Weibull Distribution for Estimating the Parameters and Application of Hilbert Transform in case of a Low Wind Speed at Kolaghat

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ABSTRACT

The wind resource varies with of the day and the season of the year and even some extent from year to year. Wind energy has inherent variances and hence it has been expressed by distribution functions. In this paper, we present some methods for estimating Weibull parameters in case of a low wind speed characterization, namely, shape parameter (k), scale parameter (c) and characterize the discrete wind data sample by the discrete Hilbert transform. We know that the Weibull distribution is an important distribution especially for reliability and maintainability analysis. The suitable values for both shape parameter and scale parameters of Weibull distribution are important for selecting locations of installing wind turbine generators. The scale parameter of Weibull distribution also important to determine whether a wind farm is good or not. Thereafter the use of discrete Hilbert transform (DHT) for wind speed characterization provides a new era of using DHT besides its application in digital signal processing. Basically in this paper, discrete Hilbert transform has been applied to characterize the wind sample data measured on College of Engineering and Management, Kolaghat, East Midnapore, India in January 2011.

Keywords: Wind Speed, Probability Distribution, Weibull Distribution, Linear Least Square Method, Maximum Likelihood Method, Discrete Hilbert Transform.

1. INTRODUCTION

After the Oil Crisis in 1973, the demand of wind energy like other non renewable energy is increasing day by day due to have the benefit of pollution free energy resources. The basic problem of using the wind energy resources is these resources are vary with few parameters like temperature, humidity etc. Today, most of the electrical energy is generated by burning huge fossil fuels and special weather conditions such as acid rain and snow, climate change, urban smog, regional haze, several tornados, etc., have happened around the whole world. It is now clear that the installation of a number of wind turbine generators can effectively reduce environmental pollution, fossil fuel consumption, and the costs of overall electricity generation. Although wind is only an intermittent source of energy, it represents a reliable energy resource from a long-term energy policy viewpoint. Among various renewable energy resources, wind power energy is one of the most popular and promising energy resources in the whole world today. In this paper we basically discuss about the low wind speed available in the month of January, 2011 on College of Engineering and Management, Kolaghat, East Midnapore, India. Actually here at first we addresses the relations among Mean wind speed (MWS), its standard deviation, and two important parameters of Weibull distribution [1,4,9]. In the month of January we generally observe the low wind speed in Kolaghat, India; not exceeding the speed of 8 km/hr. Due to less Convection process in Winter, the less agitation in wind occurs.we consider it to be as horizontal to the ground and the phase deflection is negligible between the discrete wind data sample. As we know that the electric field of the Electromagnetic wave is perpendicular to the ground, while two harmonic data are sent to judge the output to predict wind characteristic, it is better to use the Discrete Hilbert transformer [5,7,8] because polarization effect [2] occurs between vertical electric field of electromagnetic wave and horizontal two harmonic charged data sequences for predicting wind characteristic, as a result 90° phase delay occurs. The DHT of an individual input sample can really reflect the local variation performance, since it is the convolution with the reciprocal of time and the input data sequence, but there exists phase shift. For harmonic signals, the output signal holds a 90° phase delay.

2. WEIBULL DISTRIBUTION

The Weibull distribution is characterized by two parameters, one is the shape parameter k (dimensionless) and the other is the scale parameter c (m/s)

The cumulative distribution function is given by

$$F(v) = 1 - \exp\left[-\left(\frac{v}{c}\right)^k\right] \tag{1}$$

And the probability function is given by

$$f(v) = \frac{dF(v)}{dv} = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^k\right] \tag{2}$$

The average wind speed can be expressed as

$$\bar{v} = \int_0^\infty v f(v)dv = \int_0^\infty \frac{vk}{c}\left[\left(\frac{v}{c}\right)^{k-1}\right]\exp\left[-\left(\frac{v}{c}\right)^k\right]dv \tag{3}$$

Let $x = \left(\frac{v}{c}\right)^k$, $x^{\frac{1}{k}} = \frac{v}{c}$ and $dx = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} dv$

Equation (3) can be simplified as

$$\bar{v} = c\int_0^\infty x^{\frac{1}{k}} \exp(-x)dx \tag{4}$$

By substituting a Gamma Function

$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx$ into (4) and let $y = 1 + \frac{1}{k}$ then we have

$$\bar{v} = c\Gamma\left(1 + \frac{1}{k}\right) \quad (5)$$

The standard deviation of wind speed v is given by $\sigma = \sqrt{\int_0^\infty (v - \bar{v})^2 f(v) dv}$ (6)

i.e.

$$\begin{aligned} \sigma &= \sqrt{\int_0^\infty (v^2 - 2v\bar{v} + \bar{v}^2) f(v) dv} \\ &= \sqrt{\int_0^\infty v^2 f(v) dv - 2\bar{v}\int_0^\infty v f(v) dv + \bar{v}^2} \\ &= \sqrt{\int_0^\infty v^2 f(v) dv - 2\bar{v}.\bar{v} + \bar{v}^2} \end{aligned} \quad (7)$$

Use

$$\int_0^\infty v^2 f(v) dv = \int_0^\infty v^2 \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} dv = \int_0^\infty c^2 x^{\frac{2}{k}} \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} dv = \int_0^\infty c^2 x^{\frac{2}{k}} \exp(-x) dx \quad (8)$$

And put $y = 1 + \frac{2}{k}$, then the following equation can be obtained

$$\int_0^\infty v^2 f(v) dv = c^2 \Gamma\left(1 + \frac{2}{k}\right) \quad (9)$$

Hence we get

$$\begin{aligned} \sigma &= \left[c^2\Gamma\left(1 + \frac{2}{k}\right) - c^2\Gamma^2\left(1 + \frac{1}{k}\right)\right]^{\frac{1}{2}} \\ &= c\sqrt{\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)} \end{aligned} \quad (10)$$

3. LINEAR LEAST SQUARE METHOD (LLSM)

Least square method is used to calculate the parameter(s) in a formula when modeling an experiment of a phenomenon and it can give an estimation of the parameters. When using least square method, the sum of the squares of the deviations S which is defined as below, should be minimized.

$$S = \sum_{i=1}^{n} w_i^2 \left[y_i - g(x_i) \right]^2 \tag{11}$$

In the equation, x_i is the wind speed, y_i is the probability of the wind speed rank, so (x_i, y_i) mean the data plot, w_i is a weight value of the plot and n is a number of the data plot. The estimation technique we shall discuss is known as the Linear Least Square Method (LLSM), which is a computational approach to fitting a mathematical or statistical model to data. It is so commonly applied in engineering and mathematics problem that is often not thought of as an estimation problem. The linear least square method (LLSM) is a special case for the least square method with a formula which consists of some linear functions and it is easy to use. And in the more special case that the formula is line, the linear least square method is much easier. The Weibull distribution function is a non-linear function, which is

$$F(v) = 1 - \exp\left[-\left(\frac{v}{c} \right)^k \right] \tag{12}$$

$$\text{i.e. } \frac{1}{1 - F(v)} = \exp\left[\left(\frac{v}{c} \right)^k \right] \tag{13}$$

$$\text{i.e. } \ln\{\frac{1}{1 - F(v)}\} = \left[\left(\frac{v}{c} \right)^k \right] \tag{14}$$

But the cumulative Weibull distribution function is transformed to a linear function like below:

$$\text{Again } \ln\ln\left\{ \frac{1}{1 - F(v)} \right\} = k\ln v - k\ln c \tag{15}$$

Equation (15) can be written as $Y = bX + a$

where $Y = \ln\ln\left\{ \frac{1}{1 - F(v)} \right\}$, $X = \ln v$, $a = -k \ln c$, $b = k$

By Linear regression formula

$$b = \frac{n\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{n\sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i \right)^2} \tag{16}$$

$$a = \frac{\sum_{i=1}^{n} X_i^2 \sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} X_i Y_i}{n\sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i \right)^2} \tag{17}$$

4. MAXIMUM LIKELIHOOD ESTIMATOR(MLE)

The method of maximum likelihood (Harter and Moore (1965a), Harter and Moore (1965b), and Cohen (1965)) is a commonly used procedure because it has very desirable properties.

Let $x_1, x_2, \ldots\ldots\ldots\ldots\ldots\ldots x_n$ be a random sample of size n drawn from a probability density function $f(x, \theta)$ where θ is an unknown parameter. The likelihood function of this random sample is the joint density of the n random variables and is a function of the unknown parameter. Thus

$$L = \prod_{i=1}^{n} f_{X_i}(x_i, \theta) \tag{18}$$

is the Likelihood function. The Maximum Likelihood Estimator (MLE) of θ, say $\bar{\theta}$, is the value of θ, that maximizes L or, equivalently, the logarithm of L . Often, but not always, the MLE of q is a solution of

$$\frac{d\text{Log}L}{d\theta} = 0 \tag{19}$$

Now, we apply the MLE to estimate the Weibull parameters, namely the shape parameter and the scale parameters. Consider the Weibull probability density function (pdf) given in (2), then likelihood function will be

$$L(x_{1,} x_2, .., x_n, k, c) = \prod_{i=1}^{n} \left(\frac{k}{c}\right)\left(\frac{x_i}{c}\right)^{k-1} e^{-\left(\frac{x_i}{c}\right)^k} \tag{20}$$

On taking the logarithms of (20), differentiating with respect to k and c in turn and equating to zero, we obtain the estimating equations

$$\frac{\partial \ln L}{\partial k} = \frac{n}{k} + \sum_{i=1}^{n} \ln x_i - \frac{1}{c}\sum_{i=1}^{n} x_i^k \ln x_i = 0 \tag{21}$$

$$\frac{\partial \ln L}{\partial c} = \frac{-n}{c} + \frac{1}{c^2}\sum_{i=1}^{n} x_i^k = 0 \tag{22}$$

On eliminating c between these two above equations and simplifying, we get

$$\frac{\sum_{i=1}^{n} x_i^k \ln x_i}{\sum_{i=1}^{n} x_i^k} - \frac{1}{k} - \frac{1}{n}\sum_{i=1}^{n} \ln x_i = 0 \tag{23}$$

which may be solved to get the estimate of k. This can be accomplished by Newton-Raphson method. Which can be written in the form

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{24}$$

Where

$$f(k) = \frac{\sum_{i=1}^{n} x_i^k \ln x_i}{\sum_{i=1}^{n} x_i^k} - \frac{1}{k} - \frac{1}{n}\sum_{i=1}^{n} \ln x_i \tag{25}$$

And

$$f'(k) = \sum_{i=1}^{n} x_i^k (\ln x_i)^2 - \frac{1}{k^2}\sum_{i=1}^{n} x_i^k (k \ln x_i - 1) - \left(\frac{1}{n}\sum_{i=1}^{n} \ln x_i\right)\left(\sum_{i=1}^{n} x_i^k \ln x_i\right) \tag{26}$$

Once k is determined, c can be estimated using equation (22) as

$$c = \frac{\sum_{i=1}^{n} x_i^k}{n} \tag{27}$$

5. HILBERT T-RANSFORMATION

The impulse response $h(t)$ of linear analog time invariant system and its transfer function $H(jw)$ are given by the Fourier pair,

$h(t) \overset{F}{\Leftrightarrow} H(jw)$ where $H(jw)$ is the transfer function which is defined as the ratio of output signal to the input signal. The delta pulse is represented by

$$\delta(i) = 1, i = 0$$
$$\delta(i) = 0, i \neq 0$$

The impulse response of a linear time invariant system is $h(t)$ which is actually a response of a unit impulse sample $\delta(i)$ analogously, $h(t) \overset{F}{\Leftrightarrow} H(k)$. The Hilbert transform of a unit impulse is $h(\delta(t)) = 1/\pi t$, which is a non causal physically unrealizable Hilbert transform filter of the transfer function is given by the Fourier transform

pair $h(\delta(t)) = 1/\pi t \overset{F}{\Leftrightarrow} H(jw) = -j\,\mathrm{sgn}(w)$. The discrete equivalent of the transfer function of an ideal discrete Hilbert filter is given by

$H(k) = -j$ for $k = 1, 2, \ldots\ldots..(N-1)/2$.

$H(k) = 0$ for $k = 0$.

$H(k) = j$ for $k = (N+1)/2, (N+1)/2+1, \ldots\ldots.., (N-1)$.

where N is odd. If N is even, one has

$H(k) = -j$ for $k = 1,2,\ldots\ldots..N/2 - 1$

$H(k) = 0$ for $k = 0$.

$H(k) = j$ for $k = N/2 + 1, N/2 + 2\ldots\ldots.., (N - 1)$.

The discrete transfer function may be written in the closed form as $H(k) = -j\,\mathrm{sgn}(N/2 - k)\,\mathrm{sgn}(k)$. The impulse response of this transfer function can be obtained by the inverse discrete Fourier transform (IDFT) of $H(k)$ is $h(i) = -1/N\sum_{k=0}^{N-1} H(k)e^{jw} = 2/N\sum_{k=1}^{(N-1)/2} \sin 2\pi ik/N$ with w = 2pik/N, whose closed form is $h(i) = -2/N\sin^2(\pi i/2)/\tan(\pi i/N)$. Consequently by using the notion of unit impulse sample, any input sequence may be written in the form of $u(i) = \sum_{m=0}^{N-1} a(m)\delta(i-m)$, which is a linear sum of successive samples of values given by the coefficients $a(m)$. Since the discrete Hilbert transform (DHT) is a linear operation, the sequence consisting of a single delta sample and zeros and its DHT is

$$\delta(i-m) \overset{DHT}{\Leftrightarrow} 2/N\sin^2[\pi(i-m)/2]/\tan[\pi(i-m)/N]$$

This yields the DHT of the input signals $u(i)$

$$Hu(i) = u(i) \oplus \delta(i-m)$$

$$= \sum_{m=0}^{N-1} a(m)2/N\sin^2[\pi(i-m)/2]/\tan[\pi(i-m)/N] \tag{28}$$

no matter whether N is odd/even.

6. GRAPHICAL MODELING FOR WIND SPEED FORECASTING

The effectiveness of a discrete Hilbert transformer should be confirmed before its application to estimate wind speed. For this purpose, it is appropriate to use the harmonic input sample, whose DHT can be directly derived from theoretical analysis. Thus, two sequences generated by two harmonic functions with different periods are employed. The input signal for Fig. A is cos(8×2π/N) with sin(3×2π/N) for Fig. B

From the fig .1(a) and 1.(b) the DHT filter verification is successfully done here.

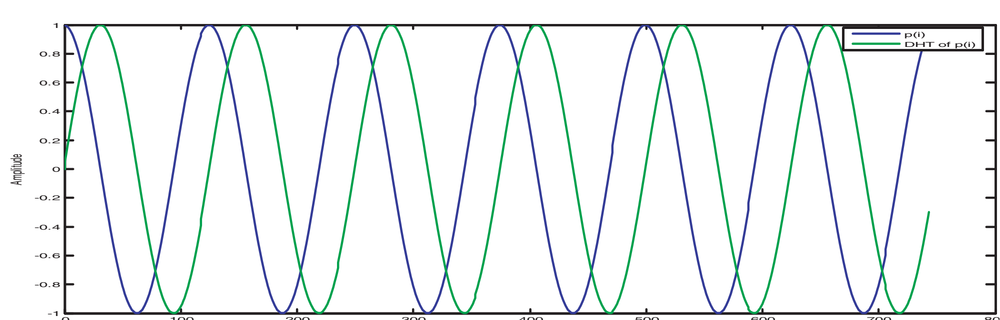

Figure A: The cosinoidal input sample with eight periods

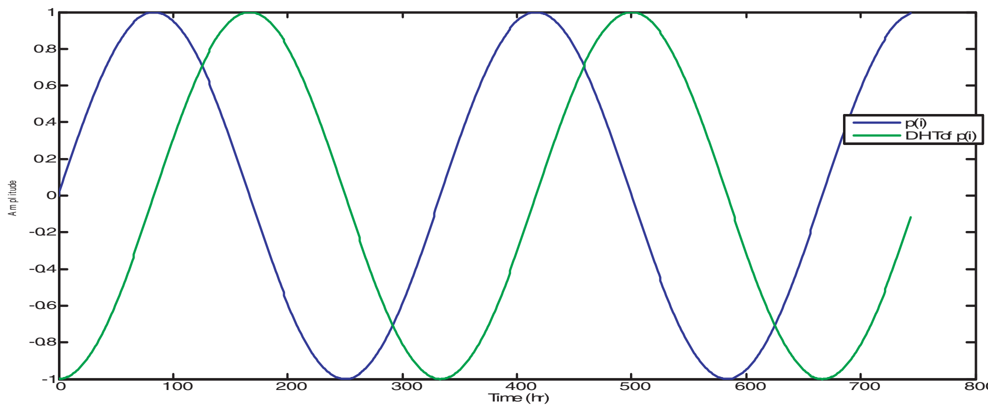

Figure B: The sinusoidal input sample with 3 periods.

7. RESULTS AND DISCUSSIONS

When a location has c = 6 the pdf under various values of k are shown in Fig. 1. A higher value of k such as 2.5 or 4 indicates that the variation of Mean Wind speed is small. A lower value of k such as 1.5 or 2 indicates a greater deviation away from Mean Wind speed.

When a location has k = 3 the pdf under various values of c are shown in Fig. 2. A higher value of c such as 11 or 12 indicates that the variation getting smaller. A lower value of c such as 9 or 10 indicates a greater value of deviation mean wind speed.

Fig. 3 represents the characteristic curve of $\Gamma\left(1+\frac{1}{k}\right)$. versus shape parameter k. The values of $\Gamma\left(1+\frac{1}{k}\right)$. varies around .889 when k is between 1.9 to 2.6.

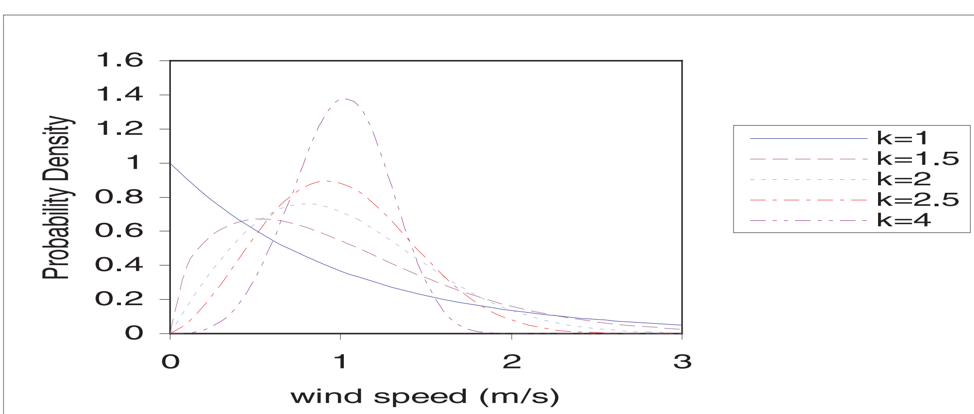

Figure 1: Weibull Distribution Density versus wind speed under a constant value of *c* and different values of *k*

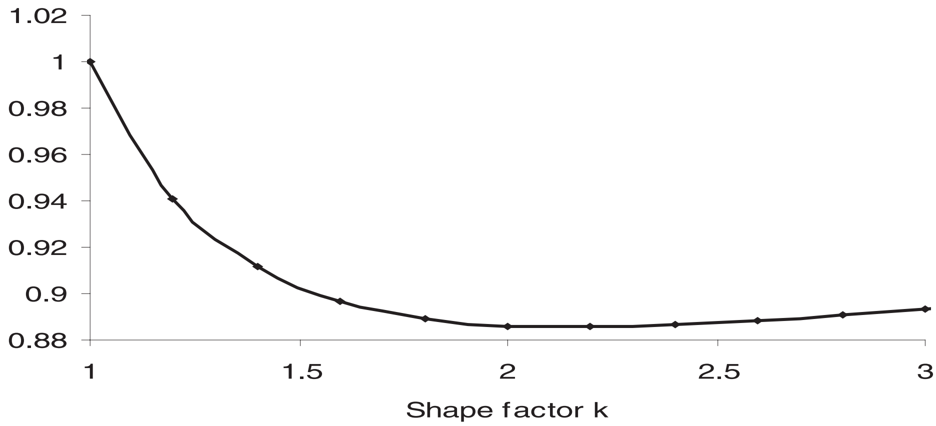

Figure 2: Weibull Distribution Density versus wind speed under a constant value of $k = 3$ and different values of c

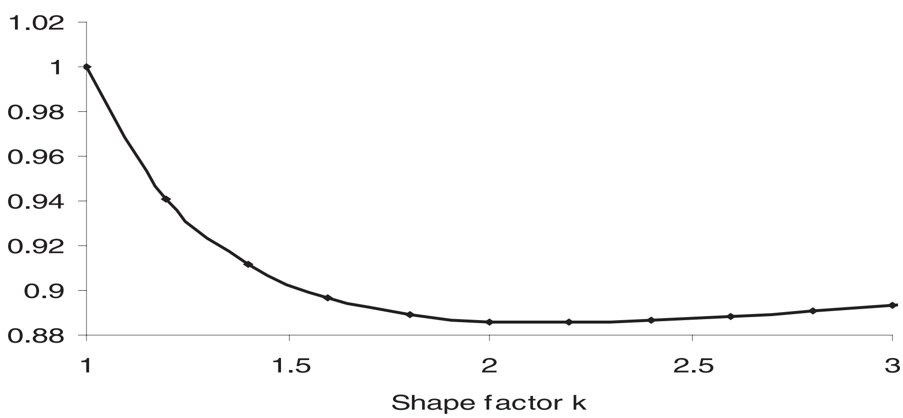

Figure 3: Characteristic curve of $\Gamma(1 + 1/k)$ versus Shape parameter k

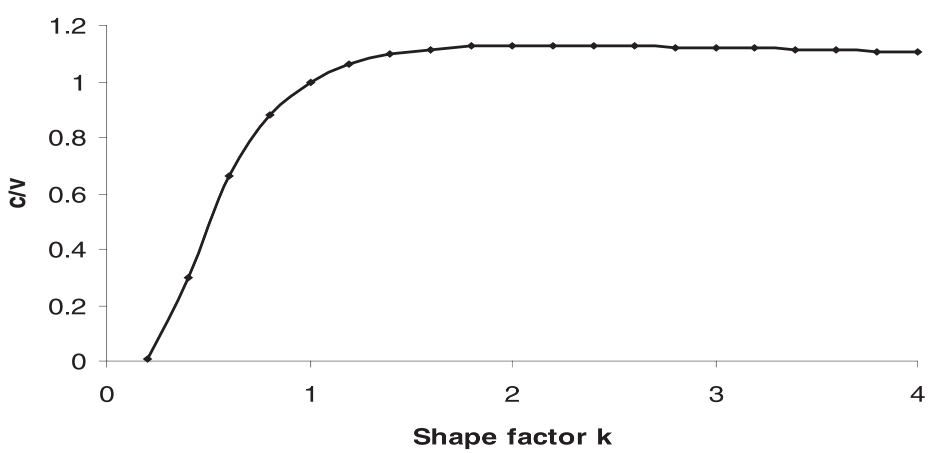

Figure 4: Characteristics curve of $c/\tilde{v}$ versus shape parameter k

Fig. 4 represents the characteristic curve of $\frac{c}{\overline{v}}$ versus shape parameter k. Normally the wind speed data collected at a specified location are used to calculate Mean Wind speed. A good estimate for parameter c can be obtained from Fig.4 as $c = 1.128\overline{v}$ = where k ranges from 1.6 to 4. If the parameter k is less than unity, the ratio $\frac{c}{\overline{v}}$ decrease rapidly. Hence c is directly proportional to Mean Wind speed for $1.6 \leq k \leq 4$ and Mean Wind speed is mainly affected by c. The most good wind farms have k in this specified range and estimation of c in terms of $\overline{v}$ may have wide applications.

Also let $F(x_i) = \frac{i}{n+1}$ and using equations (16) and (17) we get $k = 1.013658$ and $c = 29.9931$

But if we apply maximum Likelihood Method we get k = 1.912128 and c = 1.335916. There is a huge difference in value of c by the above two methods. This is due to the mean rank of $F(x_i)$ and k value is tends to unity.

Here the Wind data samples are measured from the Anemometer available at College of Engineering & Management, Kolaghat (CEMK), where $U(i)$ represents the Average Monthly Wind Speed (km/hr) at kolaghat (from 1st January, 2011 to 31st January, 2011) for characterization of Wind speed. The discrete Wind data samples are shown in below charts.

January, 2011	Wind Speed (km/hr)	January, 2011	Wind Speed (km/hr)
1	3	17	5
2	9	18	2
3	6	19	2
4	8	20	3
5	8	21	3
6	3	22	2
7	3	23	2
8	3	24	2
9	5	25	2
10	8	26	2
11	3	27	3
12	3	28	5
13	2	29	3
14	2	30	3
15	2	31	6
16	8		

The picture of the Anemometer at CEMK is given below Fig. 5.

Figure 5: The Operational Anemometer at CEMK

After taking the day basis reading from the Anemometer, we take the DHT of it to characterize the wind speed nature in winter.Here we denote the wind speed sample as $u(i)$ and find the DHT of $u(i)$ by (28).Then we obtain the following graph shown in Fig. 6.

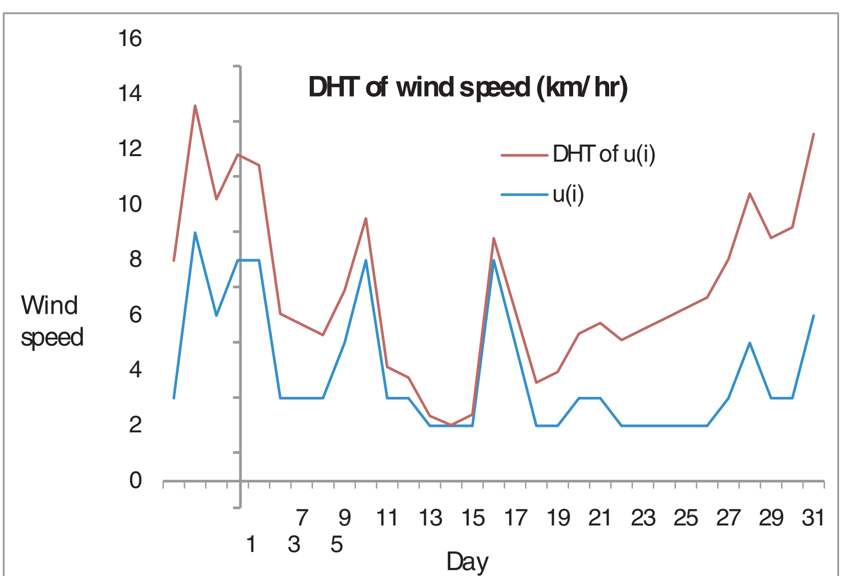

Figure 6: the sample of wind speed and the estimated wind speed by the DHT.

8. CONCLUSIONS

In this paper, we have presented two analytical methods for estimating the Weibull distribution parameters in case of the low wind speed characterization. The above results will help the scientists and the technocrats to select the location for Wind Turbine Generators. By applying Discrete Hilbert Transform (DHT) we estimate the wind speed with the sample data sequence selected from the data record observed by the observatory in Kolaghat in January 2011, during which the data pertain to deep valleys of the Rupnarayan river and sharp crests due to manifold weather conditions in this region. To confirm the performance of the discrete Hilbert transformer, two harmonic input sequences were used to inspect the output signals, whether good agreement with the theoretical results is obtained. A knowledge of how to estimate the wind speed is useful for the wind engineering and building services engineering. The discrete Hilbert transform method, which was used to propose a new view in nonlinear waves successfully with a broadening area found in digital data processing, was applied for wind speed estimation. Not only the wind amplitude but also the instant phase angle of the wind speed can be treated with the DHT filter. Therefore the instantaneous frequency can easily be obtained by taking the derivative of that instant phase.

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