Sound Radiation of Cylindrical Shells

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ABSTRACT
The acoustic signature of submarines is very critical in such high performance structure. Submarines are not only required to sustain very high dynamic loadings at all time, but also being able maneuver and perform their functions under sea without being detected by sonar systems. Submarines rely on low acoustic signature level to remain undetected.

Reduction of sound radiation is most efficiently achieved at the design stage. Acoustic signatures may be determined by considering operational scenarios, and modal characteristics. The acoustic signature of submarines is generally of two categories; broadband which has a continuous spectrum; and a tonal noise which has discrete frequencies.

The nature of sound radiation of submarine is fiction of its speed. At low speed the acoustic signature is dominated by tonal noise, while at high speed, the acoustic signature is mainly dominated by broadband noise. Submarine hulls are mainly constructed of circular cylindrical shells.

Unlike that of simpler structures such as beams and plates, the modal spectrum of cylindrical shell exhibits very unique characteristics. Mode crossing, the uniqueness of modal spectrum, and the redundancy of modal constraints are just to name a few.

In cylindrical shells, the lowest natural frequency is not necessarily associated with the lowest wave index. In fact, the natural frequencies do not fall in ascending order of the wave index either. Solution of the vibration problem of cylindrical shells also indicates repeated natural frequencies. These modes are referred to as double peak frequencies. Mode shapes associated with each one of the natural frequencies are usually a combination of Radial (flexural), Longitudinal (axial), and Circumferential (torsional) modes.

In this paper, the wave equation will be set up in terms of the pressure fluctuations, $p(x,t)$. It will be demonstrated that the noise radiation is a fluctuating pressure wave.

1. INTRODUCTION

Noise and vibration are often treated as two separate entities in study of mechanical dynamics, however the two are inter-related and must be treated as such. They simply relate to the transfer of molecular motion energy in different media usually fluid or solid. The concept of sound radiation require one to think in terms of wave of sound and also in terms of modes of vibration.

The concept of the sound can be defined in simple terms as pressure waves that propagate through an elastic medium at some characteristic speed. It is the molecular transfer of motional energy and cannot therefore pass through a vacuum. To have a wave motion, the medium has to posse's inertia and elasticity. The two fundamental mechanisms responsible for sound generation are:

1. The vibration of solid bodies resulting in the generation and radiation of sound energy usually referred to as structure-borne sound.
2. The flow-induced noise resulting from pressure fluctuations induced by turbulence and unsteady flow this concept is usually referred as aerodynamic sound.

The region of interest in structure born sound is usually in some kind of fluid in most instances air at some distance from a vibrating structure. The sound wave propagates through the stationary fluid (the fluid has a finite particle velocity due to the sound wave but zero mean velocity) from a readily identifiable source to the receiver. The region of interest does not contain any source of sound energy. This means that the sources which generate the acoustic disturbance are external to it.

The classical acoustic theory (the analysis of homogeneous wave equation) can be used for the analysis of sound wave generated by these types of sources. The solution for acoustic pressure fluctuation, *p*, describes the wave field external to the source. This wave field can be modeled in terms of combination of simple sound sources

In the aerodynamic sound, the source of sound is not so readily identifiable and the region of the fluid can be within the region of interest or external to it. When they are within the region of interest they contain the source of the sound energy. The sources are continuously being generated or converted with the flow. These aerodynamic sources must be included in the wave equation for any subsequent analysis of the sound wave in order that they can be correctly identified. The wave equation therefore is no longer homogeneous, and its solution is different than the homogeneous version.

In acoustics three different methods can be used to solve problems and are as follow: (*i*)acoustics(*ii*) ray acoustics, and (*iii*) energy acoustics. Wave acoustics is a description of wave propagation using either molecular or particular model. This is the approach that was employed in this papert. Ray acoustics is description of wave propagation over large distances, one example includes the atmosphere. The energy acoustics describe the propagation of sound waves in terms of the transfer of energy of various statistical parameters where techniques referred to as statistical energy analysis.

2. THEORETICAL FORMULATION

The study of sound waves includes for variable which are pressure P, velocity U, density ρ, and temperature T. Examining these variables we see that pressure, density and temperature are scalar quantities where velocity is a vector quantity. We know that each of these variables has a mean and a fluctuating component. Thus,

$$P(\vec{x},t) = P_0(\vec{x}) + p(\vec{x},t)$$
$$\vec{U}(\vec{x},t) = \vec{U}_0(\vec{x}) + \vec{u}(\vec{x},t)$$
$$\rho(\vec{x},t) = \rho_0(\vec{x}) + \rho'(\vec{x},t)$$
$$T(\vec{x},t) = T_0(\vec{x}) + T'(\vec{x},t)$$

The wave equation can then be set up in terms of any of those four variables. In acoustics it is the pressure fluctuations, $p(x,t)$ that are of primary concern. Thus it is common for the acousticians to solve wave equation in terms of the pressure as a dependent variable. Also one can solve the wave equation in terms of other three variables as well without any issues. It's a common practice to set $\vec{U}_0(\vec{x}) = 0$, the ambient fluid is not moving, therefore $\vec{U}(\vec{x},t) = \vec{u}(\vec{x},t)$. As the wave propagates thru solid medium we need to make several assumptions:

- The fluid is an ideal gas
- The fluid is perfectly elastic (Hooke's Law holds)
- The fluid is homogeneous and isotropic
- The fluid is inviscid
- The wave propagation through the fluid media is adiabatic and reversible
- Gravitational effects are neglected
- The fluctuations are assumed to be small (the system behaves linearly)

From conservation of mass, momentum and the thermodynamic equation of state we derive the linearised acoustic wave equation as follow:

Conservation of mass commonly known as continuity equation:

$$\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial u_x}{\partial x} + \rho_0 \frac{\partial u_y}{\partial y} + \rho_0 \frac{\partial u_z}{\partial z} = 0$$

Simplifying it further we have:

$$\frac{\partial \rho'}{\partial t} + \rho_0 \vec{\nabla} \cdot \vec{u} = 0$$

Conservation of Momentum:

$$\rho_0 \left\{ \frac{\partial u_x}{\partial t} \hat{i} + \frac{\partial u_y}{\partial t} \hat{j} + \frac{\partial u_z}{\partial t} \hat{k} \right\} + \left\{ \frac{\partial_p}{\partial x} \hat{i} + \frac{\partial_p}{\partial y} \hat{j} + \frac{\partial_p}{\partial z} \hat{k} \right\} = 0$$

Simplifying it further:

$$\rho_0 \frac{\partial \vec{u}}{\partial t} + \vec{\nabla} p = 0$$

The Thermodynamic Equation of State:

$$p(x,t) = B \left\{ \frac{\rho'}{\rho_0} \right\}$$

Where B is the adiabatic bulk modulus

$$B = \rho_0 \left\{ \frac{\partial P}{\partial \rho} \right\}$$

To obtain the linearized acoustic wave equation conservation of mass and conservation of momentum can be combined into a single expression with one dependent variable. The dependent variable of interest in acoustics is the fluctuating pressure.

Taking a derivative of conservation of mass equation we have the following equation:

$$\frac{\partial^2 \rho'}{\partial t^2} + \rho_0 \vec{\nabla} \cdot \frac{\partial \vec{u}}{\partial t} = 0$$

Divergence of conservation of momentum:

$$\rho_0 \vec{\nabla} \cdot \frac{\partial \vec{u}}{\partial t} + \nabla^2 p = 0$$

$$\rho_0 \vec{\nabla} \cdot \frac{\partial \vec{u}}{\partial t} + \nabla^2 p - \frac{\partial^2 \rho'}{\partial t^2} + \rho_0 \vec{\nabla} \cdot \frac{\partial \vec{u}}{\partial t} = \nabla^2 p - \frac{\partial^2 \rho'}{\partial t^2}$$

$$\nabla^2 p = \frac{\partial^2 \rho'}{\partial t^2}$$

One can now substitute it into thermodynamic equation of state and get the following equation:

$$\nabla^2 p = \frac{\rho_0}{B} \frac{\partial^2 p}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

This is linearized homogeneous acoustic wave equation with the fluctuating pressure as the dependent variable. The constant c is the velocity of propagation of the wave and is therefore the speed of sound. Some useful approximation can now be made in relation to the speed of sound by assuming that the sound propagation medium is a perfect gas.

$$p = \frac{p_0 \rho^{\gamma}}{\rho_0^{\gamma}}$$

Thus

$$\frac{\partial p}{\partial \rho} = \frac{\gamma p}{\rho}$$

$$c = \left(\frac{\gamma p}{\rho}\right)^{\frac{1}{2}}$$

$$c = \left(\gamma R T_k\right)^{\frac{1}{2}}$$

Where R is the universal gas constant

For small fluctuation we can approximate the speed of sound as:

$$c \approx c_0 \approx \left(\frac{\gamma p_0}{\rho_0}\right)^{\frac{1}{2}}$$

Using vector theory, it can be demonstrated that the acoustic particle velocity is irrotational. From vector theory it can be shown that if a vector function is the gradient of a scalar function, its curl is the zero vector. The curl of the moment equation:

$$\rho_0 \frac{\partial\left(\vec{\nabla} x \vec{u}\right)}{\partial t} + \left(\vec{\nabla} x \vec{\nabla} p\right) = 0$$

$$\vec{\nabla} x \vec{u} = 0$$

Hence the introduction of the concept of the acoustic velocity potential φ and $\vec{u} = \vec{\nabla}\phi$. The physical interpretation of the above result is that the acoustical excitation of an inviscid fluid does not produce rotational flow; there are no boundary layers, shear stresses or turbulence generated.

Substituting into the equation for velocity potential into momentum equation and get $\nabla\left\{\rho_0 \frac{\partial\phi}{\partial t} + p\right\} = 0$.

The acoustic quantities inside the bracket have to vanish if there is no acoustic disturbance present thus the integration constant has to be zero

$$p = -\rho_0 \frac{\partial\phi}{\partial t}$$

Substituting for p into the wave equation yields:

$$\nabla^2 \frac{\partial\phi}{\partial t} = \frac{1}{c^2} \frac{\partial^z\phi}{\partial t^2}$$

Thus

$$\nabla^2\phi = \frac{1}{c^2} \frac{\partial^z\phi}{\partial t^2}$$

This satisfies the wave equation.Three additional sound parameters that play an important role in acoustics are the sound intensity, the sound energy density and radiation sound power. The sound intensity is defined as the rate of flow of energy through a unit area which is normal to the direction of propagation. Power = Force x Velocity. For acoustic process, the instantaneous power is:

$$P = F \cdot u$$

The power per unit normal area is the instantaneous sound intensity vector *I’* where

$$\vec{I}' = p\vec{u}$$

The time average of the instantaneous power flow thru a unit area is the mean intensity vector, *I* where

$$\vec{I} = \frac{1}{T}\int_0^T p\vec{u}\,dt = \frac{1}{2} Re\left[p\vec{u}^*\right]$$

This equation is used when the acoustic power fluctuation and the particle velocities are treated as complex, harmonic variables. For a plane wave traveling in the positive *x*-direction pressure with the respect to position and time can be approximated as:

$$p(\vec{x},t) = Re\left[A_I e^{i(\omega t - kx)}\right] = \hat{p}(\cos(\omega t - kx))$$

And

$$u(x,t) = Re\left[\frac{A_I}{\rho_0 c} e^{i(\omega t - kx)}\right] = \frac{\hat{p}}{\rho_0 c}\cos(\omega t - kx)$$

To obtain the sound intensity *I* one will need to substitute the two equations above into the intensity power equation and evaluate the integral.

Consider a cylindrical shell of finite length *L*, radius *R* and thickness *h*, with shell co-ordinates *x*, ϕ and *w* as shown in Figure 1. The orthogonal components of displacements of the mid-surface of the shell are represented by *u*, *v*, and *w* corresponding to *x*, ϕ and radial directions respectively.

For shear diaphragm condition the boundary conditions are:

$$w = v = 0, \qquad \text{and } M_x = N_x = 0 \qquad \text{at } x = 0, L.$$

These conditions are closely approximated in physical application by means of rigidly attaching a thin, flat, circular cover plate at each end [12].

The strain components can be expressed using the Donnell-Mushtari [3, 9] shell theory as:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \quad \varepsilon_{\varphi\varphi} = \frac{1}{R}\frac{\partial \upsilon}{\partial \varphi} + \frac{w}{R}, \quad \varepsilon_{x\varphi} = \frac{1}{R}\frac{\partial u}{\partial \varphi} + \frac{\partial \upsilon}{\partial x} \tag{1}$$

$$k_{xx} = \frac{\partial^2 w}{\partial x^2}, \quad k_{\varphi\varphi} = \frac{1}{R^2}\frac{\partial^2 w}{\partial \varphi^2}, \quad k_{x\varphi} = \frac{2}{R}\frac{\partial^2 w}{\partial x \partial \varphi} \tag{2}$$

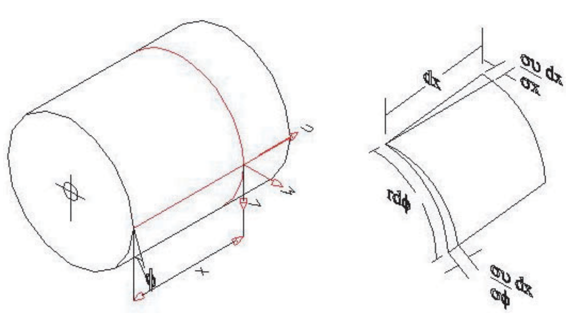

Figure 1 Displacement Components

Where, 'ε' is the strain displacement and 'k' is the curvature displacement in the particular plane. For shear diaphragm boundary condition, the orthogonal components of the displacements for the middle surface can be satisfied using Rayleigh-Ritz displacement functions:

$$\begin{aligned} u &= A\cos\frac{m\pi x}{L}\cos n\phi\cos\omega t \\ \upsilon &= B\sin\frac{m\pi x}{L}\sin n\phi\cos\omega t \\ \omega &= C\sin\frac{m\pi x}{L}\cos n\phi\cos\omega t \end{aligned} \quad (3)$$

These functions imply separation between time and the spatial variables, i.e. the shell will undergo simple harmonic motion in which both period and phase are identical at all points of the shell [12]. Constants A, B and C describe the amplitude of the axial (u), circumferential or tangential (v), and radial (w) deformations of the shell. The mode shapes are identified by n and m. Where, 'n' is the number of circumferential waves in the mode shape, n = 1,2,3... and 'm' is the number of longitudinal half-waves in the mode shape, m = 1,2,3... as shown in Figure (2).

The total strain energy (V) of a cylindrical shell of radius R is expressed in terms of the strain components as a combination of membrane or stretching strain energy (V_m) and bending strain energy (V_b).

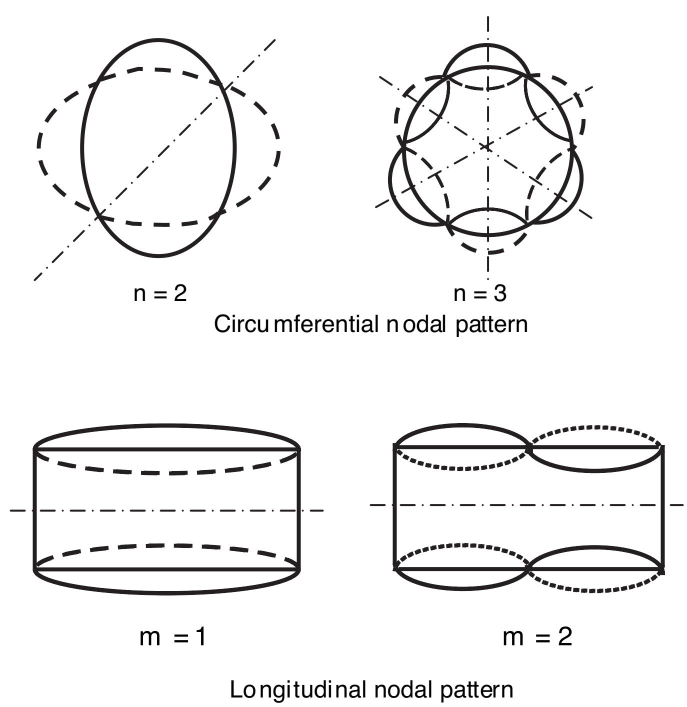

Figure 2: Normal Mode Patterns

$$V = \frac{E}{2(1-v^2)} R \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^{2\pi} \int_0^L \begin{bmatrix} \varepsilon_{xx}^2 + \varepsilon_{\varphi\varphi}^2 \\ +2v\varepsilon_{xx}\varepsilon_{\varphi\varphi} \\ +\frac{1-v}{2}\varepsilon_{x\varphi}^2 \end{bmatrix} dz \ dx \ d\varphi$$
$$+\frac{E}{2(1-v^2)} R \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^{2\pi} \int_0^L \begin{bmatrix} k_{xx}^2 + k_{\varphi\varphi}^2 \\ +2vk_{xx}k_{\varphi\varphi} \\ +2(1-v)k_{x\varphi}^2 \end{bmatrix} z^2 dz \ dx \ d\varphi \quad (4)$$

Using the strain components in combination with the Rayleigh-Ritz displacement functions, the bending strain energy (V_b) can be rewritten as

$$V_b = \frac{\pi ERLH^3}{48(1-v^2)} \left[C^2 \left(\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n}{R}\right)^2 \right)^2 \right]. \quad (5)$$

and in similar manner the membrane strain energy (V_m) is given as :

$$V_m = \frac{E\pi RLh}{4(1-v^2)} \begin{bmatrix} A^2 \left\{ \left(\frac{m\pi}{L}\right)^2 + \left(\frac{1-v}{2}\right)\left(\frac{n}{R}\right)^2 \right\} \\ +B^2 \left\{ \left(\frac{1-v}{2}\right)\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n}{R}\right)^2 \right\} \\ +C^2 \left(\frac{1}{R}\right)^2 - 2ACv\left(\frac{m\pi}{L}\right)\left(\frac{1}{R}\right) \\ -2AB\left(\frac{1+v}{2}\right)\left(\frac{m\pi}{L}\right)\left(\frac{n}{R}\right) \\ +2BC\left(\frac{1}{R}\right)\left(\frac{n}{R}\right) \end{bmatrix} \quad (6)$$

At a time *t* the expression for kinetic energy *T* of the vibrating shell is given by:

$$T = \frac{\rho}{2} R \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^L \begin{bmatrix} \left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \upsilon}{\partial t}\right)^2 \\ +\left(\frac{\partial \omega}{\partial t}\right)^2 \end{bmatrix} dz \ dx \ d\varphi \quad (7)$$

$$T = \omega^2 \overline{m} \left(\frac{A^2 + B^2 + C^2}{2} \right) \quad (8)$$

Where $\overline{m} = \frac{\pi \rho RLh}{2}$

Now, the equation of motion can be derived by reducing the lagrange energy functional (V-T) using Hamilton principles [12] to lead into an eigenvalue problem

$$\begin{bmatrix} K_{11}-\omega^2\overline{m} & K_{12} & K_{13} \\ K_{12} & K_{22}-\omega^2\overline{m} & K_{23} \\ K_{13} & K_{23} & K_{33}-\omega^2\overline{m} \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{9}$$

Where the stiffiness coeffiecients are

$$K_{11} = \frac{\pi ELhR}{2(1-v^2)}\left[\begin{array}{l} \left(\frac{m\pi}{L}\right)^2 + \left(\frac{1-v}{2}\right)\left(\frac{n}{R}\right)^2 \\ +\frac{h^2}{3}\left(\frac{1-v}{2}\right)\left(\frac{1}{R}\right)^2\left(\frac{n}{R}\right)^2 \end{array}\right]$$

$$K_{12} = \frac{\pi ELhR}{2(1-v^2)}\left[\begin{array}{l} \frac{h^2}{16}\left(\frac{1-v}{2}\right)\left(\frac{m\pi}{L}\right)\left(\frac{n}{R}\right)\left(\frac{1}{R}\right)^2 \\ -\left(\frac{1+v}{2}\right)\left(\frac{m\pi}{L}\right)\left(\frac{n}{R}\right) \end{array}\right]$$

$$K_{13} = \frac{\pi ELhR}{2(1-v^2)}\left[\begin{array}{l} v\left(\frac{m\pi}{L}\right)\left(\frac{1}{R}\right) \\ -\frac{h^2}{12}\left(\frac{1-v}{2}\right)\left(\frac{m\pi}{L}\right)\left(\frac{n}{R}\right)^2\left(\frac{1}{R}\right) \end{array}\right]$$

$$K_{22} = \frac{\pi ELhR}{2(1-v^2)}\left[\begin{array}{l} \left(\frac{n}{2}\right)^2 + \left(\frac{1-v}{2}\right)\left(\frac{m\pi}{L}\right)^2 \\ +3\frac{h^2}{16}\left(\frac{1-v}{2}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{1}{R}\right)^2 \end{array}\right]$$

$$K_{23} = \frac{\pi ELhR}{2(1-v^2)}\left[\begin{array}{l} -\left(\frac{n}{R}\right)\left(\frac{1}{R}\right) \\ -\frac{h^2}{4}\left(\frac{1-v}{2}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{n}{R}\right)\left(\frac{1}{R}\right) \end{array}\right]$$

$$K_{33} = \frac{\pi ELhR}{2(1-v)^2}\left[\left(\frac{1}{R}\right)^2 + \frac{h^2}{12}\left[\begin{array}{l} \left\{\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n}{R}\right)^2\right\}^2 \\ +\left(\frac{1}{R}\right)^4 - 2\left(\frac{n}{R}\right)^2\left(\frac{1}{R}\right)^2 \\ -2v\left(\frac{m\pi}{L}\right)^2\left(\frac{1}{R}\right)^2 \end{array}\right]\right]$$

The eigenvalue problem can then be solved numerically for each combination of mode shape parameters n and m to obtaine the natural frequencies. A MATLAB program was developed to calculate the natural frequencies of the circular cylindrical shell and the coressponding strain energy distribution for the each mode shape.

3. ILLUSTRATIVE EXAMPLE

A scale model of a submarine hull segment presented in [1,2] is developed. The scale model retain the same ratio of radius to length and thickness to radius ensuring similar modal spestrum. The scale model is a uniform circular cylindrical shell that has a length 754.4 mm, radius 251.5 mm, thickness 2.54 mm, Young's modulus (E) 206.81 Gpa, Poisson's ratio (v) 0.29, and material density (ρ) 7.82×10^3 kg/m^3. The natural frequency and the corresponding energy distribution were calculated using the MATLAB program. In parallel to the analytical investigation, a Finite Element Analysis is performed using [8] to find the natural frequency and strain energy distribution. These numerical results are compared to test the validity and accuracy of the analytical solution obtained using Donnell-Mushtari shell theory.

The finite element model of the cylindrical shell has 1984 four-node thin shell elements shown in Figure 3 is arranged 64 elements along the circumference and 31 elements along its length. Unigraphics NX-3 [8] was used for the modal analysis and the results were post processed using Structures P.E. to see the displaced geometry of the various mode shapes of vibration.

Due to the mesh density the results are accurate for $n < 8$.

Those results are presented graphically in Figure 4

To describe the sound field inside a cylindrical shell we shall use the inhomogeneous wave equation, and will require detail information about the acoustic source. However this would require statistical information for the internal pressure fluctuation and to obtain a generalized non-dimensional collapse of the data. It also should be noted that, when studying the interaction between sound wave within a cylindrical shell and the shell itself, it is convenient to assume rigid duct walls for the purpose of describing the sound field within the shell. When dealing with metallic structures the assumption are justified and are adequate for vibration purpose and external noise radiation purpose. In principle three specific cases

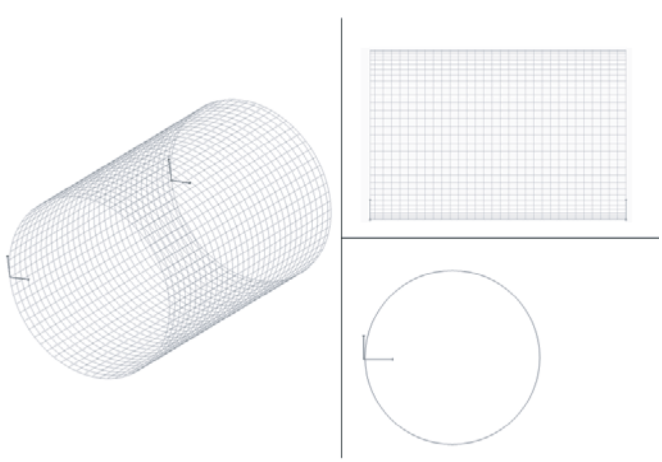

Figure 3. Finite element model of Shear-Diaphragm cylindrical shell

Table 1: Comparison of Natural Frequencies

Mode	**Frequency (Hz)**		**Difference %**
n,m	**FEA (F_f)**	**Analytical (F_a)**	**(F_a-F_f/F_f)*100**
4,1	254	252	−.079
5,1	279	276	−1.22
3,1	343	345	0.52
6,1	370	356	−3.65
7,1	496	477	−3.73
8,1	649	637	−1.77
6,2	520	528	1.56
5,2	549	551	0.36
2,1	619	621	0.40
7,2	582	574	−1.44

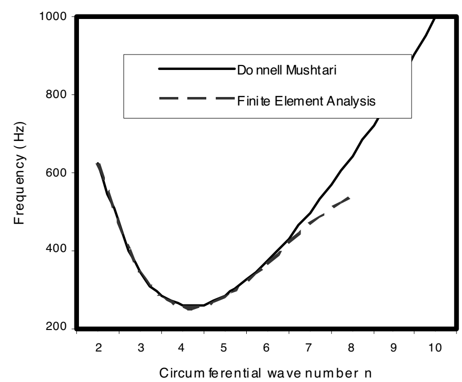

Figure 4. Frequency Vs Circumferential wave number

are possible shells with perfectly rigid walls, shells with infinitely flexible walls and shells with finitely flexible walls. For this project we will assume perfectly rigid.

For the rigid wall with radial, angular and axial coordinates *r*, |*vphi*|, *and x* the solution to homogeneous equation assuming propagation in the positive x-direction for the pressure associated with the acoustic propagation in a stationary internal fluid has the following form:

$$p(r,\theta,x) = \sum_p \sum_q (A_{pq}\cos(p\theta) + B_{pq}\sin(p\theta))J_p(\kappa_{pq}r)e^{i(k_x x-\omega t)}$$

Where:

ω = radian frequency
k_x = axial acoustic wave number
c_i = speed of sound in the internal fluid
J_p = Bessel function of the first kind of order *p*

$$\kappa_{pq}^2 + k_x^2 = k^2 = \left(\frac{\omega}{c_i}\right)^2$$

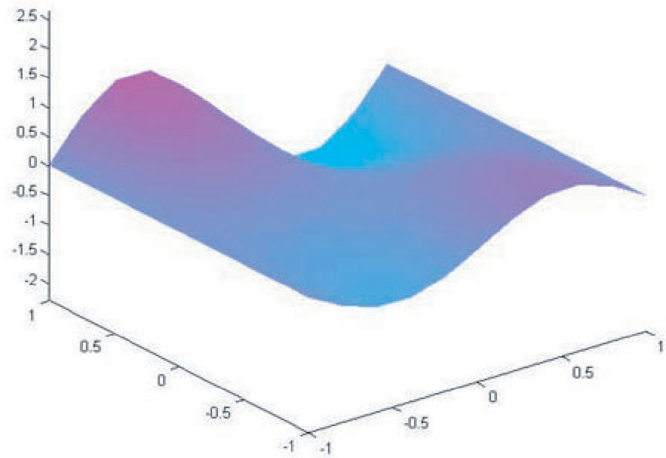

Figure 5

Cut-off frequency is defined as:

$$(\omega_{co})_{pq} = \kappa_{pq} c_i$$

Now

$$\kappa_{pq} = \frac{\pi \alpha_{pq}}{a_i}$$

Where a_i, is the internal pipe radius and the $\pi\alpha_{pq}$, are determined from eigenvalues satisfying the rigid wall boundary condition $J'_p(\kappa_{pq}, a_i) = 0$. Where J' is the first derivative of the Bessel function with respect to *r*. Thus the sound wave in a cylindrical shell can only propagate as plane wave (*p=q=0*), if $ka_i < 1.8412$, where the wave number *k* is given by $\kappa_{pq}^2 + k_x^2 = k^2 = \left(\frac{\omega}{c_i}\right)^2$, and both plane wave and higher acoustics modes if $ka_i > 1.9412$

A MATLAB was used to plot the solution for the sound propagation in the cylindrical shell. The figure below show the sound propagation as a function of r, θ, x.

4. CONCLUDING REMARKS

The analytical expressions derived can be used to reveal trends in the variation of modal frequency and strain energy distribution with cylinder geometry, that are very critical in the design of circular cylindrical shells for dynamic response.The fundamental modal characteristics of the shell is function of the unique combinations of those energy components. The sound response depends on radius of the shell, phase angle and position. In Figure 5 one can see that the shape of the sound propagating through the cylinder is of sinusoidal nature. This is because the pressure which is one of our dependent variables for sound intensity has *sin* and *cos* terms are present.

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