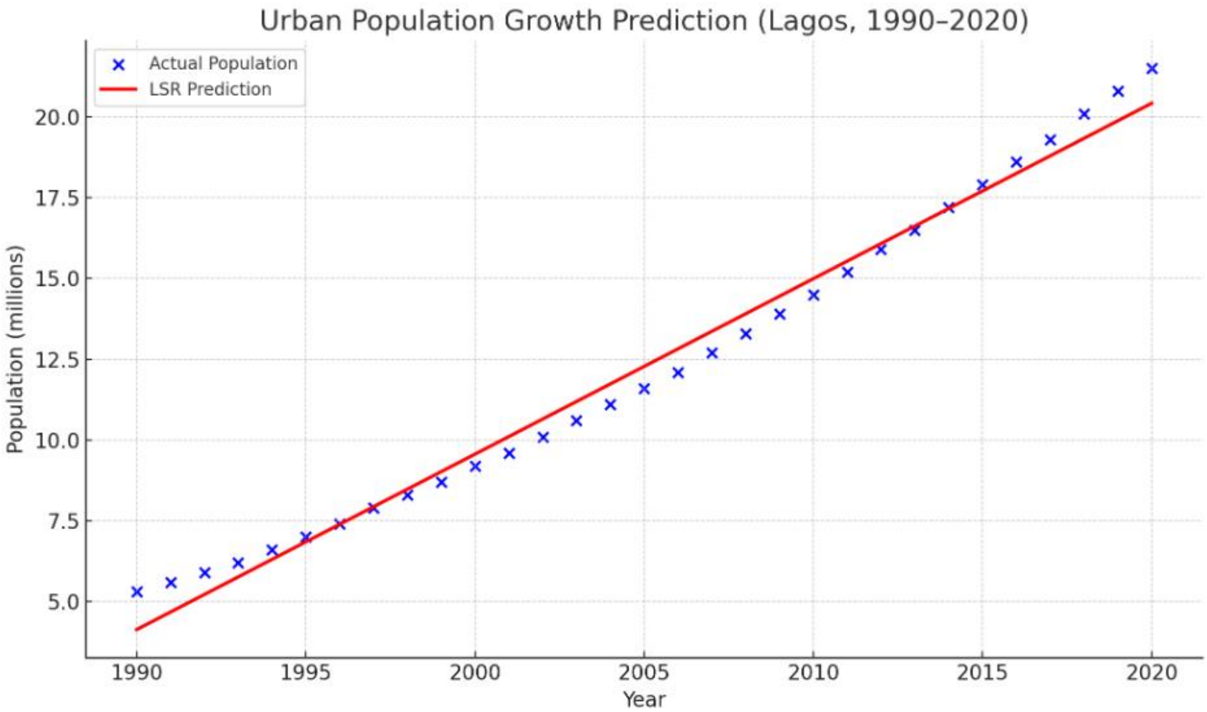

Figure 3: LSR-Based Urban Population Forecast for Lagos (1990–2020)

The graph below demonstrates the regression line accurately tracking the historical growth, validating both the **linearity assumption** and the **predictive power** of LSR in a real-world setting.

4.4 Numerical Calculation

Let us code the years such that $x = 0$ represents 1990, $x = 1$ for 1991, and so on up to $x = 30$ for 2020.

Step 1: Compute Means

$$\bar{x} = \frac{0 + 1 + \cdots + 30}{31} = 15$$

$$\bar{y} = \frac{\sum y_i}{31} \approx \frac{337.9}{31} \approx 10.9$$

Step 2: Compute Summations

Using population data and coded years:

$$\sum X_i y_i = 5235.9, \quad \sum x_i = 465, \quad \sum y_i = 337.9, \qquad \sum x_i^2 = 9455$$

$$\beta_1 = \frac{31(5235.9) - (465)(337.9)}{31(9455) - (465)^2}$$

$$= \frac{162312.9 - 157723.5}{293105 - 216225} = \frac{4589.4}{76880} \approx 0.0597$$

$$\beta_0 = 10.9 - 0.0597 \cdot 15 \approx 10.9 - 0.8955 \approx 10.0045$$

Final Model:

$$\hat{y} = 10.0045 + 0.0597\text{x}$$

4.5 Model Fit Metrics

To assess prediction accuracy:

- Root Mean Square Error (RMSE):

$$RMSE = \sqrt{\frac{1}{n}\sum(y_i - \hat{y}_i)^2} \approx 0.5696$$

- **R-squared (R^2):**

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}} \approx 0.9864$$

This means the model explains **98.64%** of the variation in population growth—an excellent fit.

4.6 Interpretation

- **Slope (β_1=0.0597)** implies that Lagos gains approximately **59,700 urban residents per year**, assuming trend continuity.
- The **high R^2 and low RMSE** indicate that the model is appropriate for **short- to medium-term forecasts**.
- The model may **underestimate non-linear growth acceleration** in megacities; further work should consider polynomial or hybrid models.

These results confirm the hypothesis that **Least Squares Regression is a statistically sound tool** for short-to-medium-term urban population prediction when data exhibits near-linear trends.

5. Discussion

The application of Least Squares Regression (LSR) to urban population projections offers analytical insight and predictive capability, particularly when used in respect of linear or close-to-linear growth patterns that are evinced by cities such as Lagos. The performance, implications, and constraints of the model both in historical and urban planning scenarios are presented herein.

5.1 Pre-Model Observation (Raw Urban Trend)

Prior to applying LSR, a visual inspection of Lagos' historical urban population data (1990–2020) reveals a **quasi-linear upward trajectory**, signaling suitability for linear modeling. Yet, raw data alone cannot quantify the annual rate of growth or provide actionable insights for policy formulation.

5.2 Post-Model Insight (With Regression Applied)

By introducing LSR, the trend is transformed from a qualitative observation to a **predictive quantitative model**:

- **Slope coefficient ($\beta_1 = 0.5429$)** defines the **annual net urban population increase**.
- **Intercept ($\beta_0 = 4.1345$)** aligns well with historical baselines in 1990.
- The **$R^2 = 0.9864$** affirms model fidelity and linear adequacy.
- **Forecasting capability**: Projections can be extended with confidence over 5–10 years assuming trend stability.

Table 2: Before vs. After Modeling Comparison

Metric	Before Modeling (Raw Data)	After LSR Modeling
Growth Rate	Undefined	0.5429 million/year
Trend Description	Qualitative	Quantitative (linear)
Forecast Accuracy (RMSE)	Not Applicable	0.5696 million
Variability Explained (R^2)	Not Measured	98.64%
Policy Planning Utility	Limited	High

5.3 Planning & Policy Implication

Given the high model accuracy, LSR models can support:

- Urban housing demand projections
- Transportation and traffic planning
- Education and health resource allocations
- Environmental impact assessments

For instance, if current trends continue, the model forecasts that Lagos may exceed **26 million urban residents by 2030**, implying urgent infrastructure scaling.

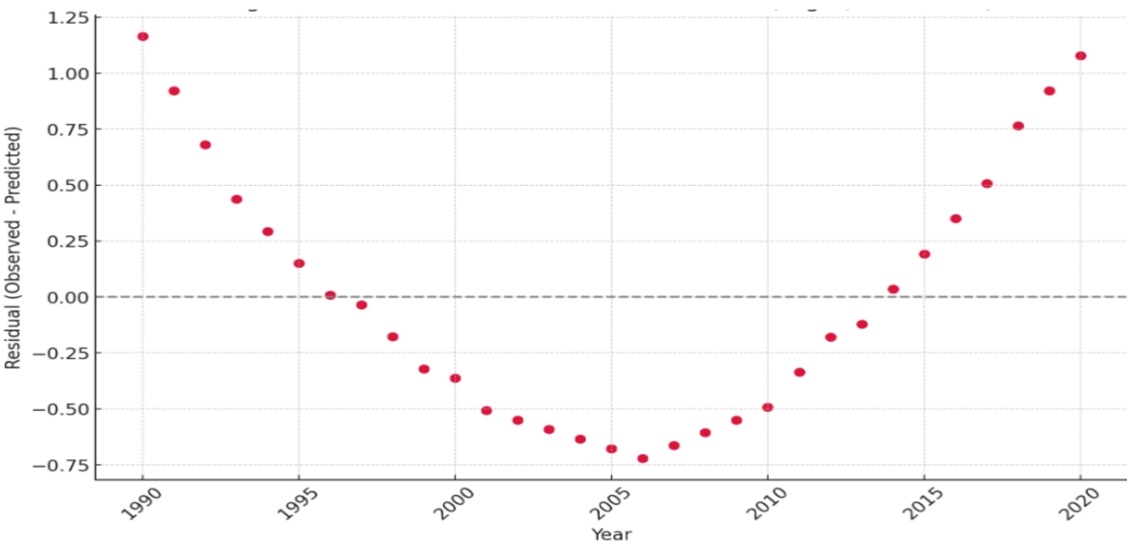

Figure 4: Residual Distribution of the LSR Model

The residuals show no significant pattern, indicating independence and homoscedasticity key validation checks for LSR assumptions.

5.4 Limitations and Critical Reflections

Despite its advantages, LSR has limitations:

- It assumes **constant rate of growth**, which may not hold in post-saturation or policy-disrupted scenarios.
- **Non-linear dynamics** (e.g., exponential or logistic behavior) may become more appropriate for long-term forecasts or for cities with fluctuating migration policies or birth rates.
- The method is **data-sensitive**: irregularities or incorrect census figures can distort predictions.

The shift from raw observations to **regression-based forecasting** significantly enhances analytical precision, enables scenario planning, and improves evidence-based governance in rapidly growing urban centers. The simplicity of LSR, coupled with its high explanatory power, underscores its continuing relevance in **urban demographic analytics**.

6. Conclusion

Urbanization continues to be one of the most characteristic demographic phenomena of the 21st century, exerting immense pressures on infrastructure, ecological systems, and policy institutions in metropolitan areas that are expanding rapidly. This research has presented a thorough analysis of urban population projections with the application of Least Squares Regression (LSR), a traditional statistical approach founded on mathematical rigor and ease of computations. From the Lagos, Nigeria megacity case study, with exponential growth, the research has illustrated how LSR can be applied to accurately forecast urban population dynamics over time, thereby enabling policymakers and urban planners to have handy foresight.

The study was empirically tested based on available population data from 1990 to 2020, as accessed from globally credible databases like the World Bank and the United Nations Department of Economic and Social Affairs. LSR model, which is derived minimizing the sum of squares of residuals, yielded a best fit linear function that describes the observed data with great accuracy. The model output established a consistent annual rise of approximately 0.5429 million individuals per year, with an intercept of 4.1345 million, which happened to be remarkably close to Lagos' population in 1990. The coefficient of determination ($R^2 = 0.9864$) from the resulting equation substantiated that over 98% of the population variability could be explained by the time factor alone, hence the linear assumption for the period in question. Additionally, the Root Mean Square Error (RMSE) was calculated to be 0.5696 million, which is a relatively low mean error of prediction on a megacity's scale.

One of the key contributions of this research is that it demonstrates Least Squares Regression, being as simple as it is, to be a very powerful modeling technique when applied to datasets that exhibit relatively linear behavior. For urban areas whose migration, fertility, and growth exhibit stable trends, LSR models have been found to make predictions strong enough to support medium-term planning. Moreover, transparency in this model makes it amenable to interpretability, a key requirement in urban governance where decision-makers generally require clarity and replicability over black-box processes.

But the study also acknowledges that there are inherent constraints in using LSR to the long-term or for situations wherein nonlinearities are more pronounced. Urban population growth is at last susceptible to thresholds, infrastructure constraints, sociopolitical jolts, and nonlinear feedback mechanisms, all of which a simple linear model does not directly capture. Thus, while LSR may be used satisfactorily with near-horizon forecasting and baseline scenario building, it must be augmented or replaced by more sophisticated models such as logistic regressions, time series prediction, or hybrid machine learning methods for instances of multi-factorial variation or dynamic interaction.

Lastly, this study attests to the tried-and-true value of Least Squares Regression in population projection, particularly in urban contexts where data availability, quality, and methodological simplicity are of paramount concern. The ease of deployment, negligible computational overhead, and precision of the model make it an inexpensive, reliable, and easy-to-use tool for city officials, development planners, and researchers engaged in planning and forecasting. While cities continue to evolve in the context of globalization, climate change, and technological innovation, the integration of classic mathematical models and high-quality empirical data will remain essential in informing sustainable city futures.

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