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On The Structure of Some Lower Intervals in the Subgroup Lattices of 2×2 Non-Singular Matrices Overz5

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Abstract

In this paper, we display the lattice structure of some lower intervals whose upper bounds are 12 element-subgroups in the lattice of subgroups of the group of 2x2 non-singular matrices over **Z5**.

Keywords: Matrix group, Subgroups, Lagrange's theorem, Poset, Lattice, Atom.

1.Introduction

Let $G = \left\{ \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} : a_1, a_2, a_3, a_4 \in Z_p \ and \ \begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix} \neq 0 \right\}$. Then G is a group under the binary operation of matrix multiplication modulo p and $o(G) = (p^2 - 1)(p^2 - p)$. Let L(G) denote the lattice formed by all subgroups of G. In the case, when p=5, $o(G) = (5^2 - 1)(5^2 - 5) = 24 \times 20 = 480 = 2^5 \times 3 \times 5$. In this paper, we display the structure of some intervalswhose upper bounds are 12 element-subgroups in the lattice of subgroups of the group of 2x2 non-singular matrices over **Z5**.

2. Preliminaries

In this section, we give somedefinitions and theorems for the development of the paper.

Definition 2.1 A partial order on a non-empty set P is a binary relation $\leq$ on P that is reflexive, anti-symmetric and transitive. The pair (P, $\leq$) is called a partially ordered set or poset. A Poset(P, $\leq$) is totally ordered if every x , $y \in$ P are comparable, that is either $x \leq y$ or $y \leq x$. A non-empty subset S of P is a chain in P if S is totally ordered by $\leq$.

Definition 2.2 Let (P, $\leq$) be a poset and let S$\subseteq$ P. An upper bound of S is an element $x \in$ P for which $s \leq x$ for all $s \in$S. The least upper bound of S is called the supremum or join of S. A lower bound of S is an element $x \in$ P for which $x \leq s$ for all $s \in$ S. The greatest lower bound of S is called the infimum or meet of S.

Definition 2.3 A Poset (P, $\leq$) is called a lattice if every pair $x, y$of elements of P have supremum and infimum, which are denoted by x $\vee$ y and $x \wedge y$ respectively.

Definition 2.4 For two elements a and b in P, a is said to cover b or b is said to be covered by a (in notation $a \succ b$ or $\prec a$) if only if $b<a$ and for no $x \in$ P, $b<x<a$ holds.

Definition 2.5 An element $a \in$ P is called an atom, if $a \succ$0 and it is a dual atom, if $a \prec$1.

Definition 2.6 Let L be a lattice. A subset I of L is called a lattice interval if there exist elements $a, b \in L$ such that I= $\{t \ \in L: a \leq t \leq b\} = [a, b]$. The elements a, b are called the end points of I.

Theorem 2.7 If G is a finite group and $a\in$ G, then the order of 'a' is a divisor of the order of G.

Theorem 2.8 Let G be a finite group and let p be any prime number that divides the order of G. then G contains an element of order p.

3. The order-wise arrangement of elements ofG

In table 3.1, we produce the list of elements according to their orders.

Order	**Elements**
1	e
2	$\alpha_{1,}\alpha_{2,}, \alpha_{3,}, \dots \dots \dots, \alpha_{31}$
3	$\beta_{1,}\beta_{2,}, \beta_{3,}, \dots \dots \dots, \beta_{20}$
4	$\gamma_{1,}\gamma_{2,}, \gamma_{3,}, \dots \dots \dots, \gamma_{152}$
5	$\delta_{1,}\delta_{2,}, \delta_{3,}, \dots \dots \dots, \delta_{24}$
6	$\mu_{1,}\mu_{2,}, \mu_{3,}, \dots \dots \dots, \mu_{20}$
8	$\omega_{1,}\omega_{2,}, \omega_{3,}, \dots \dots \dots, \omega_{40}$

10	$\lambda_1,\lambda_2,\lambda_3,\dots\dots\dots,\lambda_{24}$
12	$\eta_1,\eta_2,\eta_3,\dots\dots\dots,\eta_{40}$
20	$\xi_1,\xi_2,\xi_3,\dots\dots\dots,\xi_{48}$
24	$\rho_1,\rho_2,\rho_3,\dots\dots\dots,\rho_{80}$

4. Subgroups of Gof various orders

In this section we find all the subgroups of G of various orders. Based on Lagrange's theorem, we have to lookonly among the divisors of 480 for identifying the subgroups of G.

4.1 Subgroups of G which have order 2

Let A denote an arbitrary subgroup of G which has order 2. Then all the subgroups of order 2 are $A_1=\{e,\alpha_1\}$, $A_2=\{e,\alpha_2\}$, … …… ...$A_{31}=\{e,\alpha_{31}\}$.

4.2 Subgroupsof G which have order 3

Since $o(G) = 2^5 \times 3 \times 5$, $3 \mid o(G)$ and $3^2 \nmid o(G)$, by Sylow's theorem, G has a 3- Sylow subgroup which has order 3. Hence, the number of 3 – Sylow subgroups of G is of the form 1+3m and we have $1+3m \mid o(G)$.

That is, $1+3m \mid 2^5 \times 3 \times 5$. Then, $1+3m \mid 2^5 \times 5$. Therefore, the probable values for m = 0, 1, 3, 5,13. As the number of elements of order 3 is 20, we have atmost ten 3-Sylow subgroups corresponding to m = 3. The subgroups are

$B_1=\{e,\beta_1,\beta_{20}\}$, $B_2=\{e,\beta_2,\beta_{19}\}$, $B_3=\{e,\beta_3,\beta_{18}\}$, $B_4=\{e,\beta_4,\beta_{17}\}$, $B_5=\{e,\beta_5,\beta_{16}\}$, $B_6=\{e,\beta_6,\beta_{15}\}$, $B_7=\{e,\beta_7,\beta_{14}\}$, $B_8=\{e,\beta_8,\beta_{13}\}$, $B_9=\{e,\beta_9,\beta_{12}\}$, $B_{10}=\{e,\beta_{10},\beta_{11}\}$.

4.3 Subgroups of G which have order 4

Consider an arbitrary subgroup C of G which has order 4. Then C consists of elements of orders 1, 2 or 4. If C consists of an element which has order 4, then C is generated by that element. We get the following subgroups of order4.

$C_1 = \{e,\alpha_1,\alpha_4,\alpha_{23}\}$, $C_2 = \{e,\alpha_2,\alpha_3,\alpha_{23}\}$, $C_3 = \{e,\alpha_5,\alpha_{22},\alpha_{23}\}$, $C_4 = \{e,\alpha_6,\alpha_{23},\alpha_{27}\}$,$C_5 = \{e,\alpha_7,\alpha_{23},\alpha_{26}\}$,$C_6 = \{e,\alpha_8,\alpha_{23},\alpha_{25}\}$,$C_7 = \{e,\alpha_9,\alpha_{23},\alpha_{24}\}$, $C_8 = \{e,\alpha_{10},\alpha_{23},\alpha_{31}\}$, $C_9 = \{e,\alpha_{11},\alpha_{23},\alpha_{30}\}$, $C_{10} = \{e,\alpha_{12},\alpha_{23},\alpha_{29}\}$, $C_{11} = \{e,\alpha_{13},\alpha_{23},\alpha_{28}\}$, $C_{12} = \{e,\alpha_{14},\alpha_{21},\alpha_{23}\}$, $C_{13} = \{e,\alpha_{15},\alpha_{20},\alpha_{23}\}$, $C_{14} = \{e,\alpha_{16},\alpha_{19},\alpha_{23}\}$, $C_{15} = \{e,\alpha_{17},\alpha_{18},\alpha_{23}\}$, $C_{16} = \{e,\alpha_{14},\gamma_1,\gamma_{80}\}$, $C_{17} = \{e,\alpha_{17},\gamma_2,\gamma_{116}\}$,$C_{18} = \{e,\alpha_{19},\gamma_3,\gamma_{38}\}$,$C_{19} = \{e,\alpha_{20},\gamma_4,\gamma_{140}\}$, $C_{20} = \{e,\alpha_{23},\gamma_5,\gamma_{16}\}$, $C_{21} = \{e,\alpha_{15},\gamma_6,\gamma_{71}\}$, $C_{22} = \{e,\alpha_{16},\gamma_7,\gamma_{107}\}$,$C_{23} = \{e,\alpha_{23},\gamma_8,\gamma_{13}\}$, $C_{24} = \{e,\alpha_{21},\gamma_9,\gamma_{50}\}$, $C_{25} = \{e,\alpha_{18},\gamma_{10},\gamma_{152}\}$, $C_{26} = \{e,\alpha_{18},\gamma_{11},\gamma_{33}\}$, $C_{27} = \{e,\alpha_{21},\gamma_{12},\gamma_{135}\}$, $C_{28} = \{e,\alpha_{16},\gamma_{14},\gamma_{85}\}$, $C_{29} = \{e,\alpha_{15},\gamma_{15},\gamma_{121}\}$, $C_{30} = \{e,\alpha_{20},\gamma_{17},\gamma_{45}\}$,$C_{31} = \{e,\alpha_{19},\gamma_{18},\gamma_{147}\}$, $C_{32} = \{e,\alpha_{17},\gamma_{19},\gamma_{76}\}$, $C_{33} = \{e,\alpha_{14},\gamma_{20},\gamma_{112}\}$,

$C_{34} = \{e, \alpha_5, \gamma_{21}, \gamma_{22}\}$, $C_{35} = \{e, \alpha_8, \gamma_{23}, \gamma_{26}\}$, $C_{36} = \{e, \alpha_9, \gamma_{24}, \gamma_{27}\}$, $C_{37} = \{e, \alpha_6, \gamma_{25}, \gamma_{30}\}$, $C_{38} = \{e, \alpha_7, \gamma_{28}, \gamma_{29}\}$, $C_{39} = \{e, \alpha_{12}, \gamma_{31}, \gamma_{36}\}$, $C_{40} = \{e, \alpha_{23}, \gamma_{34}, \gamma_{151}\}$, $C_{41} = \{e, \alpha_2, \gamma_{35}, \gamma_{111}\}$, $C_{42} = \{e, \alpha_{10}, \gamma_{36}, \gamma_{47}\}$, $C_{43} = \{e, \alpha_4, \gamma_{39}, \gamma_{122}\}$, $C_{44} = \{e, \alpha_{23}, \gamma_{40}, \gamma_{145}\}$, $C_{45} = \{e, \alpha_{13}, \gamma_{41}, \gamma_{32}\}$, $C_{46} = \{e, \alpha_{11}, \gamma_{42}, \gamma_{46}\}$, $C_{47} = \{e, \alpha_{23}, \gamma_{43}, \gamma_{142}\}$, $C_{48} = \{e, \alpha_1, \gamma_{44}, \gamma_{106}\}$, $C_{49} = \{e, \alpha_3, \gamma_{48}, \gamma_{117}\}$, $C_{50} = \{e, \alpha_{23}, \gamma_{49}, \gamma_{136}\}$, $C_{51} = \{e, \alpha_{22}, \gamma_{51}, \gamma_{87}\}$, $C_{52} = \{e, \alpha_{23}, \gamma_{52}, \gamma_{89}\}$, $C_{53} = \{e, \alpha_{23}, \gamma_{53}, \gamma_{88}\}$, $C_{54} = \{e, \alpha_{22}, \gamma_{54}, \gamma_{90}\}$, $C_{55} = \{e, \alpha_{26}, \gamma_{55}, \gamma_{94}\}$, $C_{56} = \{e, \alpha_{23}, \gamma_{56}, \gamma_{101}\}$, $C_{57} = \{e, \alpha_{24}, \gamma_{57}, \gamma_{99}\}$, $C_{58} = \{e, \alpha_{24}, \gamma_{58}, \gamma_{100}\}$, $C_{59} = \{e, \alpha_{23}, \gamma_{59}, \gamma_{98}\}$, $C_{60} = \{e, \alpha_{25}, \gamma_{60}, \gamma_{93}\}$, $C_{61} = \{e, \alpha_{27}, \gamma_{61}, \gamma_{91}\}$, $C_{62} = \{e, \alpha_{23}, \gamma_{62}, \gamma_{95}\}$, $C_{63} = \{e, \alpha_{26}, \gamma_{63}, \gamma_{102}\}$, $C_{64} = \{e, \alpha_{25}, \gamma_{64}, \gamma_{97}\}$, $C_{65} = \{e, \alpha_{23}, \gamma_{65}, \gamma_{92}\}$, $C_{66} = \{e, \alpha_{27}, \gamma_{66}, \gamma_{96}\}$, $C_{67} = \{e, \alpha_{30}, \gamma_{67}, \gamma_{108}\}$, $C_{68} = \{e, \alpha_{23}, \gamma_{68}, \gamma_{119}\}$, $C_{69} = \{e, \alpha_{28}, \gamma_{69}, \gamma_{115}\}$, $C_{70} = \{e, \alpha_4, \gamma_{70}, \gamma_{146}\}$, $C_{71} = \{e, \alpha_{28}, \gamma_{69}, \gamma_{115}\}$, $C_{72} = \{e, \alpha_{23}, \gamma_{73}, \gamma_{114}\}$, $C_{73} = \{e, \alpha_{29}, \gamma_{74}, \gamma_{105}\}$, $C_{74} = \{e, \alpha_3, \gamma_{75}, \gamma_{137}\}$, $C_{75} = \{e, \alpha_{31}, \gamma_{77}, \gamma_{103}\}$, $C_{76} = \{e, \alpha_{23}, \gamma_{78}, \gamma_{109}\}$, $C_{77} = \{e, \alpha_{30}, \gamma_{79}, \gamma_{120}\}$, $C_{78} = \{e, \alpha_2, \gamma_{81}, \gamma_{150}\}$, $C_{79} = \{e, \alpha_{29}, \gamma_{82}, \gamma_{113}\}$, $C_{80} = \{e, \alpha_{23}, \gamma_{83}, \gamma_{104}\}$, $C_{81} = \{e, \alpha_{31}, \gamma_{84}, \gamma_{110}\}$, $C_{82} = \{e, \alpha_1, \gamma_{86}, \gamma_{141}\}$, $C_{83} = \{e, \alpha_5, \gamma_{123}, \gamma_{124}\}$, $C_{84} = \{e, \alpha_6, \gamma_{125}, \gamma_{130}\}$, $C_{85} = \{e, \alpha_7, \gamma_{126}, \gamma_{127}\}$, $C_{86} = \{e, \alpha_9, \gamma_{128}, \gamma_{131}\}$, $C_{87} = \{e, \alpha_8, \gamma_{129}, \gamma_{132}\}$, $C_{88} = \{e, \alpha_{10}, \gamma_{133}, \gamma_{144}\}$, $C_{89} = \{e, \alpha_{11}, \gamma_{134}, \gamma_{138}\}$, $C_{90} = \{e, \alpha_{13}, \gamma_{139}, \gamma_{148}\}$, $C_{91} = \{e, \alpha_{12}, \gamma_{143}, \gamma_{149}\}$.

Here the last 76 subgroups are cyclic and the remaining 15 subgroups are non-cyclic.

4.4 Subgroups of G which have order 6

Let E denote an arbitrary subgroup of G which has order 6. Since o(E) = 2 x 3, by Sylow's theorem E has only one subgroup of order 3. Further, if E consists of an element which has order 6, then E is generated by an element of order 6. Then the subgroups of order 6 are

$$E_1 = \{e, \alpha_1, \alpha_9, \alpha_{31}, \beta_1, \beta_{20}\},$$
$$E_2 = \{e, \alpha_4, \alpha_{10}, \alpha_{24}, \beta_1, \beta_{20}\}$$

$$E_3 = \{e, \alpha_2, \alpha_7, \alpha_{30}, \beta_2, \beta_{19}\}$$

$$E_4 = \{e, \alpha_3, \alpha_{11}, \alpha_{26}, \beta_2, \beta_{19}\}$$

$$E_5 = \{e, \alpha_2, \alpha_{12}, \alpha_{25}, \beta_3, \beta_{18}\}$$

$$E_6 = \{e, \alpha_3, \alpha_8, \alpha_{29}, \beta_3, \beta_{18}\}$$

$$E_7 = \{e, \alpha_1, \alpha_{13}, \alpha_{27}, \beta_4, \beta_{17}\}$$

$$E_8 = \{e, \alpha_4, \alpha_6, \alpha_{28}, \beta_4, \beta_{17}\}$$

$$E_9 = \{e, \alpha_7, \alpha_{10}, \alpha_{21}, \beta_5, \beta_{16}\}$$

$$E_{10} = \{e, \alpha_{14}, \alpha_{26}, \alpha_{31}, \beta_5, \beta_{16}\}$$

$$E_{11} = \{e, \alpha_6, \alpha_{11}, \alpha_{20}, \beta_6, \beta_{15}\}$$

$$E_{12} = \{e, \alpha_{15}, \alpha_{27}, \alpha_{30}, \beta_6, \beta_{15}\}$$

$$E_{13} = \{e, \alpha_9, \alpha_{12}, \alpha_{19}, \beta_7, \beta_{14}\}$$

$$E_{14} = \{e, \alpha_{16}, \alpha_{24}, \alpha_{29}, \beta_7, \beta_{14}\}$$

$$E_{15} = \{e, \alpha_8, \alpha_{13}, \alpha_{18}, \beta_8, \beta_{13}\}$$

$$E_{16} = \{e, \alpha_{17}, \alpha_{25}, \alpha_{28}, \beta_8, \beta_{13}\}$$

$$E_{17} = \{e, \alpha_5, \alpha_{14}, \alpha_{17}, \beta_9, \beta_{12}\}$$

$$E_{18} = \{e, \alpha_{18}, \alpha_{21}, \alpha_{22}, \beta_9, \beta_{12}\}$$

$$E_{19} = \{e, \alpha_5, \alpha_{15}, \alpha_{36}, \beta_{10}, \beta_{11}\}$$

$$E_{20} = \{e, \alpha_{19}, \alpha_{20}, \alpha_{22}, \beta_{10}, \beta_{11}\}$$

$$E_{21} = \{e, \alpha_{23}, \beta_1, \beta_{20}, \mu_4, \mu_5\}$$

$$E_{22} = \{e, \alpha_{23}, \beta_2, \beta_{19}, \mu_3, \mu_6\}$$

$$E_{23} = \{e, \alpha_{23}, \beta_3, \beta_{18}, \mu_2, \mu_7\}$$

$$E_{24} = \{e, \alpha_{23}, \beta_4, \beta_{17}, \mu_1, \mu_8\}$$

$$E_{25} = \{e, \alpha_{23}, \beta_5, \beta_{16}, \mu_9, \mu_{20}\}$$

$$E_{26} = \{e, \alpha_{23}, \beta_6, \beta_{15}, \mu_{10}, \mu_{19}\}$$

$$E_{27} = \{e, \alpha_{23}, \beta_7, \beta_{14}, \mu_{11}, \mu_{18}\}$$

$$E_{28} = \{e, \alpha_{23}, \beta_8, \beta_{13}, \mu_{12}, \mu_{17}\}$$

$$E_{29} = \{e, \alpha_{23}, \beta_9, \beta_{12}, \mu_{13}, \mu_{16}\}$$

$$E_{30} = \{e, \alpha_{23}, \beta_{10}, \beta_{11}, \mu_{14}, \mu_{15}\}$$

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Here each of the last ten subgroups has two elements of order 6 and find that all the 20 elements of order 6 have been taken care of and we note that every subgroup of order 6 contains exactly two elements of order 3. Then there is no other possibility for any other subgroups.

4.5 Subgroups of G which have order 12

Consider an arbitrary subgroup I of G of order 12 and o(I)=12=2^2x3. By Sylow's theorem, I has a 3-Sylow subgroup which has order 3. The number of 3-Sylow subgroup of order 3 is 1+3m and we have 1+3m | 2^2x3. Then 1+3m | 2^2. The probable values for m are 0,1. Hence, the number of subgroups of I of order 3 is either 1 or 4.

Also,I has a 2-Sylow subgroups which has order 4. The number of 2- Sylow subgroups of order 4 is 1+2m and 1+2m | 3. The probable values for mare 0,1. Hence the number of subgroups of I of order 4 is either 1 or 3.

Four cases arise:

i. Four subgroups of order 3 and three subgroups of order 4.
ii. One subgroup of order 3 and three subgroups of order 4.
iii. Only one subgroup of order 3 and one subgroup of order 4.
iv. Four subgroups of order 3 and one subgroup of order 4.

Case(i): Will not exist, since containing 4 subgroups of order 3 and three subgroups of order 4 exceeds 12 elements.

Case(ii): At a time, combining a subgroup of order 3 combining with three subgroups of order 4, we get the subgroups of order 12 as given below:

$$I_1 = \{e, \alpha_1, \alpha_4, \alpha_9, \alpha_{10}, \alpha_{23}, \alpha_{24}, \alpha_{31}, \beta_1, \beta_{20}, \mu_4, \mu_5\}\}$$

$$I_2 = \{e, \alpha_2, \alpha_3, \alpha_7, \alpha_{11}, \alpha_{23}, \alpha_{26}, \alpha_{30}, \beta_2, \beta_{19}, \mu_3, \mu_6\}$$

$$I_3 = \{e, \alpha_2, \alpha_3, \alpha_8, \alpha_{12}, \alpha_{23}, \alpha_{25}, \alpha_{29}, \beta_3, \beta_{18}, \mu_2, \mu_7\}$$

$$I_4 = \{e, \alpha_1, \alpha_4, \alpha_6, \alpha_{13}, \alpha_{23}, \alpha_{27}, \alpha_{28}, \beta_4, \beta_{17}, \mu_1, \mu_8\}$$

$$I_5 = \{e, \alpha_7, \alpha_{10}, \alpha_{14}, \alpha_{21}, \alpha_{23}, \alpha_{26}, \alpha_{31}, \beta_5, \beta_{16}, \mu_9, \mu_{20}\}$$

$$I_6 = \{e, \alpha_6, \alpha_{11}, \alpha_{15}, \alpha_{20}, \alpha_{23}, \alpha_{27}, \alpha_{30}, \beta_6, \beta_{15}, \mu_{10}, \mu_{19}\}$$

$$I_7 = \{e, \alpha_9, \alpha_{12}, \alpha_{16}, \alpha_{19}, \alpha_{23}, \alpha_{24}, \alpha_{29}, \beta_7, \beta_{14}, \mu_{11}, \mu_{18}\}$$

$$I_8 = \{e, \alpha_8, \alpha_{13}, \alpha_{17}, \alpha_{18}, \alpha_{23}, \alpha_{25}, \alpha_{28}, \beta_8, \beta_{13}, \mu_{12}, \mu_{17}\}$$

$$I_9 = \{e, \alpha_5, \alpha_{14}, \alpha_{17}, \alpha_{18}, \alpha_{21}, \alpha_{22}, \alpha_{23}, \beta_9, \beta_{12}, \mu_{13}, \mu_{16}\}$$

$$I_{10} = \{e, \alpha_5, \alpha_{15}, \alpha_{16}, \alpha_{19}, \alpha_{20}, \alpha_{22}, \alpha_{23}, \beta_{10}, \beta_{11}, \mu_{14}, \mu_{15}\}$$

$$I_{11} = \{e, \alpha_{23}, \beta_1, \beta_{20}, \gamma_8, \gamma_{13}, \gamma_{62}, \gamma_{73}, \gamma_{95}, \gamma_{114}, \mu_4, \mu_5\}$$

$$I_{12} = \{e, \alpha_{23}, \beta_2, \beta_{19}, \gamma_5, \gamma_{16}, \gamma_{65}, \gamma_{83}, \gamma_{92}, \gamma_{104}, \mu_3, \mu_6\}$$

$$I_{13} = \{e, \alpha_{23}, \beta_3, \beta_{18}, \gamma_5, \gamma_{16}, \gamma_{56}, \gamma_{68}, \gamma_{101}, \gamma_{119}, \mu_2, \mu_7\}$$

$$I_{14} = \{e, \alpha_{23}, \beta_4, \beta_{17}, \gamma_8, \gamma_{13}, \gamma_{59}, \gamma_{78}, \gamma_{98}, \gamma_{109}, \mu_1, \mu_8\}$$

$$I_{15} = \{e, \alpha_{23}, \beta_5, \beta_{16}, \gamma_{43}, \gamma_{65}, \gamma_{73}, \gamma_{92}, \gamma_{114}, \gamma_{142}, \mu_9, \mu_{20}\}$$

$$I_{16} = \{e, \alpha_{23}, \beta_6, \beta_{15}, \gamma_{34}, \gamma_{59}, \gamma_{83}, \gamma_{98}, \gamma_{104}, \gamma_{151}, \mu_{10}, \mu_{19}\}$$

$$I_{17} = \{e, \alpha_{23}, \beta_7, \beta_{14}, \gamma_{49}, \gamma_{62}, \gamma_{68}, \gamma_{95}, \gamma_{119}, \gamma_{136}, \mu_{11}, \mu_{18}\}$$

$$I_{18} = \{e, \alpha_{23}, \beta_8, \beta_{13}, \gamma_{40}, \gamma_{56}, \gamma_{78}, \gamma_{101}, \gamma_{109}, \gamma_{145}, \mu_{12}, \mu_{17}\}$$

$$I_{19} = \{e, \alpha_{23}, \beta_9, \beta_{12}, \gamma_{40}, \gamma_{43}, \gamma_{53}, \gamma_{88}, \gamma_{142}, \gamma_{145}, \mu_{13}, \mu_{16}\}$$

$$I_{20} = \{e, \alpha_{23}, \beta_{10}, \beta_{11}, \gamma_{34}, \gamma_{49}, \gamma_{53}, \gamma_{88}, \gamma_{136}, \gamma_{151}, \mu_{14}, \mu_{15}\}$$

Case(iii): Let B be a one subgroup of order 3 and C bea one subgroup which has order 4 in I. But B and C are normal subgroups in I. Therefore I=BCshould be abelian. But we find thatBC$_{52}$ only in abelian and for no other C. Hence,in this case we find the following subgroups of order12.

$$I_{21} = \{e, \alpha_{23}, \beta_1, \beta_{20}, \gamma_{52}, \gamma_{89}, \mu_4, \mu_5, \eta_4, \eta_5, \eta_{19}, \eta_{30}\}$$

$$I_{22} = \{e, \alpha_{23}, \beta_2, \beta_{19}, \gamma_{52}, \gamma_{89}, \mu_3, \mu_6, \eta_1, \eta_8, \eta_{24}, \eta_{25}\}$$

$$I_{23} = \{e, \alpha_{23}, \beta_3, \beta_{18}, \gamma_{52}, \gamma_{89}, \mu_2, \mu_7, \eta_2, \eta_7, \eta_{17}, \eta_{32}\}$$

$$I_{24} = \{e, \alpha_{23}, \beta_4, \beta_{17}, \gamma_{52}, \gamma_{89}, \mu_1, \mu_8, \eta_3, \eta_6, \eta_{22}, \eta_{27}\}$$

$$I_{25} = \{e, \alpha_{23}, \beta_5, \beta_{16}, \gamma_{52}, \gamma_{89}, \mu_9, \mu_{20}, \eta_{13}, \eta_{20}, \eta_{29}, \eta_{36}\}$$

$$I_{26} = \{e, \alpha_{23}, \beta_6, \beta_{15}, \gamma_{52}, \gamma_{89}, \mu_{10}, \mu_{19}, \eta_{10}, \eta_{23}, \eta_{26}, \eta_{39}\}$$

$$I_{27} = \{e, \alpha_{23}, \beta_7, \beta_{14}, \gamma_{52}, \gamma_{89}, \mu_{11}, \mu_{18}, \eta_{15}, \eta_{18}, \eta_{31}, \eta_{34}\}$$

$$I_{28} = \{e, \alpha_{23}, \beta_8, \beta_{13}, \gamma_{52}, \gamma_{89}, \mu_{12}, \mu_{17}, \eta_{12}, \eta_{21}, \eta_{28}, \eta_{37}\}$$

$$I_{29} = \{e, \alpha_{23}, \beta_9, \beta_{12}, \gamma_{52}, \gamma_{89}, \mu_{13}, \mu_{16}, \eta_{11}, \eta_{14}, \eta_{35}, \eta_{38}\}$$

$$I_{30} = \{e, \alpha_{23}, \beta_{10}, \beta_{11}, \gamma_{52}, \gamma_{89}, \mu_{14}, \mu_{15}, \eta_9, \eta_{16}, \eta_{33}, \eta_{40}\}$$

Case(iv): Let $\boldsymbol{\mathcal{B}}$ be a collection of four number of subgroups of order 3. Consider C be a subgroup which has order 4.Clearly, C is a normal subgroup of I, because C is the only one subgroup in I of order 4. Hence, $iri^{-1} \in I$ for every $r \in C, i \in$ I. At a time, consider a subgroup of order 4 combining it with four subgroups of order 3,we find that this condition is not true. Hence, this case does not arise at all.

5. Structure of intervals [{e}, I_i], i=1,2,3,…20, [{e}, I_j], j=21,22, …30 in L(G).

Each of the first 10 subgroups I_i, i= 1.2, …10 in case(ii) above contains 3 subgroups of order 4 and 3 subgroups of order 6 and the remaining ten subgroups I_i, i=11,12,…20 each contains three subgroups of order 4 and one subgroup of order 6.

Each subgroup I_j, j=21, 22, …30 in case (iii) contains the four -element subgroup C_{52} and exactly one subgroup of order 6. Therefore, the lattice structure of lower intervals below I_i's above are of three types namely, for $I_1, I_2, \ldots I_{10}$; $I_{11}, I_{12}, \ldots I_{20}$; $I_{21}, I_{22}, \ldots I_{30}$ which we typically display for I_1, I_{11}, I_{21}.

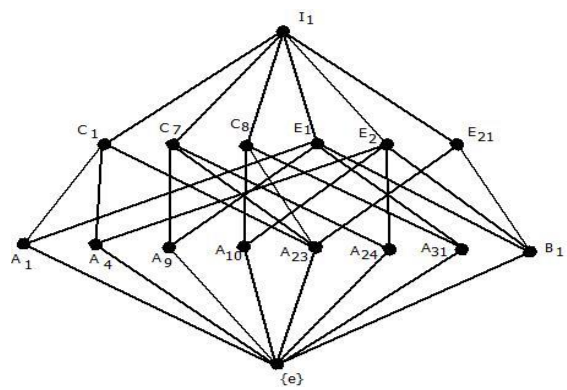

Figure 1

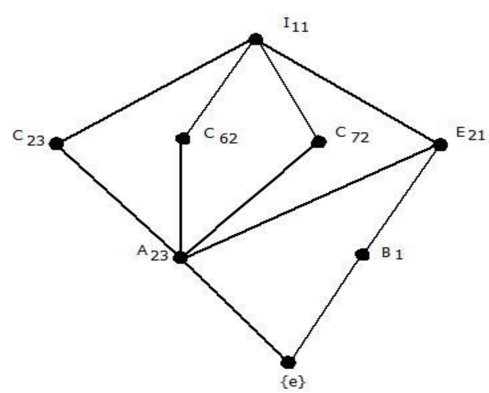

Figure 2

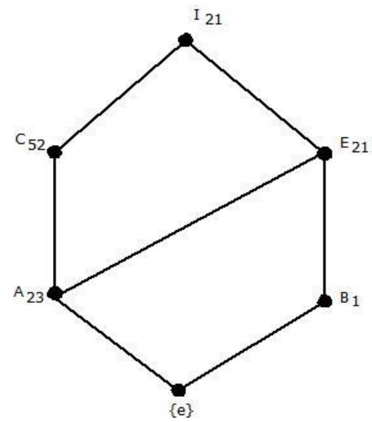

Figure 3

6. Conclusion

In this paper, we displayed the structure of lower intervals whose upper bounds are the12 element-subgroups only containing the elements of order 2, 4,6 or 12 in the lattice of subgroups of the group of 2x2 non-singular matrices over $\mathbf{Z}_5$. Similar, the display is also possible for subgroups of other orders.

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