A model for unsteady sediment transport in degrading alluvial channels

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The collection of data for unsteady/non-uniform sediment transport rates in a river below a reservoir, which arrests sediment from upstream, is very difficult. The establishment of a model for the determination of these sediment transport rates is equally difficult but important for the safety of a structure below the reservoirs. An indirect technique based on continuity equation for sediment transport has been used to obtain such data for unsteady/non-uniform flow conditions. Based on these data and dimensional analysis, a relationship for determination of sediment transport rate below high capacity reservoirs under unsteady-non-uniform conditions has been proposed which gave very high efficiency when checked with the available data for such conditions. As a consequence, a relationship for extent of degradation has also been established.

Keywords: degradation, dimensional analysis, high capacity reservoir, sediment transport, unsteady/non-uniform flow.

1. INTRODUCTION

Whenever the sediment transport rate at a section is suddenly decreased (Figure1) without changing the discharge and sediment size - a situation commonly encountered below high capacity reservoirs that trap more or less all the sediment carried by the streams-sudden discontinuity occurs in the variation of sediment transport rate along the length. As a result the equilibrium of the river is disturbed. To reestablish equilibrium the slope of the bed must decrease. The reduction in the sediment load, called sediment deficit, which increases the stream power of channel, is replenished due to erosion on the bed downstream of the dam, causing the bed to degrade. This process will continue until the new slope with prevailing water discharge is such that the sediment transport in the reach is equal to the reduced sediment transport rate at each point. The channel in which this phenomenon takes place is called degraded channel.

A good amount of literature is available to determine the sediment transport rate under uniform flow conditions prevailing in the alluvial stream-an excellent review of which is available in Task Committee report (1971) and Garde and Ranga Raju (2000). It has been found on the basis of the laboratory and field data that though in steady uniform flow conditions, the mean flow velocity can be determined with a degree of accuracy of 30% yet the steady transport rate when calculated from various available formulae is more difficult to predict. The variation in results can go up to 300 to 500 percent.

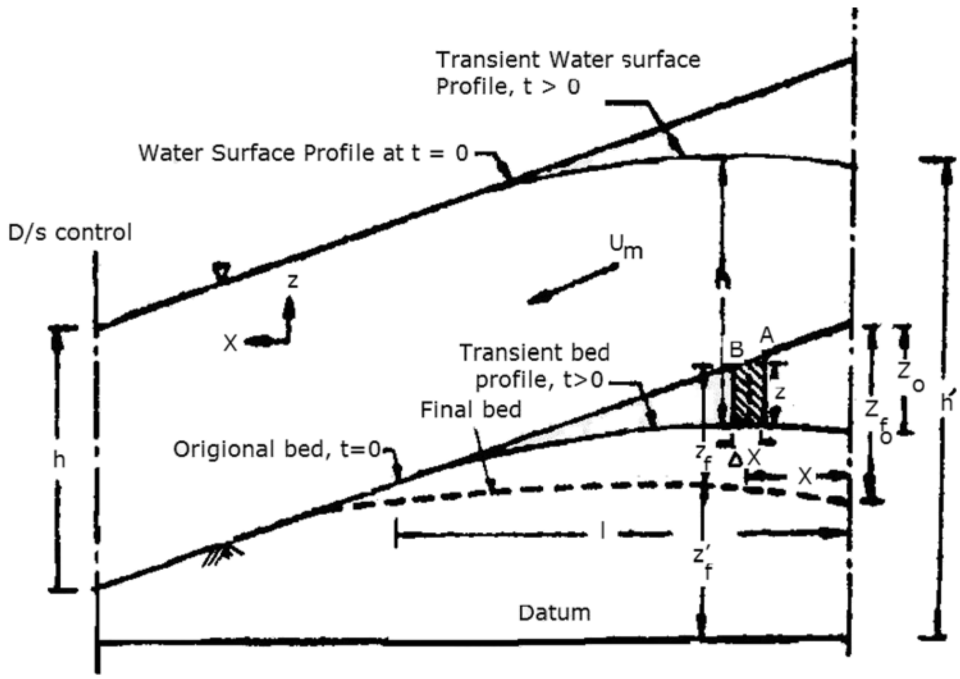

Figure 1 Definition sketch of a degradation process.

Unfortunately, the situation is much more complicated in case of non-uniform and/or unsteady flow conditions existing, for example, in alluvial channels during degradation. Very little is known about the variation of sediment transport rate as well as the sediment transport relations in such types of flow conditions. Sarda and Soni (1984) showed that, though the resistance law valid for uniform flow can be applied to unsteady/non-uniform flow conditions provided the local friction slope velocity and depth are used in this law, the sediment transport law valid for uniform flow cannot be applied directly to unsteady/non-uniform flow conditions prevailing during degradation.

While lot of database is available on sediment transport under steady uniform flow conditions, there is almost a complete lack of data under non-uniform/unsteady flow conditions. An attempt is made in this paper to develop a sediment transport law in alluvial channels which degrade due to complete stoppage of sediment supply. In the process of development, the data for unsteady sediment transport flow conditions will be determined indirectly using the data relating the progress of degradation with space and time.

2. THEORETICAL CONSIDERATION

Under non-uniform and unsteady flow conditions prevailing during degradation, the volumetric sediment transport rate per unit width depends on various factors like depth of flow, h, velocity of flow, U_m, time, t, water density, ρ, dynamic viscosity of fluid, μ, acceleration due to gravity, g, extent of degradation, L, equilibrium sediment transport rate, G_e and longitudinal distance, x. However, assuming that (i) sediment is arrested completely by the reservoirs, (ii) the river reach between the toe of the dam and control section is straight, the banks are not erodible, (iii) the variation of discharge does not occur, (iv) river bed is homogeneous with depth i.e. no armouring takes place, (v) the sediment transportation occurs only as bed load, (vi) there is no vegetation growth, (vii) there is no tributary flow and, (viii) the river section approximates a rectangle and (ix) the influence of change in bed form on the bed resistance is neglected, one can write for quasi-steady flow i.e. movement of water is much faster than that of bed, one can write:

$$G = f\left(h, U_m, t, L, G_e, x\right) \qquad \text{Equation (1)}$$

Using Buckingham π theorem Equation (1) can be written as:

$$\frac{G}{U_m h} = f\left(\frac{x}{h}, \frac{L}{h}, \frac{U_m t}{h}, \frac{G_e}{U_m h},\right) \qquad \text{Equation (2)}$$

For the boundary conditions of the problem and rearranging the π terms, Equation (2) can be finally written as:

$$\frac{G_e - G}{G_e} = f\left(\frac{x}{L}, \frac{U_m t}{h}\right) \qquad \text{Equation (3)}$$

In Equation (3), the determination of extent of degradation, L, is very important for accurate determination of sediment transport at any point and at any time downstream of section of complete stoppage of sediment. The various parameters influencing the extent of degradation, L, are: depth of flow, h; velocity of flow, U_m; time, t; water density, ρ; dynamic viscosity of fluid, μ; acceleration due to gravity, g and mean sediment size, d_{50}.

With the assumptions and conditions of the present study, one can write:

$$L = f\left(h, U_m, t, d_{50}\right) \qquad \text{Equation (4)}$$

Using the Buckingham pi-theorem, one can write Equation (4) as:

$$\frac{L}{h} = f\left(\frac{U_m t}{h}, \frac{d_{50}}{h}\right) \qquad \text{Equation (5)}$$

which can be modified by combining various pi-terms as

$$\frac{L}{d_{50}} = f\left(\frac{U_m t}{h}\right) \qquad \text{Equation (6)}$$

3. DATA USED

Newton (1951) and Suryanarayana (1969) conducted degradation experiments in the laboratory flume when degradation takes place in a homogeneous channel due to complete stoppage of sediment at the upstream control section for uniform sediment size. In the case of experiments conducted by Newton (1951), the water and bed surface profiles were not recorded simultaneously. For recording a bed profile the run was stopped and the undulations on the upstream sections were leveled with the help of a trowel before recording bed undulations. Since the transient water surface profiles were recorded only when a test was in operation and the transient bed profile while the test was stopped, the data for the transient profile could not be obtained simultaneously. To obtain water surface profiles that would correspond in time to a particular bed profile, water surface elevations and slopes were plotted against time on work sheet and required values were determined. However, in case of Suryanarayana (1969) experiments, water surface and bed surface elevations were recorded simultaneously. The complete details of the experimental set up and procedures are given elsewhere [Newton, 1951 and Suryanarayana, 1969].

Gessler (1971) while dealing with his numerical model, carried out the computation in an assumed flume with a length of 40.0 m and width 8.0 m. The material in the flume was assumed to have a maximum grain size of 6.0 m and a mean grain size (as defined by Meyer, Peter and Muller) of 1.5 mm.

The data for non-uniform flow conditions as collected above were plotted (not shown here) and average transient bed and water surface profiles were drawn. It was observed from these plots that the entire degradation reach is a region of non-uniform/unsteady flow. In the present investigation, the degradation flow depths and sediment transport rate at various sections were taken only from such averaged profiles which do not reach downstream sections and used in the analysis, which follows. The ranges of various relevant parameters which have been used in the analysis are given in Table 1.

Table 1 Range of data used

Sl. No.	Parameter	Range
1.	Depth, h, (m)	0.0336 – 0.158
2.	Mean Velocity U_m, (m/sec)	0.375 – 0.586
3.	Slope; S_f	0.0014 – 0.0066
4.	Temperature, T °C	68 – 70
5.	Discharge of water, q, (m^3/s/m)	$1.135 \times 10^{-3} – 3.90 \times 10^{-2}$
6.	Equilibrium sediment transport rate, G_e, (m^3/s/m)	$3.289 \times 10^{-6} – 1.997 \times 10^{-5}$

4. ANALYSIS OF DATA

(I) VARIATION OF SEDIMENT TRANSPORT RATE ALONG DEGRADED REACH:

The validity of a sediment transport law e.g. Equation (3), can be checked if the sediment transport rates at various sections along the transient bed profiles are known. The direct measurement of the sediment transport rate on the degraded reach is very difficult and was not made in the studies from where the present data have been taken. However, the sediment transport rate at various sections downstream of the section of sediment stoppage for t = t_1, t_2, t_3...........etc., can be determined from the sediment continuity equation reduced in the finite difference form and then applying it to two consecutive sections A and B, Δx apart [Figure 2].

$$\forall = \left\lfloor \int G.dt \right\rfloor_A - \left\lfloor \int G.dt \right\rfloor_B \qquad \text{Equation (7)}$$

Here, $\forall$ is absolute volume under the transient bed profile [= $z.\Delta x.(1 - P)$], z is the degradation depth and P is porosity of the bed material. In the present study, the porosity of sediment bed was calculated from Komura (1961).

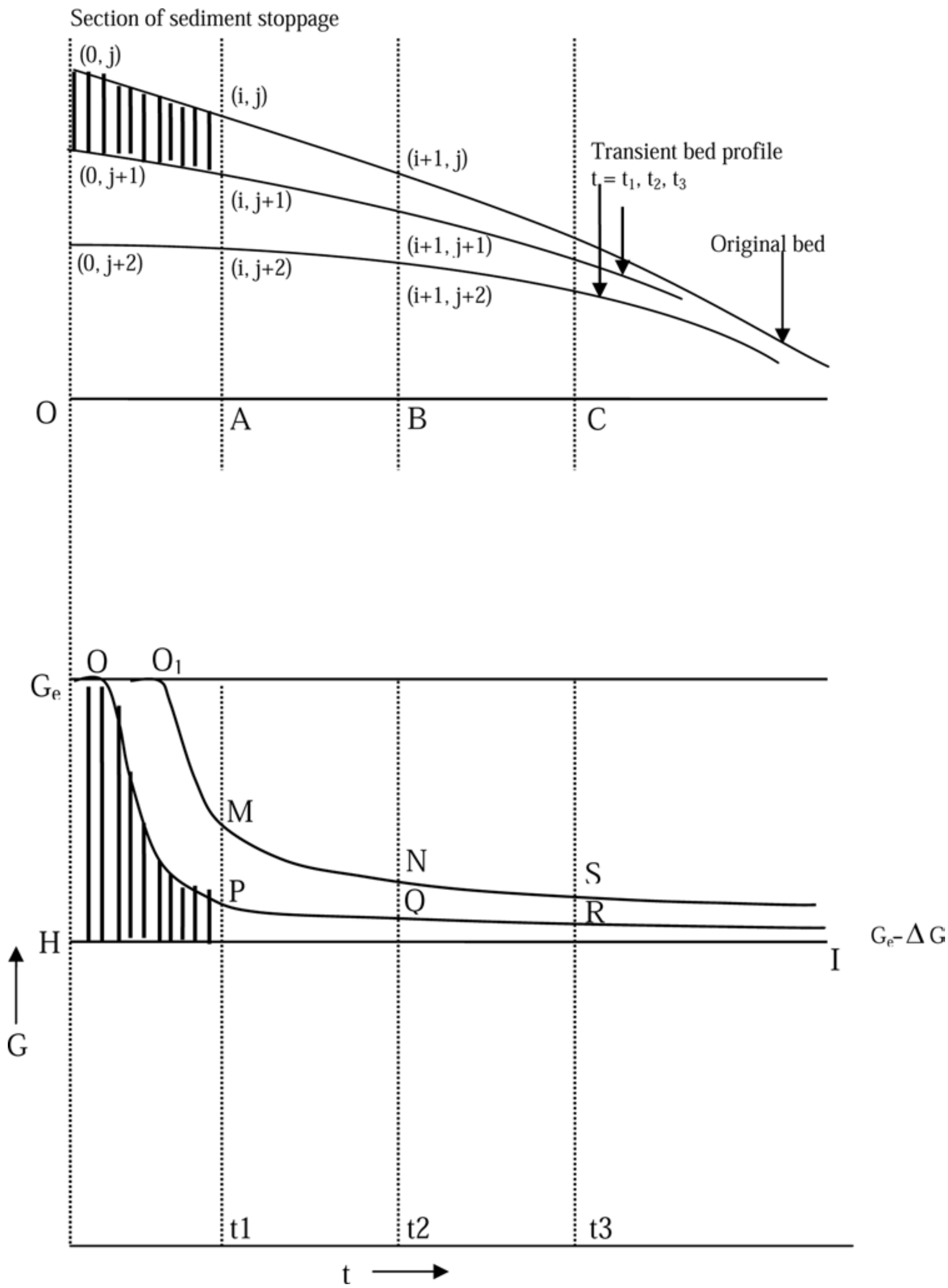

Figure 2 Definition sketch for computing sediment transport rate along transient bed profiles.

$$P = 0.2475 + 0.14 d_{50}^{-0.21} \quad \text{Equation (8)}$$

For the data used in the present study, the value of P calculated from Equation 8 varied from 0.39 to 0.41. The average value of 0.40, which is customarily the same value as used by Soni (1975) for degradation study, was adopted in the present study.

The manner of variation of G with respect to time at both the section being unknown, Equation 7 can not be solved directly. At the section x = 0, the sediment stoppage is taking place continuously at a constant rate. The sediment transport rate at this section is thus constant and is equal to ($G_e - \Delta G$), where $\Delta G = G_e$ for complete stoppage of sediment. So if the section of sediment stoppage is chosen as one of the sections, then only one unknown is left in Equation 7. Even then the direct solution of Equation 7 is not possible because of the lack of knowledge of variation of G with respect to time. However, starting from the section of sediment stoppage, Equation 7 can be solved by the following graphical trial and error procedure which was adopted by Soni (1975) for his aggradation study.

On a G – t plot, (Figure 2) draw a horizontal line H – I at the known specified transport rate equal to $G_e - \Delta G$. Assume a trial G – t curve O – P – Q – R for a particular section say A. The area between the assumed curve and line H – I is calculated and compared with the area under the transient bed profile for the same time interval and between the section O and A. In other words, the two shaded areas must give the same value after taking into account the porosity of the sand mass. In case of a difference, the assumed G – t curve is adjusted in such a manner that the areas become equal. The point P on the G – t curve represents the sediment transport rate at the point (i, j + 1) on the transient bed profile.

The trial G – t curve O_1 – M – N – S is assumed for the section B, the variation of the sediment transport rate at section A now being known. The area between the curves O – P – Q – R and O_1 – M – N – S between the time t = 0 and $t = t_1$ is then compared with the area under the transient bed profile between sections A and B. In case of difference, the trial G – t curve namely O_1 – M – N – S is adjusted till the areas are balanced. The point M on the G – t curve now represents the sediment transport rate at the point (i + 1, j + 1) on the transient bed profile. Proceed in the same manner for all the sections till the end of transient bed profile for $t = t_1$.

The sediment transport rate for the second transient bed profile for $t = t_2$ i.e. for points (i, j + 2), (i + 1, j + 2), (i + 2, j + 2).........etc., is determined by extending and adjusting assumed G –t curves in such a manner that the requirements of continuity are satisfied. The transient bed profile for $t = t_1$ serves as a base for working out sediment transport rates for the transient bed profile at $t = t_2$. The sediment transport rates at various locations for all the subsequent profiles are worked out in a similar manner. It may be noted that during the process of adjustment, (i) it was ensured that the calculated G – t curves are smooth and that there is a general similarity in shape of these curves at various x-values., (ii.) apart from the possible errors in averaging, these computed values may be prone to some error because the above procedure is based on the assumption that the suspended load varies little with x- and t-. While it was observed that most of the ejected sediment moves as suspended load in short length and then moves as bed load, variations in suspended load along the length (and with time) can not be ruled out. Since this variation is likely to be more significant close to the sections of sediment stoppage, computed values of G at small x values are likely to be less accurate than those at large distances and (iii) further some inaccuracy may also creep in due to error in measurements towards the tail end of the transient bed profile where very small erosion depths are involved.

Nevertheless, these computations provide useful information regarding the variation of sediment transport rate along the length of the channel.

Plots of the variation of G with x, with t as the third parameter, were made (not shown here) for all the data of Newton (1951) and Suryanarayana (1969). It was observed from these plots that the sediment transport at the particular section decreases very rapidly from the equilibrium values once the degradation front reaches that section, then gradually decreases and approaches ($G_e - \Delta G$) [equal to zero transport rate in the present study], asymptotically. The trend of variation of G with respect to x and t over the degraded reach was found to be same for all the authors.

For the further analysis of data, the sediment transport values for various values of x and t were read from these plots and are tabulated in Table 2. The data from Newton (1951) and Suraynarayana (1969) was used for establishing the sediment transport law while those of Gessler (1971) were used for checking the validity of the established law.

(II) SEDIMENT TRANSPORT LAW UNDER NON-UNIFORM FLOW CONDITIONS

In the experiments of Newton (1951) and Suraynarayana (1969), the sidewalls were made of plexiglass and hence considerably smoother than the bed. In the present analysis, hydraulic radius, R_b, as calculated using the following equations [Garde and Ranga Raju, 2000], was used in place of depth of flow, h, to take account of side wall effects as:

$$R_w = \left(\frac{V_w n_w}{S_f^{0.5}}\right)^{1.5} \qquad \text{Equation (9)}$$

$$R_b = \left(1 - \frac{2R_w}{B}\right)h \qquad \text{Equation (10)}$$

In these equations, R_w, V_w and n_w are the hydraulic radius, velocity and Manning's roughness coefficient with respect to walls, S_f is the energy slope and B is the width of the width of the channel. So that Equation 3 now can be rewritten as:

$$\frac{G_e - G}{G_e} = f\left(\frac{x}{L}, \frac{U_m t}{R_b}\right) \qquad \text{Equation (11)}$$

In order to study the effect of various parameters on the R-H-S of Equation 11, the data of sediment transport rate tabulated in the Table 2 are used to plot $\frac{(G_e - G)}{G_e}$ against $\frac{x}{L}$ as shown in Figure 3. It may be seen from this figure that the data do not scatter much. The third parameter $\frac{U_m t}{R_b}$ did not improve the plot and no well defined variation could be found. Hence, the effect of $\frac{U_m t}{R_b}$ on this plot has been considered to be negligible.

A mean line was drawn through the plotted data points. This graph shows that with the increase of distance the sediment transport rate decreases. The mean line has been found to follow error function and the following equation is found to fit this mean line.

Table 2 Sediment transport rate computed from transient bed profiles obtained during degradation experiments

Source	Run No.	$G_e \cdot 10^5$ ($m^3/s/m$)	Time, t (hrs)	$G \cdot 10^5$ (Distance, x, m) $m^3/s/m$									
Newton (1951)	1	0.62	1.00	0.000 (0.000)	0.117 (0.610)	0.218 (1.220)	0.310 (1.833)	0.390 (2.440)	0.465 (3.050)	0.525 (3.660)	0.577 (4.270)	0.620 (4.880)	—
	2	1.234	0.25	0.000 (0.000)	0.302 (0.305)	0.590 (0.610)	0.830 (0.915)	1.050 (1.220)	1.234 (1.525)	—	—	—	—
			0.50	0.000 (0.000)	0.250 (0.610)	0.484 (1.220)	0.700 (1.830)	0.870 (2.440)	1.020` (3.050)	1.140 (3.660)	1.234 (4.270)	—	—
			1.00	0.000 (0.000)	0.200 (0.610)	0.400 (1.220)	0.580 (1.830)	0.720 (2.440)	0.860 (3.050)	0.970 (3.660)	1.060 (4.270)	1.200 (5.490)	1.234 (6.100)
	3	1.283	0.25	0.000 (0.000)	0.242 (0.305)	0.490 (0.610)	0.720 (0.915)	0.930 (1.220)	1.110 (1.525)	1.280 (1.830)	—	—	—
			0.50	0.000 (0.000)	0.220 (0.610)	0.420 (1.220)	0.610 (1.830)	0.770 (2.440)	0.917 (3.050)	1.037 (3.660)	1.140 (4.270)	1.220 (4.880)	1.283 (5.490)
	4	1.230	0.25	0.000 (0.000)	0.164 (0.305)	0.338 (0.610)	0.500 (0.915)	0.660 (1.220)	0.800 (1.525)	0.950 (1.830)	1.082 (2.135)	1.230 (2.440)	—
			0.50	0.000 (0.000)	0.232 (0.610)	0.460 (1.220)	0.658 (1.830)	0.834 (2.440)	0.990 (3.050)	1.128 (3.660)	1.230 (4.270)	—	—
Suryanarayana (1969)	22	0.896	1.50	0.000 (0.000)	0.110 (0.763)	0.200 (1.526)	0.290 (2.289)	0.374 (3.052)	0.450 (3.815)	0.520 (4.578)	0.590 (5.341)	0.760 (7.630)	0.896 (9.910)
			4.00	0.000 (0.000)	0.090 (0.763)	0.170 (1.526)	0.310 (3.052)	0.490 (5.341)	0.590 (6.867)	0.680 (8.393)	0.780 (10.682)	0.842 (12.971)	0.896 (15.260)
	24	1.99	1.00	0.000 (0.000)	0.200 (0.763)	0.406 (1.526)	0.604 (2.289)	0.800 (3.052)	0.980 (3.815)	1.310 (5.341)	1.600 (6.867)	1.860 (8.393)	1.990 (9.156)

			2.50	0.000 (0.000)	0.280 (1.526)	0.430 (2.289)	0.570 (3.052)	0.710 (3.815)	0.850 (4.578)	1.110 (6.104)	1.460 (8.393)	1.680 (9.919)	1.990 (12.208)
			4.50	0.000 (0.000)	0.410 (2.289)	0.670 (3.815)	0.920 (5.341)	1.150 (6.867)	1.350 (8.393)	1.520 (9.919)	1.670 (11.445)	1.850 (13.734)	1.990 (16.023)
	26	1.44	1.20	0.000 (0.000)	0.190 (0.763)	0.358 (1.526)	0.660 (3.052)	0.920 (4.578)	1.121 (6.104)	1.280 (7.630)	1.340 (8.393)	1.400 (9.156)	1.440 (9.919)
			3.15	0.000 (0.000)	0.136 (0.763)	0.270 (1.526)	0.390 (2.289)	0.510 (3.052)	0.724 (4.578)	0.910 (6.104)	1.080 (7.630)	1.281 (9.919)	1.440 (12.208)
			6.00	0.000 (0.000)	0.097 (0.763)	0.200 (1.526)	0.480 (3.815)	0.660 (5.341)	0.969 (8.393)	1.100 (9.919)	1.220 (11.445)	1.331 (12.971)	1.440 (14.497)
Gessler (1971)	Numerical Model	2.15	0.50	0.000 (0.000)	0.700 (2.000)	1.249 (4.000)	1.651 (6.000)	1.950 (8.000)	2.150 (10.000)	—	—	—	—
			2.00	0.000 (0.000)	0.420 (2.000)	0.750 (4.000)	1.040 (6.000)	1.300 (8.000)	1.490 (10.000)	1.850 (14.000)	2.050 (18.000)	2.110 (20.000)	2.150 (22.000)
			5.00	0.000 (0.000)	0.280 (2.000)	0.510 (4.000)	1.120 (10.000)	1.470 (14.000)	1.720 (18.000)	1.930 (22.000)	2.050 (26.000)	2.120 (30.000)	2.150 (36.000)

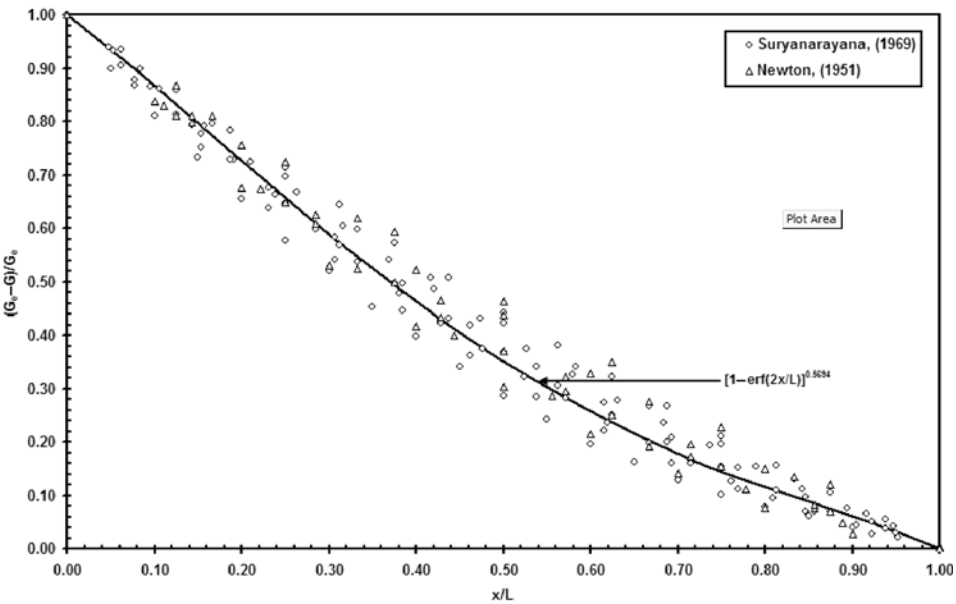

Figure 3 Variation of $(G_e - G)\big/ G_e$ with $x\big/ L$.

$$\frac{(G_e - G)}{G_e} = \left[1 - erf\left(2\frac{x}{L}\right)\right]^{0.57} \qquad \text{Equation (12)}$$

(III) DETERMINATION OF EXTENT OF DEGRADATION, L

Equation 12 will be useful if the extent of degradation L is known. The functional relationship for the extent of degradation as given in Equation 6 can be re-written as:

$$\frac{L}{d_{50}} = f\left(\frac{U_m t}{R_b}\right) \qquad \text{Equation (13)}$$

The dimensionless parameter $\frac{L}{d_{50}}$ and $\frac{U_m t}{R_b}$ were computed for all degradation runs of Newton (1951) and Suryanarayana (1969) and plotted as shown in Figure 4. The data are well described by the following equation:

$$\frac{L}{d_{50}} = 54.23 \times \left(\frac{U_m t}{R_b}\right)^{0.49} \qquad \text{Equation (14)}$$

With the help of Equations (12) and (14), the sediment transport rate for the numerical data of Gessler (1971) were computed and plotted against the observed data as shown in Figure 5. It can be seen that the calculated data by the model fall within 30% error line which is acceptable.

5. CONCLUSIONS

The sediment transport rates during non-uniform/unsteady transport conditions prevailing below high capacity reservoir have been studied. It has been found that at any section on the

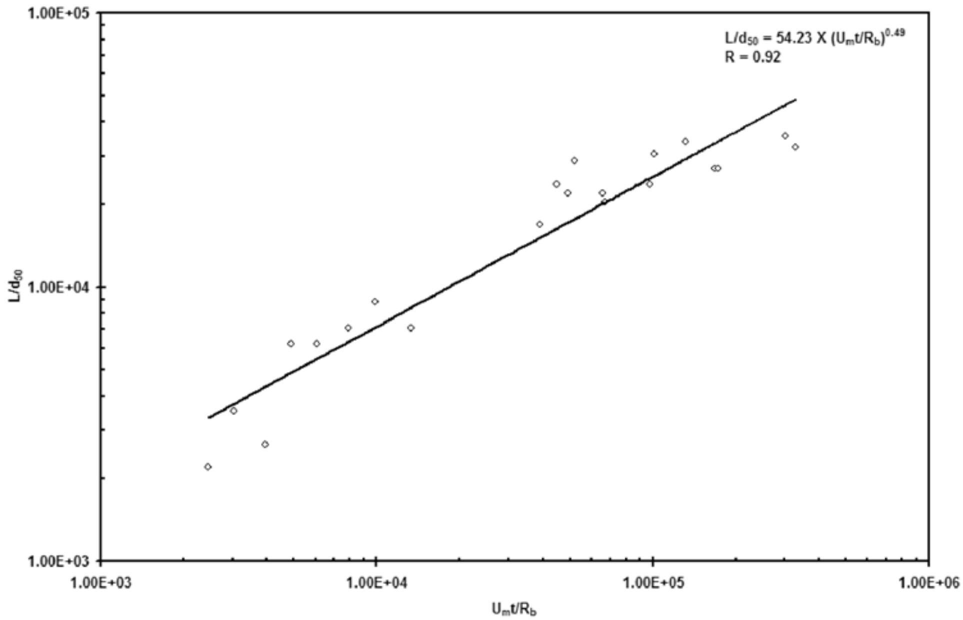

Figure 4 Variation of L/d_{50} with $U_m t/R_b$.

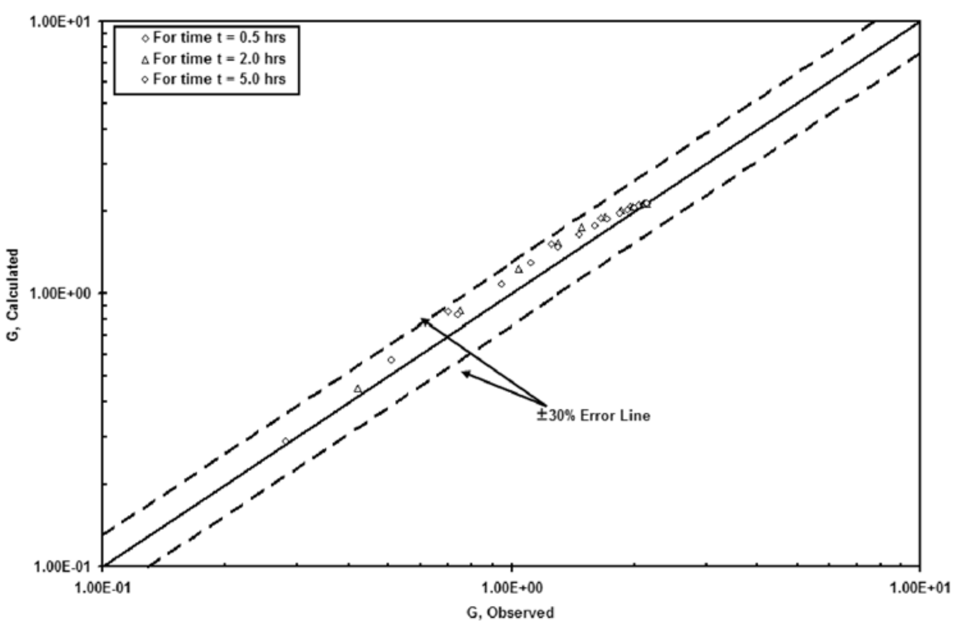

Figure 5 Comparison of observed values with calculated for Gessler (1971) data.

degrading reach, the sediment transport rate initially decreases very fast from the equilibrium value, then decreases gradually later on and approaches zero sediment transport rate asymptotically. The value of sediment transport rate downstream of the section of complete stoppage of sediment can be computed from Equation 12 along with Equation 14. The results obtained from the model lie within ±30%.

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