Hydromagnetic thermosolutal instability of compressible walters' (model B′) rotating fluid permeated with suspended particles in porous medium

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ABSTRACT
The thermosolutal instability of compressible Walters' (model B') elastico-viscous rotating fluid permeated with suspended particles (fine dust) in the presence of vertical magnetic field in porous medium is considered. By applying normal mode analysis method, the dispersion relation has been derived and solved analytically. It is observed that the rotation, magnetic field, suspended particles and viscoelasticity introduce oscillatory modes. For stationary convection the Walters' (model B') fluid behaves like an ordinary Newtonian fluid and it is observed that the rotation and stable solute gradient has stabilizing effects and suspended particles are found to have destabilizing effect on the system, whereas the medium permeability has stabilizing or destabilizing effect on the system under certain conditions. The magnetic field has destabilizing effect in the absence of rotation, whereas in the presence of rotation, magnetic field has stabilizing or destabilizing effect under certain conditions.

Keywords: Walters' (model B') fluid, thermosolutal instability, suspended particles, magnetic field, rotation , porous medium.

1. INTRODUCTION

A detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [1]. Chandra [2] observed a contradiction between the theory and experiment for the onset of convection in fluids heated from below. He performed the experiment in an air layer and found that the instability depended on the depth of the layer. A Benard-type cellular convection with the fluid descending at a cell centre was observed when the predicted gradients were imposed for layers deeper than 10 mm. A convection which was different in character from that in deeper layers occurred at much lower gradients than predicted if the layer depth was less than 7 mm, and called this motion, Columnar instability . He added an aerosol to mark the flow pattern.

Bhatia and Steiner [3] have studied the thermal instability of a Maxwellian visco-elastic fluid in the presence of magnetic field while the thermal convection in Oldroydian visco-elastic fluid has been considered by Sharma [4]. Veronis [5] has investigated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. The buoyancy forces can arise not only from density differences due to variations in solute concentration. Thermosolutal convection problems arise in oceanography, limnology and engineering.

The medium has been considered to be non-porous in all the above studies. Lapwood [6] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [7] whereas Scanlon and Segel [8] have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by the particles. The suspended particles were thus found to destabilize the layer.

Sharma and Sunil [9] have studied the thermal instability of an Oldroydian viscoelastic fluid with suspended particles in hydromagnetics in a porous medium. There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. One such class of fluids is Walters' (model B$'$) elastico-viscous fluid. Walters' [10] reported that the mixture of polymethyl methacrylate and pyridine at 25°C containg 30.5 g of polymer per litre with density 0.98 g per litre behaves very nearly as the Walters (Model B') elastico-viscous fluid. Walters' (Model B') elastico-viscous fluid form the basis for the manufacture of many important polymers and useful products. This and other class of polymers is used in the manufacture of parts of space-crafts, aeroplanes, tyres, belt conveyers, ropes, cushions, foam, plastics, engineering equipments etc. Recently, polymers are used in agriculture, communication appliances and biomedical applications.

Stommel and Fedorov [11] and Linden [12] have remarked that the length scalar characteristic of double diffusive convecting layers in the ocean may be sufficiently large that the Earth s rotation might be important in their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. Brakke [13] explained a double-diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose. The problem of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat transfer mechanism in the development of young oceanic crust (Lister, [14]). Generally, it is accepted that comets consist of a dusty snowball of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice-versa. The physical properties of comets, meteorites and inter-planetary dust strongly suggest the importance of porosity in the astrophysical context (McDonnel [15]).

When the fluids are compressible, the equations governing the system become quite complicated. Spiegel and Veronis [16] simplified the set of equations governing the flow of compressible fluids under the assumption that the depth of the fluid layer is much smaller than the scale height as defined by them, and the motions of infinitesimal amplitude are considered. Thermal instability of compressible finite-Larmor-radius Hall plasma was studied by Sharma and Sunil [17] in a porous medium.

A porous medium is a solid with holes in it, and is characterized by the manner in which the holes are imbedded, how they are interconnected and the description of their location, shape and interconnection. However, the flow of a fluid through a homogeneous and isotropic porous medium is governed by Darcy s law which states that the usual viscous term in the equations of motion of Walters (model B′) fluid is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\right]\boldsymbol{q}$, where μ and μ' are the viscosity and viscoelasticity of the compressible Walters' (model B') fluid, k_1 is the medium permeability and q is the Darcian (filter) velocity of the fluid. Such and other polymers are used in the manufacture of space crafts, aero planes, tyres, ropes, cushions, seats, foam, plastics, engineering equipments, adhesives, contact lenses etc. Recently, polymers are used in agriculture, communications appliances and in bio medical applications. Examples of these applications are filtration processes, packed bed reactors, insulation system, ceramic processing, enhanced oil recovery, chromatography etc.

Sharma and Rana [18] have studied thermal instability of incompressible Walters (Model *B′*) elastico-viscous in the presence of variable gravity field and rotation in porous medium. Sharma and Rana [19] have also studied the thermosolutal instability of incompressible Walters (Model *B′*) rotating fluid permeated with suspended particles and variable gravity field in porous medium.

The Bénard problem (the onset of convection in a horizontal layer uniformly heated from below) for for incompressible Rivlin-Ericksen rotating fluid permeated with suspended particles and variable gravity field in porous medium was studied by Rana and Kumar [20]. Recently, Rana and Kango [21] have studied thermal instability of compressible Walters' (Model B') elastico-viscous rotating fluid a permitted with suspended dust particles in porous medium. In the present paper, the study is extended to thermosolutal instability of compressible Walters' (model B′) rotating fluid permeated with suspended particles and uniform vertical magnetic field in porous medium.

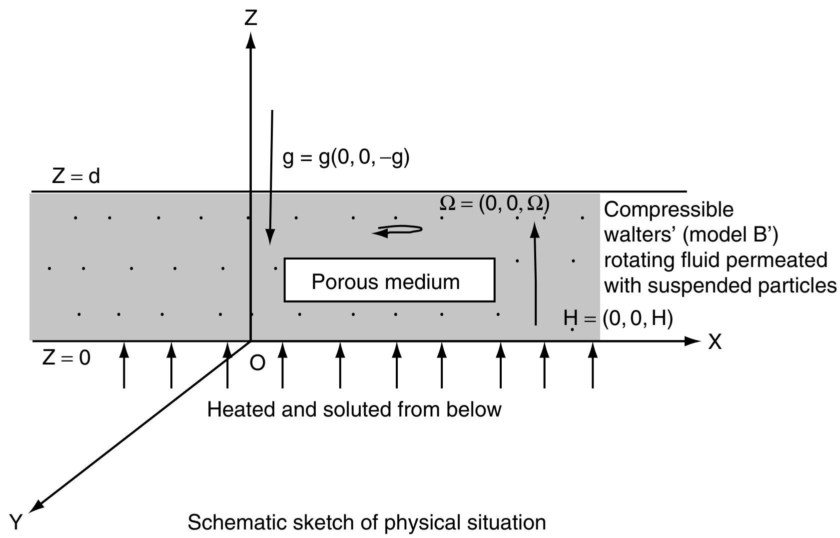
2. MATHEMATICAL MODEL

Consider an infinite horizontal layer of an electrically conducting Walters (model B′) elastico-viscous fluid of depth d in a porous medium bounded by the planes z = 0 and z = d in an isotropic and homogeneous medium of porosity $\in$ and permeability k_1, which is acted upon by a uniform rotation *Ω(*0, 0, *Ω),* uniform vertical magnetic field *H(*0, 0, *H)* and variable gravity *g(*0, 0, *-g).* This layer is heated and soluted from below such that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ and a uniform solute gradient $\beta'\left(=\left|\frac{dC}{dz}\right|\right)$ are maintained. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.

The hydromagnetic equations in porous medium (Chandrasekhar [1], Walters (model B′) [10], Sharma and Rana [19]) relevant to the problem are

$$\frac{1}{\in}\left[\frac{\partial q}{\partial t}+\frac{1}{\in}(q\cdot\nabla)q\right]=-\frac{1}{\rho_m}\nabla p+g\left(1+\frac{\delta\rho}{\rho_m}\right)-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)q+\frac{2}{\in}(q\times\Omega)$$
$$+\frac{K'N}{\rho_m\in}(q_d-q)+\frac{\mu_e}{4\pi\rho_m}(\nabla\times H)\times H, \tag{1}$$

$$\nabla\cdot q=0, \tag{2}$$



Schematic sketch of physical situation

$$\frac{\partial T}{\partial t}+(q\cdot\nabla)T+\frac{mNC_{pt}}{\rho_0 C_f}\left[\in\frac{\partial}{\partial t}+q_d\cdot\nabla\right]T=\kappa\nabla^2 T, \tag{3}$$

$$\frac{\partial C}{\partial t}+(q\cdot\nabla)C+\frac{mNC'_{pt}}{\rho_0 C'_f}\left[\in\frac{\partial}{\partial t}+q_d\cdot\nabla\right]T=\kappa'\nabla^2 C \tag{4}$$

$$\nabla\cdot H=0, \tag{5}$$

$$\in\frac{\partial H}{\partial t}=\nabla\times(q\times H)+\in\eta\nabla^2 H\ , \tag{6}$$

Here $q_d(\overline{x},t)$ and $N(\overline{x},t)$ denote the velocity and number density of the particles respectively, c_f, c_{pt}', denote respectively, the heat capacity of pure fluid, heat capacity of the particles and c_f', c_{pt}' heat capacities analogous to solute. $K' = 6\pi\eta\rho\upsilon$, where η is particle radius, is the Stokes drag coefficient, $q_d = (l, r, s)$ and $\overline{x} = (x, y, z)$. κ and κ' denote the thermal diffusivity and solute diffusivity respectively.

Assuming uniform particle size, spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term proportional to the velocity difference between particles and fluid and appears in the equation of motion (1). The force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign. The buoyancy force on the particles is neglected. Interparticle reactions are not considered, since we assume that the distance between the particles are quite large compared with their diameters.

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are

$$mN\left[\frac{\partial q_d}{\partial t}+\frac{1}{\in}(q_d\cdot\nabla)q_d\right]=K'N(q-q_d), \tag{7}$$

$$\in \frac{\partial N}{\partial t} + \nabla \cdot (N q_d) = 0, \tag{8}$$

The state variables pressure, density and temperature are expressed in the form (Spiegel and Veronis [16])

$$f(x,y,z,t) = f_m + f_0(z) + f'(x,y,z,t), \tag{9}$$

where f_m denotes for constant space distribution f, f_0 is the variation in the absence of motion, and $f'(x, y, z, t)$ is the fluctuation resulting from motion. The basic state of the system is

$$p = p(z),\ \rho = \rho(z),\ T = T(z),\ C = C(z),\ \ q = (0,\ 0,\ 0) \quad q_d = (0,\ 0,\ 0) \quad \text{and } N = N_0 \tag{10}$$

where

$$\begin{aligned}
&p(z) = p_m - g\int_0^z (\rho_m + \rho_0)dz,\\
&\rho(z) = \rho_m[1 - \alpha_m(T - T_0) + \alpha'(C - C_0) + K_m(p - p_m)],\\
&T = -\beta z + T_0,\ C = -\beta' z + C_0,\ \alpha_m = -\left(\frac{1}{\rho}\frac{\partial \rho}{\partial T}\right)_m,\\
&\alpha_m{}' = -\left(\frac{1}{\rho}\frac{\partial \rho}{\partial C}\right)_m,\ K_m = \left(\frac{1}{\rho}\frac{\partial \rho}{\partial p}\right)_m.
\end{aligned} \tag{11}$$

Here p_m and ρ_m denote a constant space distribution of p and ρ while T_0 and ρ_0 denote temperature and density of the fluid at the lower boundary.

3. PERTURBATION EQUATIONS

Let q (u, v, w), q_d (l, r, s), θ, γ, δp , and $\delta\rho$ denote, respectively, the perturbations in fluid velocity q(0, 0, 0), the perturbation in particle velocity q_d(0, 0, 0), temperature T, solute concentration C, pressure p and density ρ.

The change in density $\delta\rho$ caused by perturbation θ and γ in temperature and solute concentration is given by

$$\delta\rho = -\rho_m(\alpha\theta - \alpha'\gamma). \tag{12}$$

The linearized perturbation equations governing the motion of fluids are

$$\frac{1}{\in}\frac{\partial q}{\partial t} = -\frac{1}{\rho_m}\nabla\delta p - g(\alpha\theta - \alpha'\gamma) - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)q + \frac{K'N}{\in}(q_d - q) + \frac{2}{\in}(q \times \Omega) + \frac{\mu_e}{4\pi\rho_m}(\nabla \times \boldsymbol{h}) \times \boldsymbol{H} \tag{13}$$

$$\nabla \cdot q = 0, \tag{14}$$

$$\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\boldsymbol{q_d} = q, \tag{15}$$

$$(1+b\in)\frac{\partial\theta}{\partial t} = \left(\beta-\frac{g}{c_p}\right)(w+bs)+\kappa\nabla^2\theta, \tag{16}$$

$$(1+b'\in)\frac{\partial\theta}{\partial t} = \beta'(w+b's)+\kappa'\nabla^2\gamma \tag{17}$$

$$\nabla\cdot h = 0, \tag{18}$$

$$\in\frac{\partial H}{\partial t} = (H\cdot\nabla)q+\in\eta\nabla^2 H, \tag{19}$$

where $b=\dfrac{mNC_{pt}}{\rho_m C_f}$, $b'=\dfrac{mNC'_{pt}}{\rho_m C'_f}$ and w, s are the vertical fluid and particles velocity.

4. DISPERSION RELATION

Analyzing the disturbances into normal modes, we assume that the perturbation quantities have x, y and t dependence of the form

$$[w,s,\theta,\gamma,\zeta,h_z,\xi] = [W(z),S(z),\Theta(z),Z(z),\Gamma(z),\\ K(z),X(z)]\,exp\,(ik_x x+ik_y y+nt), \tag{20}$$

where k_x and k_y are the wave numbers in the x and y directions, $k=(k_x^2+k_y^2)^{1/2}$ is the resultant wave number and n is the frequency of the harmonic disturbance, which is, in general, a complex constant. Also

$\zeta = \dfrac{\partial \upsilon}{\partial x}-\dfrac{\partial u}{\partial y}$ is the z-component of vorticity

and $\xi = \dfrac{\partial h_y}{\partial x}-\dfrac{\partial h_x}{\partial_y}$ is the z-component of current density.

Using expression (20), equations (13) — (19) in non dimensional form, become

$$\left[\frac{\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right](D^2-a^2)W+\frac{ga^2d^2\alpha\Theta}{\upsilon}+\frac{2\Omega d^2}{\in\upsilon}DZ-\frac{\mu_e Hd}{4\pi\upsilon\rho_m}(D^2-a^2)DK=0, \tag{21}$$

$$\left[\frac{\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right]z=\left(\frac{2\Omega d^2}{\in\upsilon}\right)DW+\frac{\mu_e Hd}{4\pi\upsilon\rho_m}DX, \tag{22}$$

$$[D^2-a^2-p_1\sigma]X=-\left(\frac{Hd}{\in\eta}\right)DZ, \tag{23}$$

$$[D^2-a^2-p_2\sigma]K=-\left(\frac{Hd}{\in\eta}\right)DW, \tag{24}$$

$$[D^2-a^2-E_1p_1\sigma]\Theta=-\frac{gd^2}{\kappa c_p}(G-1)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \tag{25}$$

$$[D^2-a^2-E_1'p_1'\sigma]\Gamma=-\left(\frac{\beta' d^2}{\kappa'}\right)\left(\frac{B'+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \tag{26}$$

where we have put

$a = kd$, $\sigma=\frac{nd^2}{\upsilon}$, $\tau=\frac{m}{K'}$, $\tau_1=\frac{\tau\upsilon}{d^2}$, $M=\frac{mN}{\rho_m}$, $G=\left(\frac{c_p}{g}\right)\beta$, $E_1 = 1 + b \in$, $E_1' = 1 + b' \in$, B = b + 1, $B'=1+b'$ $F=\frac{\upsilon'}{d^2}$, $P_l=\frac{k_1}{d^2}$, is the dimensionless medium permeability, $p_1=\frac{\upsilon}{\kappa}$, is the thermal Prandtl number, $p_1'=\frac{\upsilon}{\kappa'}$, is the Schmidt number, $p_2=\frac{\upsilon}{\eta}$, is the magnetic Prandtl number $D^*=d\frac{d}{dz}$ and the superscript * is suppressed.

Applying the operator $(D^2 - a^2 - p_1\sigma)$ to the equation (22) to eliminate X between equations (22) and (23), we get

$$\left\{\left[\frac{\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right](D^2-a^2-p_2\sigma)+\frac{Q}{\in}D^2\right\}W=\frac{2\Omega d^2}{\upsilon}(D^2-a^2-p_2\sigma)DW. \tag{27}$$

Eliminating K, Θ and Z between equations (21) — (27), we obtain

$$\left[\frac{\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right](D^2-a^2)(D^2-a^2-E_1p_1\sigma)(D^2-a^2-p_2\sigma)(D^2-a^2-E_1'p_1'\sigma)W-Ra^2\left(\frac{G-1}{G}\right)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)(D^2-a^2-E_1'p_1'\sigma)(D^2-a^2-p_2\sigma)W+Sa^2\left(\frac{B'+\tau_1\sigma}{1+\tau_1\sigma}\right)(D^2-a^2-E_1p_1\sigma)(D^2-a^2-p_2\sigma)W+\frac{Q}{\in}(D^2-a^2)(D^2-a^2-E_1'p_1'\sigma)(D^2-a^2-E_1p_1\sigma)W+\left[\frac{\frac{T_A}{\in^2}(D^2-a^2-E_1p_1\sigma)(D^2-a^2-E_1'p_1'\sigma)(D^2-a^2-p_2\sigma)^2}{\left[\frac{\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right](D^2-a^2-p_2\sigma)+\frac{Q}{\in}D^2}\right]D^2W=0 \tag{28}$$

where $R=\dfrac{g\alpha\beta d^4}{\upsilon\kappa}$, is the thermal Rayleigh number,

$S=\dfrac{g\alpha'\beta' d^4}{\upsilon\kappa'}$, is the analogous solute Rayleigh number,

$Q=\dfrac{\mu_e H^2 d^2}{4\pi\upsilon\rho_m\eta}$, is the Chandrasekhar number

and $T_A=\left(\dfrac{2\Omega d^2}{\upsilon}\right)^2$, is the Taylor number.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries and adjoining medium is electrically non-conducting. The boundary conditions appropriate to the problem are (Chandrasekhar, [1]; Veronis, [5].

$$W = D^2W = DZ = \Gamma = \Theta = 0 \text{ at } z = 0 \text{ and } 1 \tag{29}$$

and the components of h are continuous. Since the components of the magnetic field are continuous and the tangential components are zero outside the fluid, we have

$$DK = 0, \tag{30}$$

on the boundaries. Using the boundary conditions (29) and (30), we can show that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$ and hence, the proper solution of equation (28) characterizing the lowest mode is

$$W = W_0 \sin \pi z; \quad W_0 \text{ is a constant.} \tag{31}$$

Substituting equation (31) in (28), we obtain the dispersion relation

$$R_1 x = \left(\frac{G}{G-1}\right)\left\{\left[\frac{i\sigma_1}{\in}\left(1+\frac{M}{1+\tau_1\pi^2 i\sigma_1}\right)+\frac{1-F\pi^2 i\sigma_1}{P}\right](1+x)(1+x+E_1p_1 i\sigma_1)\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right)+\frac{S_1 x\lambda(1+x+E_1p_1 i\sigma_1)}{(D^2-a^2-E_1'p_1'\sigma)}\left(\frac{B'+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right)+\frac{Q_1}{\in}\frac{(1+x)(1+x+E_1p_1 i\sigma_1)}{1+x+p_2 i\sigma_1}\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right)+\frac{\frac{T_{A_1}}{\in^2}(1+x+E_1p_1 i\sigma_1)}{\frac{i\sigma_1}{\in}\left(1+\frac{M}{1+\tau_1\pi^2 i\sigma_1}\right)+\frac{1-F\pi^2 i\sigma_1}{P}}\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right)\right\}, \tag{32}$$

where $R_1 = \dfrac{R}{\pi^4},\ S_1 = \dfrac{S}{\pi^4},\ T_{A_1} = \dfrac{T_A}{\pi^4},\ x = \dfrac{a^2}{\pi^2},\ i\sigma_1 = \dfrac{\sigma}{\pi^2},\ P = \pi^2 P_l,\ Q_1 = \dfrac{Q}{\pi^4}.$

Equation (32) is required dispersion relation accounting for the effect of suspended particles, stable solute gradient, magnetic field, medium permeability, compressibility, rotation on thermosolutal instability of compressible Walters (model B′) elastico-viscous fluid in porous medium. There is an analogous dispersion relation for thermal instability in Walters (Model *B′*) elastico-viscous rotating fluid permitted with suspended dust particles in porous medium in the absence of magnetic field and stable solute gradient as derived by Rana and Kango [21].

5. STABILITY OF THE SYSTEM AND OSCILLATORY MODES

Here we examine the possibility of oscillatory modes, if any, in Walters (model B′) elastico-viscous fluid due to the presence of suspended particles, stable solute gradient, rotation, magnetic field, viscoelasticity and variable gravity field. Multiply equation (21) by W^* the complex conjugate of W, integrating over the range of z and making use of equations (22)–(26) with the help of boundary conditions (29) and (30), we obtain

$$\left[\frac{\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right]I_1-\frac{\mu_e\in\eta}{4\pi\upsilon\rho_0}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)(I_2+p_2\sigma^*I_3)-\frac{\alpha\alpha^2 g\kappa}{\upsilon\beta}\left(\frac{G}{G-1}\right)\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)(I_4+E_1p_1\sigma^*I_5)+d^2\left[\frac{\sigma^*}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma^*}{P_l}\right]I_6+\frac{\mu_e\in\eta d^2}{4\pi\upsilon\rho_0}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)(I_7+p_2\sigma^*I_8)+\frac{\alpha'\alpha'^2 g\kappa'}{\upsilon\beta'}\left(\frac{1+\tau_1\sigma^*}{B'+\tau_1\sigma^*}\right)(I_9+E_1'p_1'\sigma^*I_{10})=0, \tag{33}$$

where $I_1 = \int_0^1 (|DW|^2 + a^2 |W|^2) dz,$

$$I_2 = \int_0^1 (|D^2 K|^2 + a^4 |K|^2 + 2a^2 |DK|^2) dz,$$

$$I_3 = \int_0^1 (|DK|^2 + a^2 |K|^2) dz,$$

$$I_4 = \int_0^1 (|D\Theta|^2 + a^2 |\Theta|^2) dz,$$

$$I_5 = \int_0^1 |\Theta|^2 \, dz,$$

$$I_6 = \int_0^1 |Z|^2 \, dz,$$

$$I_7 = \int_0^1 (|DX|^2 + a^2 |X|^2) dz,$$

$$I_8 = \int_0^1 |X|^2 \, dz,$$

$$I_9 = \int_0^1 | D\Gamma|^2 + a^2 |\Gamma|^2) dz,$$

$$I_{10} = \int_0^1 |\Gamma|^2 \, dz.$$

The integral part I_1–I_{10} are all positive definite. Putting $\sigma = i\sigma_i$ in equation (33), where σ_i is real and equating the imaginary parts, we obtain

$$\sigma_i \left\{ \left[\frac{1}{\in} \left(1 + \frac{M}{1 + \tau_1^2 \sigma_i^{\ 2}} \right) + \frac{F}{P_l} \right] (I_1 - d^2 I_4) - \frac{\mu_e \in \eta}{4\pi \upsilon \, \rho_0} \left[\left(\frac{\tau_1 (B-1)}{B^2 + \tau_1^{\ 2} \sigma_i^{\ 2}} \right) I_2 + \frac{B + \tau_1^{\ 2} \sigma_i^{\ 2}}{B^2 + \tau_1^{\ 2} \sigma_i^{\ 2}} p_2 I_3 \right] + \right.$$
$$\frac{\alpha \alpha^2 g \kappa}{\upsilon \beta'} \left(\frac{G}{G-1} \right) \left[\left(\frac{\tau_1 (B-1)}{B^2 + \tau_1^{\ 2} \sigma_i^{\ 2}} \right) I_4 + \frac{B + \tau_1^{\ 2} \sigma_i^{\ 2}}{B^2 + \tau_1^{\ 2} \sigma_i^{\ 2}} E_1 p_2 I_5 \right] -$$
$$\left. \frac{\alpha' \alpha^2 g \kappa'}{\upsilon \beta'} \left[\left(\frac{\tau_1 (B'-1)}{B'^2 + \tau_1^{\ 2} \sigma_i^{\ 2}} \right) I_9 + \frac{B' + \tau_1^{\ 2} \sigma_i^{\ 2}}{B'^2 + \tau_1^{\ 2} \sigma_i^{\ 2}} E_1' p_1' I_{10} \right] + \frac{\mu_e \in \eta d^2}{4\pi \upsilon \, \rho_0} \left[\left(\frac{\tau_1 (B-1)}{B^2 + \tau_1^{\ 2} \sigma_i^{\ 2}} \right) I_6 + \frac{B + \tau_1^{\ 2} \sigma_i^{\ 2}}{B^2 + \tau_1^{\ 2} \sigma_i^{\ 2}} p_2 I_8 \right] \right\} = 0 \tag{34}$$

Equation (34) implies that $\sigma_i = 0$ or $\sigma_i \rangle\ 0$ which mean that modes may be non oscillatory or oscillatory. The oscillatory modes introduced due to presence of rotation, stable solute gradient, magnetic field, suspended particles and viscoelasticity. This result is an agreement with the result derived by Rana and Kumar [20] in the absence of compressibility, magnetic field and stable solute gradient.

6. THE STATIONARY CONVECTION

For stationary convection putting σ = 0 in equation (32) reduces it to

$$R_1 = \left(\frac{G}{G-1}\right)\left\{\frac{1+x}{xB}\left[\frac{1+x}{P} + \frac{Q_1}{\in} + \frac{T_{A_1}(1+x)P}{\{\in(1+x)+Q_1P\}\in}\right] + \frac{S_1B'}{B}\right\}, \tag{35}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters T_{A_1}, B, P, Q_1 and Walters (model B′) rotating fluid behave like an ordinary Newtonian fluid since elastico-viscous parameter F vanishes with σ.

Let the non-dimensional number G accounting for compressibility effect is kept as fixed, then we get

$$\overline{R_c} = \left(\frac{G}{G-1}\right)R_c, \tag{36}$$

where $\overline{R_c}$ and R_c denote, respectively, the critical number in the presence and absence of compressibility. Thus, the effect of compressibility is to postpone the instability on the onset of thermosolutal instability of Walters (model B′) fluid in porous medium. The cases $G = 1$ and $G < 1$ correspond to infinite and negative values of Rayleigh numbers due to compressibility which are not relevant to the present study.

To study the effects of suspended particles, rotation and medium permeability, we examine the behavior of $\frac{dR_1}{dB}, \frac{dR_1}{dT_{A_1}}, \frac{dR_1}{dQ_1}, \frac{dR_1}{dS_1}$ and $\frac{dR_1}{dP}$ analytically.

Equation (35) yields

$$\frac{dR_1}{dB} = -\left(\frac{G}{G-1}\right)\left\{\frac{1+x}{xB^2}\left[\frac{1+x}{P} + \frac{Q_1}{\in} + \frac{T_{A_1}(1+x)P}{\{\in(1+x)+Q_1P\}\in}\right] + \frac{S_1B'}{B^2}\right\}, \tag{37}$$

which is negative implying thereby that the effect of suspended particles is to destabilize the system when $G > 1$. This stabilizing effect is an agreement with the earlier work of Scanlon and Segel [8], Rana and Kumar [20].

From equation (35), we get

$$\frac{dR_1}{dT_{A_1}} = \left(\frac{1+x}{xB}\right)\left(\frac{G}{G-1}\right)\frac{(1+x)P}{\{\in(1+x)+Q_1P\}\in}, \tag{38}$$

which shows that rotation has stabilizing effect on the system. This stabilizing effect is an agreement of the earlier work of Sharma and Rana [19].

From equation (35), we get

$$\frac{dR_1}{dQ_1} = \frac{1+x}{xB}\left(\frac{G}{G-1}\right)\left[\frac{1}{\in} - \frac{T_{A_1}(1+x)P^2}{\{\in(1+x)+Q_1P\}^2\in}\right], \tag{39}$$

which implies that magnetic field stabilizes the system, if

$$\{\in(1+x)+Q_1P\}^2 > T_{A_1}(1+x)P^2,$$

and destabilizes the system, if

$$\{\in(1+x)+Q_1P\}^2 < T_{A_1}(1+x)P^2.$$

In the absence of rotation, magnetic field has stabilizing effect on the system which is identical with the result as derived by Rana and Kango. [21]. Thus rotation plays an important role on the system.

From equation (35), we get

$$\frac{dR_1}{dS_1} = \frac{B'}{B}\left(\frac{G}{G-1}\right), \tag{40}$$

which is positive implying thereby that the stable solute gradient has a stabilizing effect. This stabilizing effect is an agreement of the earlier work of Sharma and Rana [19], Rana and Kango [21].

It is evident from equation (35) that

$$\frac{dR_1}{dP} = -\frac{(1+x)^2}{xB}\left(\frac{G}{G-1}\right)\left[\frac{1}{P^2} - \frac{T_{A_1}(1+x)}{\{\in(1+x)+Q_1P\}^2}\right], \tag{41}$$

From equation (41), we observe that medium permeability has destabilizing effect when $\{\in(1+x)+Q_1P\}^2 < T_{A_1}(1+x)P^2$ and medium permeability has a stabilizing effect when $\{\in(1+x)+Q_1P\}^2 > T_{A_1}(1+x)P^2$.

In the absence of rotation, $\frac{dR_1}{dP}$ is always negative implying thereby the destabilizing effect of medium permeability which is identical with the result as derived by Rana and Kango [21].

7. CONCLUSIONS

The thermosolutal instability of compressible Walters (model B′) rotating fluid permeated with suspended particles (fine dust) in the presence of vertical magnetic field in porous medium has been investigated. The main conclusions are as follow:

(i) For the case of stationary convection, Walters (model B′) elastico-viscous fluid behaves like an ordinary Newtonian fluid as elastico-viscous parameter F vanishes with σ.

(ii) It is clear from equation (36) that the effect of compressibility is to postpone the onset of thermal instability.

(iii) The expressions for $\frac{dR_1}{dS_1}, \frac{dR_1}{dT_{A_1}}, \frac{dR_1}{dB}, \frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dP}$ are examined analytically and it has been found that the stable solute gradient and rotation have stabilizing effect on the system. The suspended particles are found to have destabilizing effect on the system whereas the medium permeability has a stabilizing / destabilizing effect on the system for $\{\in(1+x)+Q_1P\}^2 < T_{A_1}$

$(1 + x)P^2 / \{\in(1 + x) + Q_1P\}^2 > T_{A_1} (1 + x)P^2$. The magnetic field has stabilizing / destabilizing effect on the system for $\{\in(1 + x) + Q_1P\}^2 > T_{A_1} (1 + x)P^2$ / $\{\in(1 + x) + Q_1P\}^2 < T_{A_1} (1 + x)P^2$.

(iv) The presence of rotation, compressibility, medium permeability, suspended particles stable solute gradient, magnetic field and viscoelasticity introduce oscillatory modes.

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NOMENCLATURE

q	Velocity of fluid
qd	Velocity of suspended particles
p	Pressure
g	Gravitational acceleration vector
g	Gravitational acceleration
k_1	Medium permeability
T	Temperature
t	Time coordinate
C_f	Heat capacity of fluid
C_{pt}	Heat capacity of particles
mN	Mass of the particle per unit volume
k	Wave number of disturbance
k_x, k_y	Wave numbers in x and y directions
p_1	Thermal Prandtl number
P_l	Dimensionless medium permeability

Greek Symbols

$\in$	Medium porosity
$\boldsymbol{\rho}$	Fluid density
μ	Fluid viscosity
μ'	Fluid viscoelasticity
υ	Kinematic viscosity
υ'	Kinematic viscoelasticity
η	Particle radius
κ	Thermal diffusitivity
κ'	Solute diffusivity
α	Thermal coefficient of expansion
α'	Solvent coefficient of expansion
β	Adverse temperature gradient
β'	Solute gradient
θ	Perturbation in temperature
n	Growth rate of the disturbance
δ	Perturbation in respective physical quantity
ζ	z-component of vorticity
ξ	z-component of current density
$\boldsymbol{\Omega}$	Rotation vector having components (0, 0, Ω)
γ	Perturbation in solute concentration
μ_e	Magnetic permeability