

Insight into Quark Gluon Plasma Dynamics from Elliptic Flow of Heavy-Flavour Muons at the Large Hadron Collider

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Abstract

This study investigates the elliptic flow (v_2v_2) of heavy-flavour decay muons in Pb–Pb collisions at the Large Hadron Collider (LHC) as a probe into the properties and dynamics of the quark-gluon plasma (QGP). Heavy quarks (charm and bottom) are produced in the early stages of high-energy collisions and traverse the QGP, losing energy and participating in its collective motion. Measuring the anisotropic flow of heavy-flavour decay muons provides critical insights into the degree of thermalization and the transport properties of QGP. Data collected by the ALICE experiment at forward rapidity are analyzed to extract v_2v_2 using various methods. Results show a non-zero v_2v_2 for muons from heavy-flavour decays, suggesting strong interaction with the medium and supporting the collective behavior of the QGP.

Keywords: - Quark-Gluon Plasma, Heavy-Flavour Muons, Elliptic Flow (v_2v_2), LHC, ALICE Experiment, Anisotropic Flow, High-Energy Nuclear Collisions, Heavy-Ion Physics, QCD Matter, Forward Rapidity

Introduction

The study of strongly interacting matter under extreme conditions is one of the central aims of contemporary nuclear physics. Heavy-ion collisions at ultra-relativistic energies, such as those conducted at the LHC, provide the environment needed to recreate the early Universe's quark-gluon plasma. Elliptic flow, arising from the initial spatial anisotropy of non-central collisions, is a key observable in understanding the collective behavior of the QGP. The elliptic flow of heavy-flavour decay muons is particularly interesting because heavy quarks are produced early and probe the entire evolution of the QGP. Their interaction with the medium offers valuable insights into the QGP's viscosity and degree of thermalization. In the quest to understand the fundamental properties of matter under extreme conditions, the study of the **Quark-Gluon Plasma (QGP)** has emerged as a central theme in high-energy nuclear physics. The QGP is a state of deconfined quarks and gluons believed to have existed microseconds after the Big Bang, before hadronization formed the protons and neutrons that constitute ordinary matter. Recreating and studying this primordial phase of matter is made possible through **ultra-relativistic heavy-ion collisions**, such as those conducted at the **Large Hadron Collider (LHC)** at CERN.

Among the various detectors at the LHC, the **A Large Ion Collider Experiment (ALICE)** is specifically designed to investigate QGP properties. Through collisions of lead ions (Pb–Pb) at unprecedented energies, ALICE provides a unique platform to probe the strongly interacting matter formed during these events. A key observable emerging from such studies is **elliptic flow**, quantified by the second Fourier coefficient (v_2v_2) of the azimuthal particle distribution. This flow arises due to the pressure gradients in the initial spatial anisotropy of non-central collisions and reflects the collective expansion of the medium, thereby serving as an indicator of thermalization and fluid-like behavior of the QGP.

While early flow studies predominantly focused on light hadrons, the role of **heavy quarks**—namely charm and bottom (beauty) quarks—has become increasingly vital. Heavy quarks are primarily produced in the early stages of the collisions through hard scattering processes, and thus experience the full evolution of the QGP. As they

traverse the medium, they lose energy via both collisional and radiative mechanisms. By studying the azimuthal anisotropy of the particles they eventually decay into, researchers can infer the extent to which these heavy quarks interact with and become part of the collective flow of the medium.

One important experimental method of accessing heavy-flavour dynamics is through the detection of **muons from heavy-flavour hadron decays**, particularly at **forward rapidities** (i.e., regions away from the central collision zone). Muons are minimally affected by the strong nuclear medium and provide a clean probe of heavy-flavour production and propagation. The **ALICE Muon Spectrometer** is especially suited for this purpose, enabling the study of elliptic flow in a previously less explored kinematic region.

The study of heavy-flavour decay muon v_2v_2 at forward rapidity complements measurements at mid-rapidity and helps build a comprehensive picture of QGP dynamics. It allows researchers to test the **universality of flow phenomena** across different rapidity ranges and understand whether the collective behavior observed centrally is sustained throughout the medium. It also sheds light on important transport parameters of the QGP such as the **diffusion coefficient**, **drag**, and **energy loss rates**, which are critical for validating Quantum Chromodynamics (QCD)-based models and numerical simulations of QGP evolution.

Theoretical models, including **hydrodynamic simulations** and **Langevin-based transport approaches**, provide predictions for heavy-quark behavior and flow. These models often incorporate elements like initial state fluctuations, quark diffusion, hadronization via coalescence or fragmentation, and interactions with the medium constituents. A precise measurement of v_2v_2 of heavy-flavour decay muons enables testing these models and refining their parameters, particularly in the forward rapidity region, which is still a relatively under-explored domain in QGP physics.

Furthermore, the observed behavior of heavy-flavour particles in terms of energy loss and collective motion offers indirect evidence for the medium's **viscosity-to-entropy ratio** (η/s), one of the most crucial transport properties of the QGP. A low value of η/s close to the **Kovtun-Son-Starinets (KSS) bound** suggests that the QGP behaves almost like a perfect fluid, a result consistent with observations at both RHIC and LHC. Heavy-quark observables such as v_2v_2 and nuclear modification factors (R_{AA}) provide complementary insight into these characteristics.

This research delves deep into the measurement and interpretation of the **elliptic flow of heavy-flavour muons at forward rapidity in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV** recorded by ALICE. Using sophisticated flow analysis techniques such as the **event plane** and **scalar product methods**, and accounting for systematic effects like detector acceptance and background subtraction, the study extracts v_2v_2 values with high precision. The results are compared with theoretical predictions from models like **TAMU**, **EPOS**, and **MC@HQ**, allowing for a rigorous assessment of current QGP evolution theories.

Definitions

- **Quark-Gluon Plasma (QGP):** A state of matter in which quarks and gluons are deconfined from hadrons.
- **Elliptic Flow (v_2v_2):** The second Fourier coefficient of the azimuthal distribution of emitted particles, reflecting anisotropic collective expansion.
- **Heavy Flavour:** Quarks heavier than up and down—specifically charm and bottom.
- **Forward Rapidity:** A kinematic region at high pseudorapidity, where particle production is studied away from the central collision zone.

Need for the Study

Understanding QGP dynamics is critical to grasp the fundamental aspects of quantum chromodynamics (QCD) under extreme temperatures and densities. Studying heavy-flavour muon v_2v_2 provides complementary information to light-flavour hadron flow measurements, revealing how strongly heavy quarks interact with the QGP and testing the limits of hydrodynamic behavior.

Aims of the Study

- To evaluate the elliptic flow (v_2v_2) of heavy-flavour decay muons in Pb–Pb collisions.
- To understand the extent of heavy-quark thermalization in the QGP.
- To provide experimental input for refining QGP transport models.

Objectives

1. Analyze Pb–Pb collision data at $\sqrt{s_{NN}} = 5.02$ TeV from ALICE.
2. Extract v_2v_2 of heavy-flavour muons at forward rapidity using event plane and scalar product methods.
3. Compare results with theoretical models (e.g., hydrodynamics, transport models).
4. Correlate elliptic flow trends with collision centrality and transverse momentum.

Hypothesis

Heavy-flavour decay muons exhibit a non-zero elliptic flow at forward rapidity in Pb–Pb collisions, indicating significant interaction with the QGP and collective behavior consistent with hydrodynamic flow.

Literature Search

Recent studies from ALICE, CMS, and ATLAS have shown consistent v_2v_2 signals for heavy-flavour decay electrons and D mesons. Theoretical models like Langevin dynamics and hydrodynamic simulations predict a moderate but measurable elliptic flow for heavy quarks. However, forward rapidity results are limited, and this study seeks to fill that gap using ALICE's unique muon spectrometer capabilities.

Key sources:

- ALICE Collaboration, *JHEP*, *Phys. Rev. Lett.*, *Phys. Rev. C*
- Rapp et al. (transport models)
- Flow harmonics from RHIC and LHC data

Research Methodology

- **Data Sample:** Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV from the ALICE detector.
- **Detector Subsystems Used:** Muon Spectrometer, V0 detectors for event plane estimation.
- **Analysis Techniques:** Event Plane Method, Scalar Product Method.
- **Corrections Applied:** Detector acceptance, background subtraction, efficiency.
- **Systematic Uncertainties:** Evaluated by varying analysis cuts and comparing methods.
- **Theoretical Comparisons:** With TAMU, MC@sHQ, and EPOS models.

Strong Points of Present Research Study

1. Novelty of the Kinematic Region: Forward Rapidity Focus

- The study investigates **elliptic flow (v_2v_2) of heavy-flavour decay muons** specifically at **forward rapidity**, a relatively under-explored region in QGP studies.
- Most prior flow measurements have been centered at mid-rapidity; this focus provides **complementary insights** into QGP evolution at different longitudinal positions.

2. Use of Muons as Clean Probes

- Muons are **minimally affected by final-state hadronic interactions** and can traverse the medium with

reduced scattering compared to hadrons or electrons.

- This allows for a **clean extraction of heavy-flavour decay dynamics**, reducing background and increasing confidence in the results.

3. Contribution to Heavy-Quark Thermalization Understanding

- Heavy quarks, due to their large masses, are produced early and serve as excellent probes for the **entire lifespan of the QGP**.
- Measuring their elliptic flow helps determine the **extent to which they participate in the collective expansion**, offering insights into the degree of **thermalization** of charm and bottom quarks.

4. Relevance to QCD Transport Properties

- The v_2v_2 measurements of heavy-flavour muons help **constrain theoretical models** regarding:
 - **Drag and diffusion coefficients**
 - **Energy loss mechanisms (collisional and radiative)**
 - **Viscosity-to-entropy ratio (η/s)**
- This directly supports precision testing of **Quantum Chromodynamics (QCD)** under extreme conditions.

5. Validation of Theoretical Models

- The data is compared with state-of-the-art theoretical predictions from models like **TAMU, MC@sHQ, EPOS**, and **Langevin transport simulations**.
- The good agreement or disagreement with models helps **validate or refine key model parameters**, making this study valuable for theorists.

6. Sophisticated Flow Analysis Techniques

- The study employs robust and **well-established flow analysis methods**:
 - **Event Plane Method**
 - **Scalar Product Method**
- These multiple techniques allow for **cross-validation** of results and reduce method-dependent uncertainties.

7. High Statistical Precision

- The research benefits from **large data samples** collected during **LHC Run 2** (Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV).
- This improves the **statistical accuracy** of the measured v_2v_2 values and allows for fine-grained studies across different **centrality** and **transverse momentum (p_{Tp_TpT}) bins**.

8. Interdisciplinary Relevance

- The outcomes have implications beyond nuclear physics, including:
 - **Cosmology**: Conditions analogous to the early universe.
 - **Condensed Matter Physics**: Fluid dynamics, phase transitions, and collective behavior.
 - **Plasma Physics**: Strongly coupled plasma behavior.

9. Unique Capabilities of the ALICE Muon Spectrometer

- ALICE's **Muon Spectrometer** offers high-resolution measurement of muons in the range $2.5 < y < 42.5 <$

$$y < 42.5 < y < 4.$$

- Its **trigger and tracking system** enable high-purity muon reconstruction and **clear identification of heavy-flavour decay sources**, distinguishing them from background contributions.

10. Contribution to Global QGP Research

- Adds valuable data to the **global effort in understanding QGP** properties, complementing mid-rapidity heavy-flavour measurements from CMS and ATLAS.
- Enhances the **rich heavy-ion physics program** at LHC, strengthening international collaboration and comparative analysis.

11. Supports Multi-Differential Flow Studies

- The research provides **centrality-differentiated and pT_T-differentiated flow measurements**, which allow for precise mapping of the medium response under various initial geometries and expansion conditions.
- These results are useful for **flow harmonics studies**, especially for understanding collective behavior at various scales.

12. Cross-Validation with Other Probes

- Results can be correlated with:
 - **D-meson v₂^{v2}**
 - **Non-prompt J/ψ/ψ⁰ measurements**
 - **Heavy-flavour electron v₂^{v2}**
- These comparisons allow a **multi-channel validation** of heavy-quark flow behavior.

13. Deep Theoretical Significance

- The results help test several important QCD-based hypotheses:
 - Whether heavy quarks approach thermal equilibrium.
 - Whether they experience significant collective flow.
 - The dominance of collisional versus radiative energy loss.
 - Flavor dependence of flow in the QGP.

14. Potential to Impact Future Experimental Design

- Based on this study's results and methodology, **future upgrades** to ALICE (e.g., Muon Forward Tracker) and data-taking strategies can be optimized for:
 - Better **charm–bottom separation**
 - **Higher statistics**
 - Enhanced **muon-trigger efficiency**

15. Contribution to the Global Heavy-Flavour Working Groups

- The findings will serve as an important input for:
 - **CERN Heavy-Ion Physics Review Panels**
 - **QCD Matter Working Groups**
 - **Joint theory-experiment QGP initiatives**, especially those focused on **transport model benchmarking**

Weak Points of Present Research Study

1. Inability to Distinguish Charm and Bottom Contributions

- The muons detected originate from **both charm and bottom hadron decays**, but the current analysis **cannot separate** these contributions precisely.
- This limits the ability to study **mass-dependent flow patterns**, which are important for understanding quark-medium interactions.

2. Limited Rapidity Coverage

- The study is restricted to **forward rapidity** ($2.5 < y < 42.5$), which excludes mid-rapidity where many comparative measurements (e.g., D-meson flow) exist.
- This creates challenges in **cross-referencing results across different kinematic regions** and developing a unified picture.

3. Reduced Particle Yields at Forward Rapidity

- Particle multiplicities decrease at higher rapidities, resulting in **lower statistics and increased statistical uncertainties**.
- Some centrality or p_{Tp_TpT} bins may be poorly populated, affecting the reliability of flow extraction in those regions.

4. Absence of Direct Muon Source Tagging

- The measured muons include **background from light-flavour decays and hadronic contamination**, which can bias the $v_2^{v_2}$ measurement.
- Although background subtraction is performed, **residual contamination** introduces **systematic uncertainties**.

5. Dependence on Event Plane Resolution

- The **event plane method** depends on the accurate estimation of the reaction plane via detectors like V0.
- Poor event plane resolution, especially in **central and peripheral collisions**, may result in **underestimation or overestimation** of the true elliptic flow.

6. Model Dependence in Theoretical Comparisons

- The comparison with theoretical models (e.g., TAMU, MC@sHQ) involves **assumptions about initial conditions, hadronization mechanisms, and energy loss processes**.
- Discrepancies between model predictions and experimental data can be difficult to interpret due to these **model-dependent uncertainties**.

7. Limited Sensitivity to Early-Time QGP Dynamics

- While heavy quarks are produced early, muons are observed **only after hadronization and decay**.
- This limits direct sensitivity to **early-stage QGP evolution**, including pre-equilibrium dynamics and initial gluon fields (e.g., glasma effects).

8. Uncertainty in Muon p_{Tp_TpT} Correlation with Parent Heavy-Flavour Hadron

- The detected muon transverse momentum ($p_{T\mu p_T^{\mu p} T_\mu}$) does not directly correspond to the parent hadron or quark p_{Tp_TpT} , leading to **kinematic smearing**.
- This complicates the interpretation of results in terms of quark-level physics.

9. Lack of Flavor-Tagged Observables

- Without secondary vertex tracking in the muon arm, the experiment cannot isolate observables like:
 - **Prompt D-mesons**
 - **Non-prompt J/ψ / J/ψ from B decays**
 - **Direct charm versus bottom muon separation**
- This reduces the **granularity** and **specificity** of the heavy-flavour analysis.

10. Limited Temporal Resolution

- The study observes **final-state decay products**, but **cannot resolve the QGP temporal evolution**, such as:
 - Duration of QGP phase
 - Time dependence of energy loss
 - Quark hadronization chronology

11. Systematic Uncertainty from Detector Efficiency and Acceptance

- Detector performance variations across runs, centrality classes, and rapidity bins contribute to **non-negligible systematic uncertainties**.
- Correcting for these effects relies on simulation models, introducing potential **bias or mismodeling risks**.

12. Insufficient Sensitivity to Higher-Order Flow Harmonics

- The study focuses solely on the second-order harmonic (v_2v_2), neglecting:
 - **Triangular flow (v_3v_3)**
 - **Quadrangular flow (v_4v_4)**
- These higher harmonics contain important information about **initial-state fluctuations** and **viscous effects**.

13. Limited Use of Multivariate or Machine Learning Techniques

- The analysis employs traditional cut-based and scalar methods.
- Advanced tools like **machine learning classifiers or regression models** could improve:
 - Signal/background separation
 - Charm-bottom discrimination
 - Systematic uncertainty modeling

14. Challenges in Interpreting Non-Zero v_2v_2

- A non-zero v_2v_2 could arise from multiple effects:
 - Hydrodynamic flow
 - Path-length dependent energy loss
 - Initial state correlations
- Without additional discriminating observables, it's **difficult to unambiguously attribute the measured v_2v_2** to a specific physical mechanism.

15. Absence of Flow Correlation Measurements

- The study does not explore **flow correlations** (e.g., muon–hadron correlations, event shape engineering).
- Such analyses could **strengthen the interpretation** of collective behavior and isolate jet-like or non-flow effects.

16. Limited Comparative Analysis Across Collision Systems

- The focus is solely on **Pb–Pb collisions** at a single energy ($\sqrt{s_{NN}} = 5.02$ TeV).
- Inclusion of **p–Pb or Xe–Xe systems** would allow investigation of system-size dependence and collectivity onset.

17. Restricted Impact on Initial State Physics

- Since muons stem from heavy-flavour hadron decays, they are **less sensitive to initial nuclear effects**, such as:
 - Nuclear parton distribution functions (nPDFs)
 - Shadowing
 - Saturation (Color Glass Condensate)
- Thus, the study mainly probes **final-state effects**, not initial-state modifications.

18. No Direct Probe of Medium Opacity Gradient

- Unlike jet tomography studies, heavy-flavour muons offer **less spatial resolution** for medium opacity or density variations.

While the study offers novel insights into QGP collectivity through heavy-flavour muons at forward rapidity, it is constrained by experimental limitations (e.g., flavor separation, rapidity coverage), analysis technique boundaries (e.g., background modeling, plane resolution), and interpretational challenges (e.g., model dependence, flow attribution). Addressing these issues in future studies will enhance the precision and physical scope of heavy-ion collision research.

Current Trends of Present Research Study

1. Precision Flow Measurements at Multiple Rapidities

- Expansion of flow studies beyond mid-rapidity to include **forward and backward rapidity regions**, as in ALICE’s forward muon arm.
- Improved statistical methods and detector upgrades are enabling **differential flow analysis** across a wider kinematic range.

2. Charm–Bottom Quark Separation

- Current focus is on **disentangling charm and bottom contributions** to heavy-flavour observables using:
 - **Secondary vertexing**
 - **Impact parameter techniques**
 - **Machine learning models**
- Allows precise comparison between **mass-dependent flow behavior**, enhancing theoretical model discrimination.

3. Deployment of Machine Learning (ML) and AI Tools

- ML tools are increasingly being integrated for:
 - **Signal/background discrimination**
 - **Track reconstruction**
 - **Event classification**
 - **Heavy-flavour tagging**
- Use of **deep learning in detector calibration** and **flow harmonic decomposition** is now mainstream.

4. Transport Coefficient Extraction

- Emphasis on determining QGP transport coefficients:
 - **Heavy quark diffusion constant (D)**
 - **Shear viscosity-to-entropy density ratio (η/s)**
 - **Jet quenching parameter ($\hat{q}$)**
- Global Bayesian analyses using multi-observable inputs are used to extract these coefficients with higher precision.

5. Multi-Observable Correlations

- Studies now combine **elliptic flow (v_2)**, **triangular flow (v_3)**, and **nuclear modification factor (RAA_{AA})** to comprehensively probe:
 - **Energy loss mechanisms**
 - **Collectivity**
 - **Path-length dependence of quark propagation**

6. Flow Harmonics Beyond v_2

- Researchers are expanding focus to include:
 - **Higher-order harmonics (v_3, v_4 , etc.)**
 - **Event shape engineering (ESE)**
- This helps understand initial-state fluctuations and viscous damping in the QGP.

7. Jet–Heavy Quark Correlations

- Simultaneous study of **heavy-flavour tagged jets** and **muons from heavy-flavour decays** to map out **medium modification effects**.
- Offers insight into **energy loss hierarchy**: gluon > light quark > charm > bottom.

8. Advancements in Theoretical Modeling

- Progress in **Langevin dynamics**, **hydrodynamic simulations**, and **parton transport models**.
- Coupling of heavy-quark propagation with full QGP evolution in **3D+1 viscous hydrodynamics** frameworks.

9. Cross-Experiment Comparisons (ALICE, ATLAS, CMS, STAR, PHENIX)

- Efforts to **cross-calibrate heavy-flavour measurements** from multiple experiments at RHIC and LHC.
- Creation of standardized benchmarks for:

- v_2v_{22}
- $RAAR_{\{AA\}}RAA$
- D-meson and B-meson flow
- Non-prompt $J/\psi/\psi J/\psi$ observables

10. System Size and Energy Dependence Studies

- Study of heavy-flavour flow in **smaller systems** like p–Pb and d–Au to test the **onset of collectivity**.
- Exploration of collision energy dependence at different LHC runs ($\sqrt{s_{NN}} = 2.76, 5.02, 5.44$ TeV).

11. Upgrade of Detectors and Data Quality

- ALICE and other LHC detectors are undergoing major upgrades:
 - **ALICE ITS2 and ITS3**
 - **Muon Forward Tracker (MFT)**
 - **Fast Interaction Trigger (FIT)**
- These upgrades allow better spatial resolution, time resolution, and heavy-flavour separation.

12. Focus on Hadronization Mechanisms

- Renewed attention on **hadronization via coalescence versus fragmentation** in the QGP medium.
- Study of **flow of strange and multi-strange hadrons** alongside heavy-flavour mesons to validate hadronization models.

13. QGP Opacity and Energy Loss Tomography

- Use of **azimuthal anisotropies** of heavy particles to conduct **tomography of QGP**, i.e., mapping spatial energy density distributions.

14. Study of Non-Flow Effects and Background Suppression

- Techniques like:
 - **Mixed event subtraction**
 - **Two-particle correlations**
 - **Rapidity gap methods**
- are being refined to separate **genuine flow** from **non-flow contributions** (e.g., jets, resonance decays).

15. Bayesian Model Calibration Frameworks

- Frameworks like **JETSCAPE** and **Bayesian parameter estimation pipelines** are used to:
 - Fit data across many observables
 - Extract physical properties like **thermalization time**, **initial temperature**, etc.

16. Flow in Novel Probes

- Elliptic flow is now also being studied in:
 - **Heavy baryons** (e.g., Λ_c, Ξ_c)
 - **Exotic hadrons** (e.g., tetraquarks, pentaquarks)
 - **Open beauty hadrons (B^+, B^0)** via indirect decay channels

17. Lattice QCD Predictions and Constraints

- Lattice QCD simulations are providing **equation of state**, **screening masses**, and **spectral functions** relevant for:
 - Modelling QGP viscosity
 - Heavy-quark diffusion constants
 - Thermalization scales

18. Data-Driven Monte Carlo Event Generators

- New generations of **Monte Carlo tools** (e.g., MARTINI, DABMod, SHERPA, POWHEG) are integrated with:
 - **Heavy-flavour production and evolution**
 - **Jet-medium interactions**
 - **Medium response to quarks**

19. Real-Time QGP Imaging Techniques

- Experimental groups are developing **space-time imaging** of QGP using **femtoscopy** and **anisotropic flow cumulants**.

20. Interdisciplinary Collaborations

- Collaboration between:
 - **Theorists and experimentalists**
 - **Lattice QCD and hydrodynamics**
 - **AI/ML specialists and physicists**
- Leading to **rapid methodological advancements** and **greater predictive accuracy**.

The field of QGP and heavy-flavour muon flow is evolving rapidly with **improvements in theory, experimental resolution, computational power, and interdisciplinary integration**. Your study fits well into these modern research directions, particularly by focusing on **forward rapidity flow**, a frontier in heavy-ion collision physics.

History of Present Research Study

The concept of QGP was proposed in the 1970s, with early searches at CERN SPS and Brookhaven RHIC. LHC experiments, starting in 2010, brought unprecedented energy densities and particle multiplicities. Initial flow studies focused on light hadrons. As detectors improved, heavy-flavour measurements gained prominence, particularly via decay leptons. ALICE pioneered muon flow studies at forward rapidity, establishing the foundation for this research.

1. Theoretical Conception of Quark-Gluon Plasma (1970s–1980s)

- The concept of the **Quark-Gluon Plasma (QGP)** originated from **Quantum Chromodynamics (QCD)**—the theory of the strong interaction.
- In the **1970s**, theorists like **Collins and Perry (1975)** predicted that under extreme temperature and energy densities, normal hadronic matter would transition into a **deconfined state of quarks and gluons**.
- **Lattice QCD calculations** in the **1980s** supported this by indicating a phase transition at temperatures around **150–160 MeV**, corresponding to energy densities of $\sim 1 \text{ GeV/fm}^3$.

2. Early Experimental Efforts (1980s–1990s)

- Initial attempts to create QGP experimentally took place at fixed-target heavy-ion accelerators:

- **Bevalac** (Berkeley)
- **SPS (Super Proton Synchrotron)** at CERN
- **AGS (Alternating Gradient Synchrotron)** at Brookhaven National Laboratory (BNL)
- These facilities used collisions of light and heavy nuclei (e.g., O+O, S+S, Pb+Pb, Au+Au) at **sub-relativistic energies**.
- Observables like **strangeness enhancement**, **J/ψ suppression**, and **transverse energy distributions** were studied as potential QGP signatures.

3. Rise of Elliptic Flow Studies (1990s–2000s)

- The 1990s saw the **emergence of elliptic flow (v_2)** as a robust observable linked to the **collective behavior** of the medium formed in non-central collisions.
- The **Fourier decomposition** of azimuthal particle distributions became standard, and elliptic flow began to be recognized as a **sensitive probe of the QGP's viscosity**.
- With the **Relativistic Heavy Ion Collider (RHIC)** becoming operational at **BNL in 2000**, a new era of high-energy nuclear physics began:
 - RHIC provided collisions at $\sqrt{s_{NN}} = 200$ GeV.
 - Detectors like **PHENIX**, **STAR**, **BRAHMS**, and **PHOBOS** made significant discoveries.
 - **Elliptic flow of light hadrons** at RHIC revealed strong collective behavior consistent with **hydrodynamic expansion and low viscosity**.

4. Emergence of Heavy-Quark Flow Studies

- Initially, it was believed that **heavy quarks (charm and bottom)** would not exhibit significant flow due to their large mass and early production.
- However, **non-zero v_2 values** observed for **non-photonic electrons (from heavy-flavour decays)** at RHIC suggested that **heavy quarks interact strongly** with the medium.
- This challenged earlier assumptions and triggered the development of **transport models**, such as:
 - **Langevin dynamics**
 - **Boltzmann transport equations**
 - **Hydro+transport hybrid models**

5. LHC Era and Enhanced Energy Frontiers (2010–Present)

- With the launch of the **Large Hadron Collider (LHC)** at CERN in **2010**, ultra-relativistic heavy-ion collisions reached a new regime.
- **ALICE**, **ATLAS**, and **CMS** conducted Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV and later at **5.02 TeV**, leading to:
 - Larger QGP volumes
 - Longer lifetimes
 - Higher initial temperatures
- **Heavy-flavour studies expanded** to include:
 - **D mesons (charm)**
 - **Non-prompt J/ψ (from B decays)**

- **Heavy-flavour decay muons and electrons**

6. Focus on Forward Rapidity Muons

- The **ALICE Muon Spectrometer** was specifically designed to measure **muons from heavy-flavour decays at forward rapidity** ($2.5 < y < 4.025$ and $4.025 < y < 4.0$).
- This opened up new possibilities for studying:
 - **Azimuthal anisotropy of heavy quarks** away from mid-rapidity.
 - **Muon v_2** as a clean, less disturbed probe of QGP dynamics.
- First measurements of **elliptic flow of heavy-flavour muons at forward rapidity** were published by ALICE in 2018–2022.

7. Advancements in Theoretical Models and Simulations

- The rise of **multi-stage models** such as **TAMU**, **MC@sHQ**, **EPOS HQ**, and **PHSD** allowed better predictions of heavy-quark transport.
- Lattice QCD constraints were integrated into models for better temperature-dependent diffusion coefficients.
- Studies began focusing on:
 - **Flavor dependence of flow**
 - **Jet–medium interaction**
 - **Initial state fluctuations**

8. Technological Milestones and Detector Upgrades

- To enhance performance and resolution, ALICE and other LHC experiments undertook major detector upgrades:
 - **ALICE ITS2 (Inner Tracking System)**
 - **Muon Forward Tracker (MFT)**: Improves secondary vertexing for charm/bottom separation in the muon channel.
- These developments directly benefit future **precision measurements of v_2** and energy loss.

9. Collaborative Global Research Networks

- The study of QGP and heavy-flavour flow has become a highly collaborative global effort involving:
 - **CERN (Europe)**
 - **BNL (USA)**
 - **GSI/FAIR (Germany)**
 - **RIKEN (Japan)**
 - **VECC (India)**
- Cross-experiment comparisons and data sharing have become common to validate results across systems and energies.

10. Present-Day and Future Directions

- Current research is focused on:
 - **Precision extraction of transport coefficients** (e.g., D_s , D_{sD} , η/s , ϵ/s)

- **Charm-bottom separation** in flow observables
- **Event shape engineering** and **machine learning for flow extraction**
- **Understanding hadronization dynamics** via coalescence and fragmentation
- Future heavy-ion programs at LHC (Run 3 and Run 4) and upcoming facilities like **FAIR**, **NICA**, and the **Electron-Ion Collider (EIC)** aim to:
 - Explore the **QCD phase diagram**
 - Investigate the **critical point**
 - Provide **more differential and flavor-resolved QGP measurements**

The history of QGP and heavy-flavour flow research reflects a continuous evolution from **theoretical prediction**, to **experimental discovery**, and now to **precision measurement and modeling**. The specific focus on **elliptic flow of heavy-flavour muons at forward rapidity** is a natural progression in this journey, addressing critical questions about the **interaction of heavy quarks with the QGP**, the **universality of collective behavior**, and the **fundamental transport properties of strongly interacting matter**.

Discussion of Present Research Study

The measured $v_2v_{-2v_2}$ of heavy-flavour decay muons shows a clear positive signal in mid-central collisions, indicating substantial coupling between heavy quarks and the QGP. The trend of $v_2v_{-2v_2}$ rising with p_{Tp_TpT} and then saturating is consistent with transport models that incorporate both collisional and radiative energy loss. Comparisons with model predictions reveal that including both charm and bottom contributions is essential. These findings support the picture of partial thermalization of heavy quarks.

Results of Present Research Study

- Non-zero $v_2v_{-2v_2}$ observed for muons from heavy-flavour decays.
- Peak $v_2v_{-2v_2}$ values around 2–3% at intermediate p_{Tp_TpT} .
- Strong centrality dependence, with maximum flow in 20–40% centrality.
- Model predictions qualitatively agree but vary in magnitude.
- Results indicate strong interactions between heavy quarks and QGP medium.

Conclusion

This study provides clear evidence of the elliptic flow of heavy-flavour decay muons at forward rapidity, reinforcing the idea that heavy quarks participate in the collective dynamics of the QGP. The findings align with theoretical predictions and highlight the role of heavy-flavour probes in QGP characterization.

Suggestions and Recommendations

- Enhance charm/bottom separation via vertex detector upgrades.
- Extend flow studies to higher statistics and multiple rapidity regions.
- Cross-check results with electron and D meson measurements.
- Develop improved transport models tuned to experimental data.
- Use machine learning to refine signal extraction.

Future Scope

- Analyze Run 3 and Run 4 data with upgraded ALICE detector.
- Combine muon and electron flow measurements for comprehensive insight.
- Study higher-order flow harmonics ($v_3v_{-3v_3}$, $v_4v_{-4v_4}$) of heavy-flavour particles.

- Explore heavy-flavour correlations and jet substructure.
- Investigate flow in small systems (p–Pb, pp) to test collectivity limits.

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