

A semi-circle theorem on the onset of couple stress fluid under rotation saturated by a porous medium

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ABSTRACT

In the present paper, the thermal convection in couple stress fluid under rotation saturating a porous medium is considered. By applying linear stability theory and normal mode analysis method, a semi-circle theorem is derived. The mathematical analysis on the onset of thermal convection in a couple-stress fluid under rotation in a porous medium for the case of rigid boundaries shows that the complex growth rate σ of oscillatory perturbations, neutral or unstable for all wave numbers, must lie inside a semi-circle

$$|\sigma|^2 = \varepsilon T_A - \frac{\varepsilon}{P_l^2} \left(F^2 \pi^4 + F \pi^2 + \frac{1}{P_l} \right),$$

in the right half of a complex σ -plane, which prescribes the upper limits to the complex growth of arbitrary oscillatory motions of growing amplitude.

Keywords: Couple Stress fluid, Porous medium, Rotation, Thermal convection.

MSC 2010: 76A05; 76A10; 76E06; 76E07; 76S05.

1. INTRODUCTION

An understanding of thermal convection in a porous medium has various practical applications in geophysics, food processing, soil sciences, ground water hydrology and nuclear reactors etc. Many researchers have investigated thermal convection problems by taking different types of fluids. A detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [1]. Lapwood [2] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [3]. Benerjee et al. [4] have studied bounds for growth rate of perturbation in thermohaline convection and gave a new scheme for combining the governing equations of thermohaline convection which is shown to lead to bounds for the complex growth rate of arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries.

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In all the above studies, the fluid is considered to be Newtonian. Although the problem of thermal convection has been extensively investigated for Newtonian fluids, relatively little attention has been devoted to this problem with non-Newtonian fluids. With the growing importance of non-Newtonian fluids under rotation in modern technology and industries, the investigations on such fluids are desirable. In the recent years, considerable interest has been evinced in the study of couple-stress viscoelastic fluid because of their applications in biosciences, physical sciences, in the effective design of artificial organs in biomedical engineering, inertial fusion energy (IFE), solidification process in material science, heat transfer across barriers, friction between surfaces and their mitigation, in adhesion and failure of polymers and so on.

Stokes [5] proposed and postulated the theory of couple-stress fluid. One of the applications of couple-stress fluid is its use to the study of the mechanism of lubrication of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as lubricant. When fluid film is generated, squeeze film action is capable of providing considerable protection to the cartilage surface. The shoulder, knee, hip and ankle joints are the loaded-bearing synovial joints of human body and these joints have low-friction coefficient and negligible wear. Normal synovial fluid is clear or yellowish and is a viscous, non-Newtonian fluid.

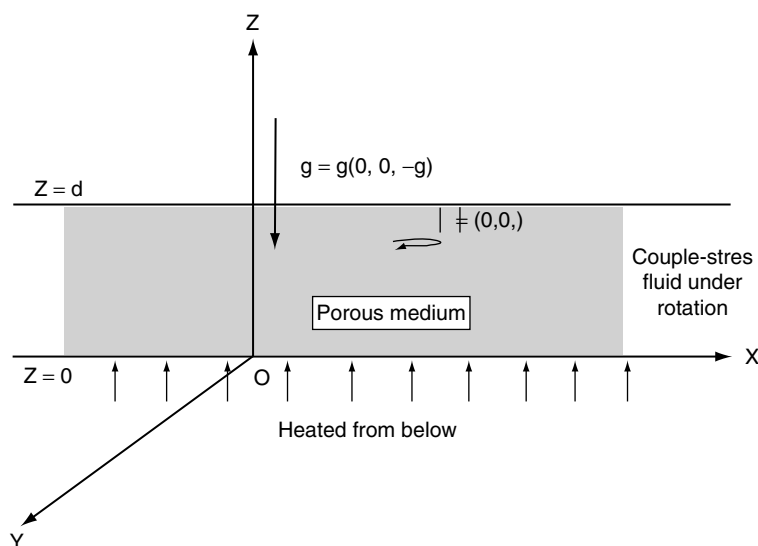
According to the theory of Stokes [5], couple-stresses are found to appear in noticeable magnitude in fluids very large molecules. Since the long chain hylauronic acid molecules are found as additives in synovial fluid. Walicki and Walicka [6] modeled synovial fluid as couple-stress fluid in human joints. Sharma and Sharma [7] have studied the couple-stress fluid heated from below in porous medium.

The investigation in porous media has been started with the simple Darcy model and gradually it was extended to Darcy-Brinkman model. A good account of convection problems in a porous medium is given by Vafai and Hadim [8], Ingham and Pop [9] and Nield and Bejan [10]. Sharma and Rana [11] have studied thermal instability of a incompressible Walters' (model B') elastico-viscous fluid in the presence of variable gravity field and rotation in porous medium whereas Rudraiah and Chandrashekara [12] studied the effects of couple stress on the growth rate of Rayleigh-Taylor instability at the interface in a finite thickness couple stress fluid. Recently, Kumar [13] studied stability of stratified couple-stress dusty fluid in the presence of magnetic field through porous medium whereas Rana and Sharma [14] studied hydromagnetic thermosolutal instability of compressible Walters' (model B') rotating fluid permeated with suspended particles in porous medium. Rana and Thakur [15] derived a mathematical theorem on the onset of couple stress fluid permeated with suspended dust particles saturating a porous medium and found that medium permeability and suspended particles have destabilizing effect on the system.

Keeping in mind the importance in various applications mentioned above, our interest, in the present paper is to study the thermal convection on the onset of couple-stress elastico-viscous fluid under rotation in a porous medium.

2. MATHEMATICAL MODEL AND PERTURBATION EQUATIONS

Here, we consider an infinite, horizontal, incompressible couple-stress viscoelastic fluid of depth d , bounded by the planes $z = 0$ and $z = d$ in an isotropic and homogeneous medium of porosity ε and permeability k_1 , which is acted upon by gravity $\mathbf{g}(0, 0, -g)$ and uniform rotation $\Omega(0, 0, \Omega)$. This layer is heated from below such that a steady adverse temperature gradient $\beta \left(= \left| \frac{dT}{dz} \right| \right)$ is maintained. The character of equilibrium of this initial static state is



Schematic Sketch of Physical Situation

determined by supposing that the system is slightly disturbed and then following its further evolution.

Let ρ , ν , μ_c , p , ε , T , α and $v(0, 0, 0)$, denote respectively, the density, kinematic viscosity, couple-stress viscosity, pressure, medium porosity, temperature, thermal coefficient of expansion and velocity of the fluid.

The equations expressing the conservation of momentum, mass, temperature and equation of state for couple-stress fluid in a porous medium (Chandrasekhar [1], Sharma and Sharma [11], Kumar [13] and Rana and Thakur [15]) are

$$\frac{1}{\varepsilon} \left[\frac{\partial v}{\partial t} + \frac{1}{\varepsilon} (v \cdot \nabla) v \right] = -\frac{1}{\rho_0} \nabla p + g \left(1 - \frac{\delta \rho}{\rho_0} \right) - \frac{1}{k_1} \left(v - \frac{\mu_c}{\rho_0} \nabla^2 v \right) + \frac{2}{\varepsilon} (v \times \Omega). \quad (1)$$

$$v = 0, \quad (2)$$

$$E \frac{\partial T}{\partial t} + (v \cdot \nabla) T = \kappa \nabla^2 T, \quad (3)$$

$$\rho = \rho_0 [1 - \alpha(T - T_0)], \quad (4)$$

where the suffix zero refers to values at the reference level $z = 0$.

Here

$$E = \varepsilon + (1 - \varepsilon) \left(\frac{\rho_s c_s}{\rho_0 c_f} \right)$$

which is constant, κ is the thermal diffusivity, ρ_s , c_s ; ρ_0 , c_f denote the density and heat capacity of solid (porous) matrix and fluid, respectively.

The initial state of the system is taken to be quiescent layer (no settling) with a uniform particle distribution number. The initial state is

$$v = (0, 0, 0), \quad T = -\beta z + T_0, \quad \rho = \rho_0(1 + \alpha\beta z), \quad (5)$$

is an exact solution to the governing equations.

Let $u, v, w, \theta, \delta p$ and $\delta\rho$ denote, respectively, the perturbations in fluid velocity $v(0, 0, 0)$, temperature T , pressure p and density ρ .

The change in density $\delta\rho$ caused by perturbation θ in temperature is given by

$$\delta\rho = -\alpha\rho_0\theta. \quad (6)$$

The linearized perturbation equations governing the motion of fluid are

$$\frac{1}{\varepsilon} \frac{\partial v}{\partial t} = -\frac{1}{\rho_0} \nabla \delta p - g \frac{\delta\rho}{\rho_0} - \frac{1}{k_1} \left(v - \frac{\mu_c}{\rho_0} \nabla^2 \right) v + \frac{2}{\varepsilon} (v \times \Omega), \quad (7)$$

$$\nabla \cdot v = 0, \quad (8)$$

$$E \frac{\partial \theta}{\partial t} = \beta w + \kappa \nabla^2 \theta. \quad (9)$$

In the Cartesian form, equations (7)–(9) with the help of equation (6) can be expressed as

$$\frac{1}{\varepsilon} \frac{\partial u}{\partial t} = -\frac{1}{\rho_0} \frac{\partial}{\partial x} (\delta p) - \frac{1}{k_1} \left(v - \frac{\mu_c}{\rho_0} \nabla^2 \right) u + \frac{2}{\varepsilon} \Omega v, \quad (10)$$

$$\frac{1}{\varepsilon} \frac{\partial v}{\partial t} = -\frac{1}{\rho_0} \frac{\partial}{\partial y} (\delta p) - \frac{1}{k_1} \left(v - \frac{\mu_c}{\rho_0} \nabla^2 \right) v + \frac{2}{\varepsilon} \Omega u, \quad (11)$$

$$\frac{1}{\varepsilon} \frac{\partial w}{\partial t} = -\frac{1}{\rho_0} \frac{\partial}{\partial z} (\delta p) + g\alpha\theta - \frac{1}{k_1} \left(v - \frac{\mu_c}{\rho_0} \nabla^2 \right) w, \quad (12)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (13)$$

$$E \frac{\partial \theta}{\partial t} = \beta w + \kappa \nabla^2 \theta. \quad (14)$$

Operating equation (10) and (11) by $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively, adding and using equation (13), we get

$$\frac{1}{\epsilon} \frac{\partial}{\partial t} \left(\frac{\partial w}{\partial z} \right) = \frac{1}{\rho_0} \left(\nabla^2 - \frac{\partial^2}{\partial z^2} \right) \delta p - \frac{1}{k_1} \left(v - \frac{\mu_c}{\rho_0} \nabla^2 \right) \left(\frac{\partial w}{\partial z} \right) - \frac{2}{\epsilon} \Omega \zeta, \quad (15)$$

where $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the z -component of vorticity.

Operating equation (12) and (15) by $\left(\nabla^2 - \frac{\partial^2}{\partial z^2} \right)$ and $\frac{\partial}{\partial z}$ respectively and adding to eliminate δp between equations (12) and (15), we get

$$\frac{1}{\epsilon} \frac{\partial}{\partial t} (\nabla^2 w) = -\frac{1}{k_1} \left(v - \frac{\mu_c}{\rho_0} \nabla^2 \right) \nabla^2 w + g \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \alpha \theta - \frac{2}{\epsilon} \Omega \frac{\partial \zeta}{\partial z}, \quad (16)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

Operating equation (10) and (11) by $-\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial x}$ respectively and adding, we get

$$\frac{1}{\epsilon} \frac{\partial \zeta}{\partial t} = -\frac{1}{k_1} \left(v - \frac{\mu_c}{\rho_0} \nabla^2 \right) \zeta + \frac{2}{\epsilon} \Omega \frac{\partial w}{\partial z}. \quad (17)$$

3. NORMAL MODE ANALYSIS

Following the normal mode analyses, we assume that the perturbation quantities have x , y and t dependence of the form

$$[w, \zeta, \theta] = [W(z), Z(z), \theta(z)] \exp(ilx + imy + nt), \quad (18)$$

where l and m are the wave numbers in the x and y directions, $k = (l^2 + m^2)^{1/2}$ is the resultant wave number and n is the frequency of the harmonic disturbance, which is, in general, a complex constant.

Using expression (18) in equations (18), (19) and (16) become

$$\frac{n}{\epsilon} \left[\frac{d^2}{dz^2} - k^2 \right] W = -gk^2 \alpha \theta - \frac{1}{k_1} \left(v - \frac{\mu_c}{\rho_0} \nabla^2 \right) \left(\frac{d^2}{dz^2} - k^2 \right) W - \frac{2\Omega}{\epsilon} \frac{dZ}{dz}, \quad (19)$$

$$\frac{n}{\epsilon} Z = -\frac{1}{k_1} \left(v - \frac{\mu_c}{\rho_0} \nabla^2 \right) Z + \frac{2\Omega}{\epsilon} \frac{dW}{dz}, \quad (20)$$

$$E \frac{\partial \theta}{\partial t} = \beta W + \kappa \left(\frac{d^2}{dz^2} - k^2 \right) \theta. \quad (21)$$

Equation (20) and (21) in non-dimensional form, become

$$\left[\frac{\sigma}{\varepsilon} + \frac{1 - F(D^2 - \alpha^2)}{P_l} \right] (D^2 - \alpha^2)W = -\frac{g\alpha^2 d^2 \alpha \theta}{\nu} - \sqrt{T_A} dDZ, \quad (22)$$

$$\left[\frac{\sigma}{\varepsilon} + \frac{1 - F(D^2 - \alpha^2)}{P_l} \right] Z = \frac{\sqrt{T_A}}{d} DW, \quad (23)$$

$$[D^2 - \alpha^2 - EP_r \sigma] \theta = -\frac{\beta d^2}{\kappa} W, \quad (24)$$

where we have put

$\alpha = kd$, $\sigma = \frac{nd^2}{\nu}$ and $P_l = \frac{k_1}{d^2}$, is the dimensionless medium permeability, $P_r = \frac{\nu}{\kappa}$, is the thermal Prandtl number, $F = \frac{\mu_c}{\mu d^2}$, is the couple-stress parameter and $D' = d \frac{d}{dz} = dD$ and dropping (') for convenience.

Substituting $W = W'$, $Z = \frac{\sqrt{T_A}}{d} Z'$ and $\theta = \frac{\beta d^2}{\kappa} \theta'$ in equations (22)–(24) and dropping (') for convenience, we obtain

$$\left[\frac{\sigma}{\varepsilon} + \frac{1 - F(D^2 - \alpha^2)}{P_l} \right] (D^2 - \alpha^2)W = -R\alpha^2 \theta - T_A DZ, \quad (25)$$

$$\left[\frac{\sigma}{\varepsilon} + \frac{1 - F(D^2 - \alpha^2)}{P_l} \right] Z = DW, \quad (26)$$

$$[D^2 - \alpha^2 - EP_r \sigma] \theta = -W, \quad (27)$$

where $R = \frac{g\alpha\beta d^4}{\nu\kappa}$, is the thermal Rayleigh number and $T_A = \left(\frac{2\Omega d^2}{\varepsilon\nu} \right)^2$, is the modified Taylor number.

Since both the boundaries are rigid and maintained at constant temperature, the perturbations in the temperature are zero at the boundaries.

The boundary conditions appropriate to the problem are (Chandrasekhar [1])

$$W = DW = \theta = Z = 0 \text{ at } z = 0 \text{ and } 1. \quad (28)$$

Then, we prove the following theorem:

THEOREM: If $R > 0$, $F > 0$, $T_A > 0$, $\sigma_r \geq 0$ and $\sigma_i \neq 0$, then the necessary condition for the existence of non-trivial solution (W , θ , Z) of equations (25)–(27) together with the boundary conditions (28) is that

$$|\sigma|^2 < \varepsilon T_A - \frac{\varepsilon}{P_l^2} \left(F^2 \pi^4 + F \pi^2 + \frac{1}{P_l} \right).$$

PROOF: Multiplying equation (22) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical rang of z , we get

$$\begin{aligned} \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l} \right) \int_0^1 W^* (D^2 - \alpha^2) W dz \\ - \frac{F}{P_l} \int_0^1 W^* (D^2 - \alpha^2)^2 W dz = -R\alpha^2 \int_0^1 W^* \theta dz - T_A \int_0^1 W^* DZ dz, \end{aligned} \quad (29)$$

Taking complex conjugate on both sides of equation (27), we get

$$[D^2 - \alpha^2 - EP_r \sigma] \theta^* = -W^*. \quad (30)$$

Integrating both sides of equation (30) over the vertical range of z , we get

$$\int_0^1 W^* \theta dz = - \int_0^1 \theta (D^2 - \alpha^2 - EP_r \sigma^*) \theta^* dz, \quad (31)$$

Now taking complex conjugate on both sides of equation (26), we get

$$\left[\frac{\sigma^*}{\varepsilon} + \frac{1 - F(D^2 - \alpha^2)}{P_l} \right] Z^* = DW^*, \quad (32)$$

Therefore, using equation (32), we get

$$\int_0^1 W^* DZ dz = - \int_0^1 DW^* Z dz = - \int_0^1 Z \left(\frac{\sigma^*}{\varepsilon} + \frac{1 - F(D^2 - \alpha^2)}{P_l} \right) Z^* dz, \quad (33)$$

Using equations (31) and (33) in the right hand side of equation (29), we obtain

$$\begin{aligned} \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l} \right) \int_0^1 W^* (D^2 - \alpha^2) W dz - \frac{F}{P_l} \int_0^1 W^* (D^2 - \alpha^2)^2 W dz \\ = R\alpha^2 \int_0^1 \theta (D^2 - \alpha^2 - EP_r \sigma^*) \theta^* dz + T_A \int_0^1 Z \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l} - \frac{F(D^2 - \alpha^2)}{P_l} \right) Z^* dz \end{aligned} \quad (34)$$

Integrating term by term on both sides of equation (34) for an appropriate number of times by making use of boundary conditions (28), we obtain

$$\begin{aligned}
 & (\sigma/\varepsilon + 1/P_l l) \int_0^1 0^{\uparrow} 1 \left(|DW|^{\uparrow} 2 + \alpha^{\uparrow} 2 |W|^{\uparrow} 2 \right) dz \\
 & + F/P_l l \int_0^1 0^{\uparrow} 1 \left(|D^{\uparrow} 2W|^{\uparrow} 2 + 2\alpha^{\uparrow} 2 |DW|^{\uparrow} 2 + \alpha^{\uparrow} 4 |W|^{\uparrow} 2 \right) dz \\
 & = R\alpha^{\uparrow} 2 \int_0^1 0^{\uparrow} 1 \left(|D\theta|^{\uparrow} 2 + 2\alpha^{\uparrow} 2 |\theta|^{\uparrow} 2 + EP_l r \sigma^{\uparrow} * |\theta|^{\uparrow} 2 \right) dz \\
 & - T_l A (\sigma/\varepsilon + 1/P_l l) \int_0^1 0^{\uparrow} 1 |Z|^{\uparrow} 2 dz + (FT_l A)/P_l l \int_0^1 0^{\uparrow} 1 (|DZ|
 \end{aligned} \tag{35}$$

Equating imaginary parts on both sides of equation (35) and cancelling $\sigma_i (\neq 0)$ throughout from the resulting equation, we get

$$\frac{1}{\varepsilon} \int_0^1 \left(|DW|^2 + \alpha^2 |W|^2 \right) dz + Ra^2 EP_r \int_0^1 |\theta|^2 dz = \frac{T_l A}{\varepsilon} \int_0^1 |Z|^2 dz. \tag{36}$$

Further, multiplying equation (26) and its complex conjugate (32), integrating by parts each term of resulting equation on both sides for an appropriate number of times and making use of boundary condition on Z , namely $Z(0) = 0 = Z(1)$ along with (28), it follows that

$$\begin{aligned}
 & \frac{F}{P_l} \int_0^1 \left(|D^2 Z|^2 + 2\alpha^2 |DZ|^2 + \alpha^4 |Z|^2 \right) dz + \left(\frac{\sigma_r}{\varepsilon} + \frac{1}{P_l} \right) \int_0^1 \left(|DZ|^2 + \alpha^2 |Z|^2 \right) dz \\
 & + \frac{1}{FP_l} \left(\frac{\sigma_r}{\varepsilon} + \frac{1}{P_l} \right) \int_0^1 |Z|^2 dz + \frac{P_l |\sigma|^2}{\varepsilon F} \int_0^1 |Z|^2 dz = \frac{P_l}{F} \int_0^1 |DW|^2 dz.
 \end{aligned} \tag{37}$$

Since W , Z and θ satisfy $W(0) = 0 = W(1)$, $Z(0) = 0 = Z(1)$ and $\theta(0) = 0 = \theta(1)$, we have by Rayleigh-Ritz inequality [9]

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz, \tag{38}$$

$$\int_0^1 |DZ|^2 dz \geq \pi^2 \int_0^1 |Z|^2 dz, \tag{39}$$

$$\int_0^1 |D\theta|^2 dz \geq \pi^2 \int_0^1 |\theta|^2 dz, \tag{40}$$

and

$$\int_0^1 |D^2 Z|^2 dz \geq \pi^4 \int_0^1 |Z|^2 dz. \quad (41)$$

Since $F > 0$ and $\sigma_r \geq 0$, therefore equation (37) gives

$$\frac{F}{P_l} \int_0^1 |D^2 Z|^2 dz + \frac{1}{P_l} \int_0^1 |DZ|^2 dz + \frac{1}{FP_l^2} \int_0^1 |Z|^2 dz + \frac{P_l |\sigma|^2}{\varepsilon F} \int_0^1 |Z|^2 dz < \frac{P_l}{F} \int_0^1 |DW|^2 dz. \quad (42)$$

Using inequalities (39) and (41) in inequality (42), we get

$$\int_0^1 |Z|^2 dz < \frac{P_l}{\frac{F^2 \pi^4}{P_l} + \frac{F \pi^2}{P_l} + \frac{1}{P_l^2} + \frac{P_l}{\varepsilon} |\sigma|^2} \int_0^1 |DW|^2 dz. \quad (43)$$

If $R > 0$ and $T_A > 0$, using the inequality (43) in equation (36), we obtain

$$\left(1 - \frac{T_A P_l}{\frac{F^2 \pi^4}{P_l} + \frac{F \pi^2}{P_l} + \frac{1}{P_l^2} + \frac{P_l}{\varepsilon} |\sigma|^2} \right) \int_0^1 |DW|^2 dz + \varepsilon \int_0^1 \alpha^2 |W|^2 dz + R \alpha^2 \varepsilon E P_r \int_0^1 |\theta|^2 dz < 0. \quad (44)$$

Therefore, we must have

$$|\sigma|^2 < \varepsilon T_A - \frac{\varepsilon}{P_l^2} \left(F^2 \pi^4 + F \pi^2 + \frac{1}{P_l} \right). \quad (45)$$

Hence, if

$$\sigma_r \geq 0 \text{ and } \sigma_i \neq 0, \text{ then } |\sigma|^2 < \varepsilon T_A - \frac{\varepsilon}{P_l^2} \left(F^2 \pi^4 + F \pi^2 + \frac{1}{P_l} \right).$$

This completes the proof of the theorem.

4. CONCLUSIONS

The effect of rotation on thermal convection in couple-stress fluid in a porous medium has been investigated. From the above theorem, the main conclusions are as follows:

- (i) The complex growth rate of an arbitrary oscillatory motions of growing amplitude, lies inside the semi-circle in the right half of the $\sigma_r \sigma_i$ -plane whose centre is at origin and

$$\text{radius is } \left[\varepsilon T_A - \frac{\varepsilon}{P_l^2} \left(F^2 \pi^4 + F \pi^2 + \frac{1}{P_l} \right) \right]^{\frac{1}{2}}.$$

- (ii) From inequality (45), a sufficient condition for the validity of ‘principle of exchange of stabilities’ in the convection of couple-stress fluid in a porous medium is that the non-

$$\text{dimensional parameter } \frac{T_A P_l}{\frac{F^2 \pi^4}{P_l} + \frac{F \pi^2}{P_l} + \frac{1}{P_l^2} + \frac{P_l}{\varepsilon} |\sigma|^2} \leq 1.$$

- (iii) The existence of oscillatory motions of growing amplitude in this problem depends

$$\text{crucially upon the magnitude of non-dimensional parameter } \frac{T_A P_l}{\frac{F^2 \pi^4}{P_l} + \frac{F \pi^2}{P_l} + \frac{1}{P_l^2} + \frac{P_l}{\varepsilon} |\sigma|^2},$$

$$\text{in the sense so long as } 0 < \frac{T_A P_l}{\frac{F^2 \pi^4}{P_l} + \frac{F \pi^2}{P_l} + \frac{1}{P_l^2} + \frac{P_l}{\varepsilon} |\sigma|^2} \leq 1 \text{ no such motions are possible.}$$

- (iv) It is clear from inequality (45) that rotation has stabilizing effect on the system which is an agreement with the earlier work of Sharma and Rana [11] and Rana and Sharma [14].
- (v) The medium permeability has a destabilizing effect on the system as can be seen from inequality (45), which is an agreement with the earlier work of Sharma and Sharma [7], Sharma and Rana [11], Kumar [13], Rana and Sharma [14] and Rana and Thakur [15].

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NOMENCLATURE

F	Couple-Stress parameter
P_l	Dimensionless medium permeability
g	Gravitational acceleration
\mathbf{g}	Gravitational acceleration vector
p	Pressure
T_A	Taylor number
P_r	Thermal Prandtl number
v	Velocity of fluid
v_d	Velocity of suspended particles
k	Wave number of disturbance

GREEK SYMBOLS

β	Adverse temperature gradient
μ_c	Couple-Stress viscosity
ρ	Fluid density
μ	Fluid viscosity
ν	Kinematic viscosity
ε	Medium porosity
δ	Perturbation in respective physical quantity
θ	Perturbation in temperature
κ	Thermal diffusivity
α	Thermal coefficient of expansion

