

Operation control of fluids pumping in curved pipes during annular flow: a numerical evaluation

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ABSTRACT

To generate projects which provide significant volume recovery from heavy oils reservoirs and improve existing projects, is important to develop new production and transport technologies, especially in the scenario of offshore fields. The core-flow technique is one of new technologies used in heavy oil transportation. This core-flow pattern is characterized by a water pellicle that is formed close or adjacent to the inner wall of the pipe, functioning as a lubricant. The oil flows in the center of the pipe causing a reduction in longitudinal pressure drop. In this sense, this work presents a numerical study of heavy oil annular flow (core-flow) assisted by computational tool ANSYS CFX[®] Release 12.0. It was used a three-dimensional, transient and isothermal mathematical model considered by the mixture and turbulence – models to address the water-heavy oil two-phase flow, assuming laminar flow for oil phase and turbulent flow for water phase. Results of the pressure, velocity and volume fraction distributions of the phases and the pressure drop for different operation conditions are presented and evaluated. It was observed that the oil core flowing eccentrically in the pipe and stops of the water flux considerably increases the pressure drop in the pipe after the restart of the pump.

Keywords: Heavy oil, Transportation, Numerical simulation

1. INTRODUCTION

The multiphase flow appears in several industrial sectors for example in the food, chemical processing of materials and oil, industries.

In an increasingly competitive scenario, the oil industry has evolved rapidly, creating the need to develop techniques that allow the specification and design of production systems applied to flow of multiphase mixtures. Such flows occur from the reservoirs to the surface facilities, through the production wells [9].

The multiphase flow in pipes involving liquid and gas or liquid and liquid, phases, depending geometric configurations, presenting interfaces whose definitions depend on the prevailing operation conditions. The different flow patterns observed depend on many factors, such as flow rates of the phases, the relative velocity between them, the pressure drop, the diameter, roughness and inclination of the pipe, the wettability of the fluid in the pipe walls, the surface tension, viscosity and density of fluids [5]. The inclination of the pipe is a factor that can change the flow pattern very so much that there are some striking differences between the horizontal and vertical flows. The flow rates of the phases are determinant in setting the pattern for the same pair of fluids in certain geometry, as well as the density and viscosity ratios between the phases [6].

Fluid injection rate is also important in determining the flow pattern. For a fixed water injection flow, different patterns are obtained by varying the quantity of oil in water. It starts with the dispersion of small bubbles of oil in water. If the oil fraction is increased, the oil droplets grow in size and become an order of magnitude comparable to the size of the radius of the pipe either becomes continuous throughout the pipe. Are also observed stratified flow regime and core annular flow (CAF), depending on operation conditions [7, 8].

Among the different methods for the production and transport of heavy oils, we can cite the core annular flow or annular flow (CAF), also known as core-flow, characterized by the smallest amount of energy required to pump heavy oil [4]. This method was idealized by Isaacs and Speed 1904 cited in U.S. Patent No. 759,374 in the United States, indicating a way of transporting heavy oils by lubrication water. However, only in 1970 was built a large industrial pipeline for the transportation of heavy oils by Shell company around Bakersfield, California, with 38 km long and a 15 cm diameter. For more than ten years a viscous crude oil was transported at a rate of 24,000 bbl/d in a system lubricated with water [5].

The core-flow technique basically consists in injecting small quantities of water at a flow rate lower than the oil causing the heavy oil is enveloped by a layer of water and flow in the center of the pipe without touching the inner wall of the pipe, thus establishing an annular pattern. A disadvantage for the use of this technique is when the oil comes in contact with the inner wall of the pipeline during transportation, as this may cause a great increase in system pressure and may cause serious environmental and transportation system problems.

The interest in heavy oil production using the core annular flow technique (CAF) has been increasing in recent years as a result of the large amount of heavy oil reserves accessible. However, this technique has brought attractive results with regard to energy consumption. This is due to the reduction of the pressure drop during the flow-core type oil-water flow when compared to heavy oil single-phase flow [14].

The core-flow technique does not modify the viscosity of the oil but the flow pattern turns and reduces friction during transport of very viscous products such as heavy oils. Prada and Bannwart [14] observed that frictional pressure drop in the core-flow is 750 to 2000 times lower than that for the flow with only oil in the pipe. It has been reported in the literature, works related to use of this technique in order to optimize the transportation of heavy oils using water as a lubricant [1, 2-5, 8, 9, 12-15]. In this study, we evaluated the effect of shutdown and restarting the water pump on the annular flow “core-flow” type, thus it was possible to analyze the behavior of two-phase (flow heavy oil and water) by decreasing the mass flow rate of water in the flow.

2. METHODOLOGY

2.1 PHYSICAL PROBLEM DESCRIPTION

In present day, the Computational Fluid Dynamics (CFD) has been increasingly used in diverse industry segments (automotive, aerospace, chemical processing, power generation, metallurgy, oil companies, etc.) for various purposes.

The physical problem evaluated in this work consists in a two-phase flow (water and heavy oil) in a curved pipe of 6 meters length and 0.15 m diameter. Figures 1 and 2 illustrate

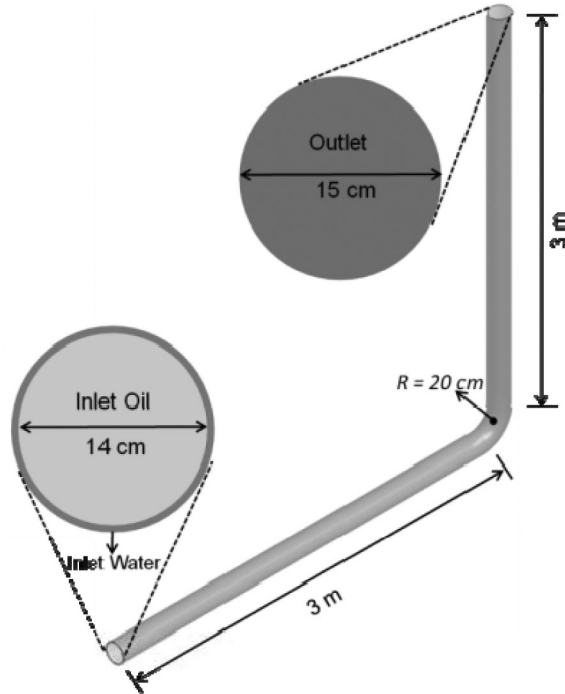


Figure 1: Domain of study with details of the inlet and outlet of fluids in curved pipe

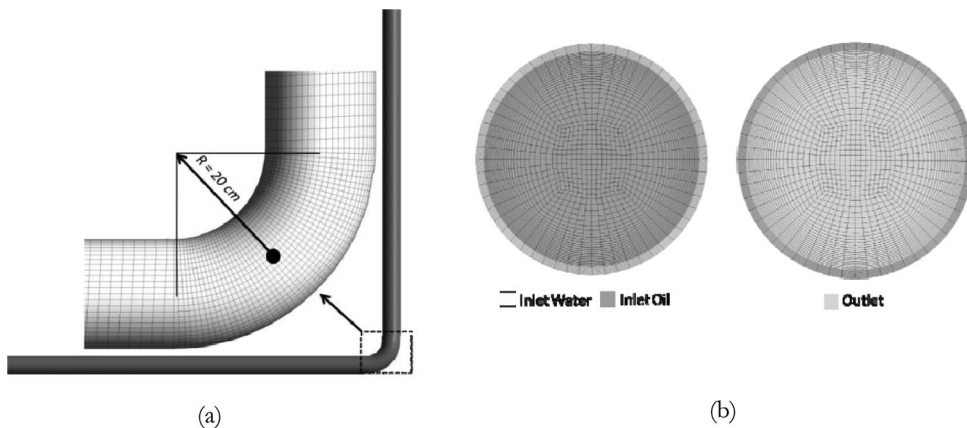


Figure 2: a) The computational grid and b) Details of the Inlet and outlet of fluids

the representation of the curved pipe as well as details of geometry and mesh, respectively, both used to study two-phase flow of water and highly-viscous oil.

Here in, mesh with 946484 control volume was created in module ANSYS ICEM CFD® Release 12.0. This domain of study was created by defining points, curves, surfaces and solids describing its size and shape. In this mesh we can observe the main details near the wall of the curved pipe and in the regions of interface, where the velocity gradients are most relevant. A further refinement of this region was done and compared to the central region of the mesh in the tentative to get results closer to reality where they have a well-defined interface. It is through the interface that exists heat exchange, dissolution, drag, i.e., the interface is the region of space where the different phases exchange information, and also where occurs transfer of heat, mass, and momentum.

2.2 MULTIPHASE MATHEMATICAL MODELING

The modeling procedure consists in the mathematical description of the physical problem to be analyzed. In the case of fluid flow, the mathematical model consists of conservation equations (mass, energy and momentum), initial and boundary conditions, and constitutive equations establishing the relationship between the stress and velocity in flow, among others. From the viewpoint of engineering, these models correspond to a set of data and abstract ideas that allow the engineer, or researcher, to propose an explanation for the phenomenon that is being studied.

To model the isothermal multiphase flow, the following equations can be used [2]:

2.2.1 Mass Conservation Equation

This equation is given as follows:

$$\frac{\partial}{\partial t}(f_\alpha \rho_\alpha) + \nabla \cdot (f_\alpha \rho_\alpha \vec{U}_\alpha) = S_{MS\alpha} + \sum_{\beta=1}^{N_p} \Gamma_{\alpha\beta} \quad (1)$$

When the terms of mass source $S_{MS\alpha}$ and the term of mass diffusivity $\Gamma_{\alpha\beta}$ are neglected, we can write the Equation 1 as:

$$\frac{\partial}{\partial t}(f_\alpha \rho_\alpha) + \nabla \cdot (f_\alpha \rho_\alpha \vec{U}_\alpha) = 0 \quad (2)$$

2.2.2 Momentum Equation

This equation is given as follows:

$$\begin{aligned} \frac{\partial}{\partial t}(f_\alpha \rho_\alpha \vec{U}_\alpha) + \nabla \cdot [f_\alpha (\rho_\alpha \vec{U}_\alpha \otimes \vec{U}_\alpha)] = & -f_\alpha \nabla p_\alpha + \nabla \cdot \left\{ f_\alpha \mu_\alpha \left[\nabla \vec{U}_\alpha + (\nabla \vec{U}_\alpha)^T \right] \right\} \\ & + \sum_{\beta=1}^{N_p} (\Gamma_{\alpha\beta}^+ \vec{U}_\beta - \Gamma_{\beta\alpha}^+ \vec{U}_\alpha) + \vec{S}_{M\alpha} + \vec{M}_\alpha \end{aligned} \quad (3)$$

In the Equations (1), (2) and (3), the sub-index α represent the phase indicator on the water-heavy oil two-phase flow; f , ρ , μ and \vec{U} are the volume fraction, density, dynamic viscosity and velocity vector, respectively; p is pressure and $\vec{S}_{M\alpha}$ represents the term of external forces which act on the system per unit volume. In the term regarding momentum transfer induced by interfacial mass transfer (third term on the right side of equality) the sub-indexes α and β correspond the phases involved, water and heavy oil. $\Gamma_{\alpha\beta}^+$ corresponds to the mass flow rate per unit volume of phase α to phase β and vice-versa; \vec{M}_α describes the total force

per unit volume (interfacial drag force, lift force, wall lubrication force, virtual mass force and turbulent dispersion force) on the α phase due to interaction with β phase.

The interfacial mass transfer term was not taken into account because the interfacial mass transfer in the momentum equation is used for disperse solid phase representing an additional force due to collisions between particles [2]. Thus, Equation (3) reduces to:

$$\frac{\partial}{\partial t}(f_\alpha \rho_\alpha \bar{U}_\alpha) + \nabla \cdot [f_\alpha (\rho_\alpha \bar{U}_\alpha \otimes \bar{U}_\alpha)] = -f_\alpha \nabla p_\alpha + \nabla \cdot \left\{ f_\alpha \mu_\alpha [\nabla \bar{U}_\alpha + (\nabla \bar{U}_\alpha)^T] \right\} + \bar{M}_\alpha \quad (4)$$

2.2.3 Turbulence Model

The k - ε turbulence model was used for the water phase flow. In this model, it is assumed that Reynolds tensors are proportional to the average velocity gradients, with the constant of proportionality being characterized by turbulent viscosity (idealization known as Boussinesq hypothesis). The characteristics of this type of model is that two transport equations modeled, separately, are solved for the turbulent length and time scale or for which either two combinations are linearly independent. The transport equations for the turbulent kinetic energy, k , and turbulent dissipation rate, ε , are respectively:

$$\frac{\partial(\rho_\alpha f_\alpha k_\alpha)}{\partial t} + \nabla \cdot \left\{ f_\alpha \left[\rho_\alpha \bar{U}_\alpha k_\alpha - \left(\mu + \frac{\mu_{t\alpha}}{\sigma_k} \right) \nabla k_\alpha \right] \right\} = f_\alpha (G_\alpha - \rho_\alpha \varepsilon_\alpha) \quad (5)$$

$$\frac{\partial(\rho_\alpha f_\alpha \varepsilon_\alpha)}{\partial t} + \nabla \cdot \left\{ f_\alpha \rho_\alpha \bar{U}_\alpha \varepsilon_\alpha - \left(\mu + \frac{\mu_{t\alpha}}{\sigma_\varepsilon} \right) \nabla \varepsilon_\alpha \right\} = f_\alpha \frac{\varepsilon_\alpha}{k_\alpha} (C_1 G_\alpha - C_2 \rho_\alpha \varepsilon_\alpha) \quad (6)$$

where G_α is the production of turbulent kinetic energy within the phase α ; C_1 and C_2 are empirical constants. Although on these equations, the dissipation rate of turbulent kinetic energy and turbulent kinetic energy for phase α , are defined by:

$$\varepsilon_\alpha = \frac{c_\mu q_\alpha^3}{l_\alpha} \quad (7)$$

$$k_\alpha = \frac{q_\alpha^2}{2} \quad (8)$$

where l_α is the length of spatial scale, q_α is the velocity range and c_μ is an empirical constant, given by:

$$c_\mu = 4c_\alpha^2 \quad (9)$$

where c_α is also an empirical constant. The turbulent viscosity $\mu_{t\alpha}$ is given as follows:

$$\mu_{t\alpha} = c_\mu \rho_\alpha \frac{k_\alpha^2}{\varepsilon_\alpha} \quad (10)$$

The constants used on previous equations are: $C_1 = 1.44$; $C_2 = 1.92$; $C_\mu = 0.09$; $\sigma_k = 1.0$ and $\sigma_\varepsilon = 1.3$.

2.2.4 Mixture Model (Constitutive equations)

For this work we adopted the Eulerian model of mixing, for the analysis of water-heavy oil two-phase flow in pipes with curved connections. This model can present more complex formulations, however, is suitable for the modeling of liquid-liquid two-phase flow, where is calculated all forces acting on the interface between the fluid phases. In this model was

considered gravity and drag effects. The total drag force exerted by the phase β to the phase α per unit volume is given as follows:

$$\vec{M}_\alpha = C_D \rho_{\alpha\beta} A_{\alpha\beta} |\vec{U}_\beta - \vec{U}_\alpha| (\vec{U}_\beta - \vec{U}_\alpha) \quad (11)$$

where C_D is the drag coefficient in which was assumed a value equal to 0.44. The density and viscosity of the mixture are given respectively by:

$$\rho_{\alpha\beta} = f_\alpha \rho_\alpha + f_\beta \rho_\beta \quad (12)$$

$$\mu_{\alpha\beta} = f_\alpha \mu_\alpha + f_\beta \mu_\beta \quad (13)$$

The mixture model treats both phases α and β symmetrically. The surface area per unit volume is given by:

$$A_{\alpha\beta} = \frac{f_\alpha f_\beta}{d_{\alpha\beta}} \quad (14)$$

where $d_{\alpha\beta} = 1$ mm is an interfacial length scale which must be specified.

The dimensionless transfer coefficient between the phases (Reynolds number of the mixture) is given by:

$$\text{Re}_{\alpha\beta} = \frac{\rho_{\alpha\beta} |\vec{U}_\beta - \vec{U}_\alpha| d_{\alpha\beta}}{\mu_{\alpha\beta}} \quad (15)$$

2.3 INITIAL AND BOUNDARY CONDITIONS

2.3.1 Continuous pumping

In the inlet section for oil was used a prescribed value for the velocity component $U_o = 1.0$ m/s and oil volume fraction $f_o = 1.0$:

- a) $u = v = 0$ and $w = U_o$ in $z = 0$ to $\forall (x, y)$;
- b) Laminar flow.

In the annular section for the inlet of water we use a prescribed value for the velocity component $U_w = 0.2, 0.4, 0.45, 0.5, 0.55, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8$ and 2.0 m/s and water volume fraction $f_w = 1.0$. Further,

- a) $u = v = 0$ and $w = U_w$ in $z = 0$ to $\forall (x, y)$;
- b) Turbulent flow.

In the wall of the pipe was used not slip condition and roughness $45 \mu\text{m}$. Further,

- a) $u = v = w = 0$ at the wall pipe;

2.3.2 Intermittent pumping

To evaluate the effect of water mass flow on the performance of heavy oil-water two-phase flow, we performed one simulation which allows evaluating the shutdown and restart the water pumping. The purpose was to observe the behavior of the pressure drop as a function of water mass flow. In this particular case was used the following boundary conditions for the curved pipe:

In the inlet section for oil was used a prescribed value for the velocity component $U_o = 1.0$ m/s and oil volume fraction $f_o = 1.0$:

$$0 < r < (R - \Delta r) \Rightarrow \begin{cases} U_o^z = U_o \\ f_o = 1 \\ U_o^r = U_A^r = U_A^z = f_A = 0 \end{cases} \quad (16)$$

In the annular section for the inlet of water we use a prescribed water mass flow \dot{m}_A rate, as follows:

$$(R - \Delta r) < r < R \Rightarrow \begin{cases} \dot{m}_A = 4.0779 \frac{\text{kg}}{\text{s}} & \because t = 1 \text{ s} \\ \dot{m}_A = 0 & \because 1 \text{ s} < t < 13 \text{ s} \\ \dot{m}_A = 4.0779 \frac{\text{kg}}{\text{s}} & \because 13 \text{ s} < t < 30 \text{ s} \\ U_o^z = f_o = 0 \end{cases} \quad (17)$$

Table 1 reports the thermal physical properties of the fluids used in all simulation.

Table 1: Thermo-physical properties of the water and oil used in the work

Physicals properties	Water	Heavy oil
Density (kg/m^3)	997	989
Dynamic viscosity (Pa.s)	8.89×10^{-4}	10.0
Surface tension (N/m)	0.072	

3. RESULTS AND DISCUSSIONS

3.1 CONTINUOUS FLOW

Figure 3 illustrates pressure drop in the curved pipe as a function of ratio between the velocities of the water and the oil U_w/U_o .

These results show a decrease in pressure drop in the pipe with increasing water velocity until reach a stable condition, which pressure drop is close to that obtained or single water flow. The reduction in the friction is due to presence of water film, thereby forming oil core

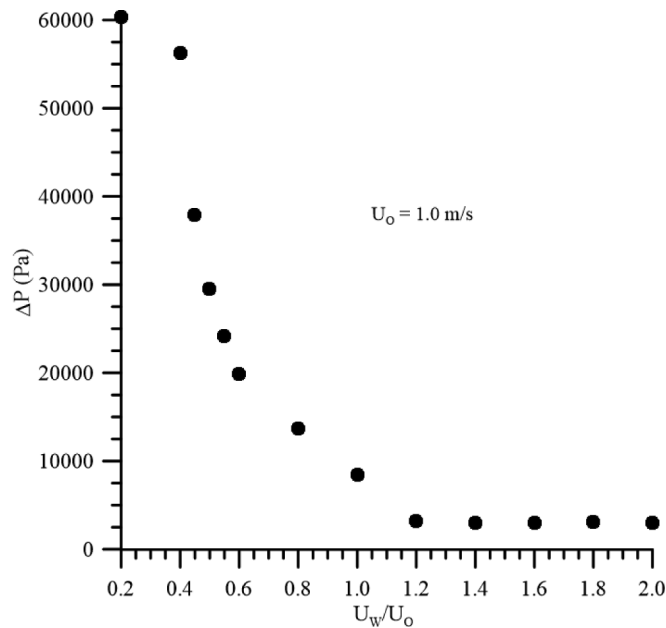


Figure 3: Pressure drop as a function of water velocity in the curved pipe

which hardly comes into contact with the pipe wall. Figure 4 illustrates the pressure field by friction along the curved pipe. We can verify that there is a higher pressure in the initial region of the pipe which decreases along the pipe, thus favoring the flow.

Further, it can be noted that in the left region of the curvature the pressure is greater than in the right region of one. This is due to the fact that fluids, both water and oil, became the inertia, effect, to continue in the horizontal direction causing the pressure to increase in this region.

At the time of approximately 5 seconds (see Figure 5) the heavy oil volume fraction already starts to increase at the outlet of the curved pipe. At this moment is notable the presence of undulations in the interface region of heavy oil-water, this event approaches well to experimental results available in the literature [4, 8, 10, 14].

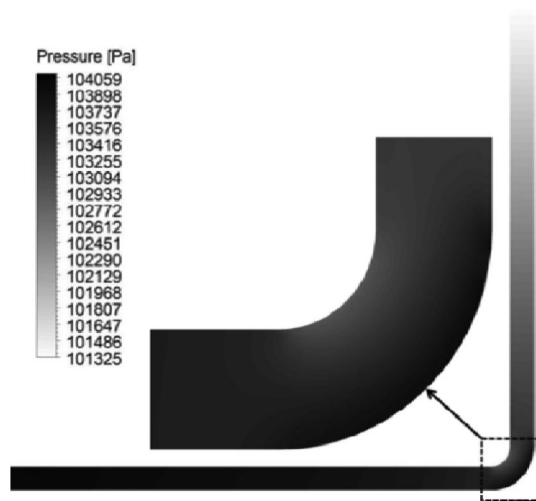


Figure 4: Pressure field in the curved pipe at the instant 13 seconds. ($U_w = 1.8$ m/s)

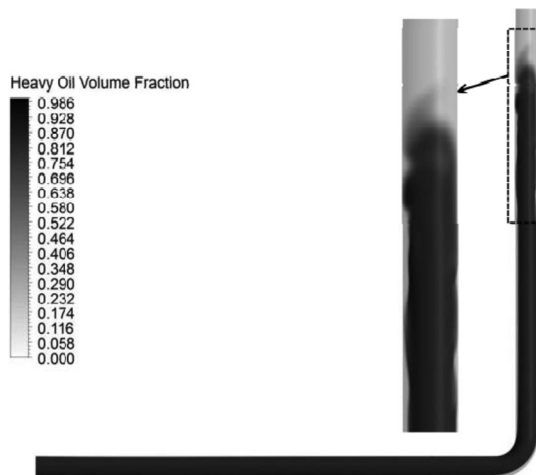


Figure 5: Heavy oil volume fraction at the instant 5 seconds

According to authors cited in this text, the core-flow technique used in vertical pipes is more favored because the center of gravity is the same for the two fluids. However, in horizontal pipes the phenomenon becomes a little more complicated due to the density difference between the fluids. In this case, the fluids of lower density tend to touch the upper wall of the pipe.

This phenomenon can be seen in Figure 6 where the heavy oil core is not centered (z-axis) on horizontal line of the curved pipe ($U_w = 1.8$ m/s).

However, for the flow in vertical ascending direction, the eccentricity of the oil core is relatively much smaller compared with the flow in the horizontal region of the pipe. Bensakhria et al. [5] evaluated the radial position of oil core in the annular flow and showed that this position depends solely on the ratio of the perimeter of contact between the pipe wall, the fluid which forms the core (heavy oil) and the circumference of the pipe. This ratio depends on the density difference between the fluids to be transported and the lubrication, as well as the amount of water injected into the pipe. Some authors state that ideal amount of water for a established water-heavy oil annular flow should be around 20% of the total mass flow rate, however, in this study, this amount is around 21%, i.e., 4.0779 kg/s of water.

Figure 7 illustrates the volume fraction of heavy oil along the entire perimeter of curvature in the total simulation time ($t = 13$ s). We can note that the heavy oil does not touch the walls in this region. Thus, the transportation of heavy oils using the core flow technique can be performed in whole pipes with curved connections as long as the radius of curvature be not smallest.

Figure 8 illustrate the oil superficial velocity at the XY plane for two axial position $Z = 1$ m and $Z = 2$ m (horizontal pipe). We can see that the oil velocity at the pipe wall increasing quickly to the center of the pipe, reaching the value $J_0 = 1.15$ m/s. By analyzing the oil volume fraction, it is verified the eccentricity of the oil flow (Figure 9).

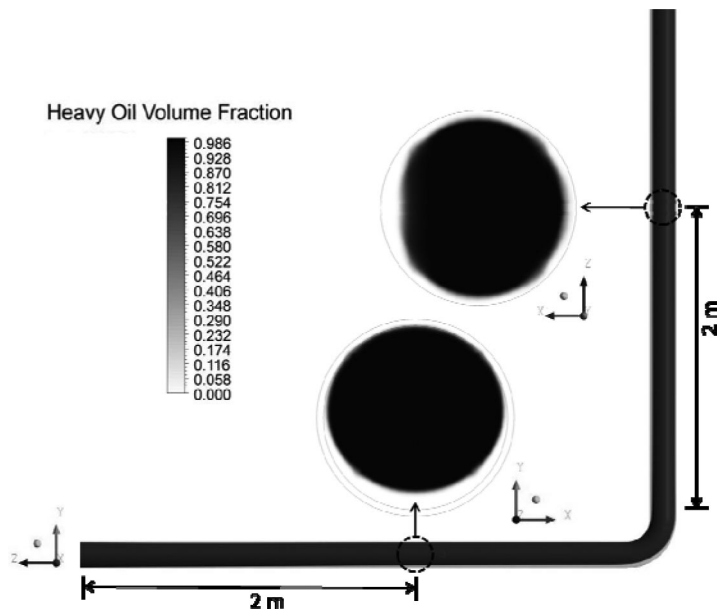


Figure 6: Heavy oil volume fraction in the horizontal and vertical extension of the curved pipe at the instants 13 seconds

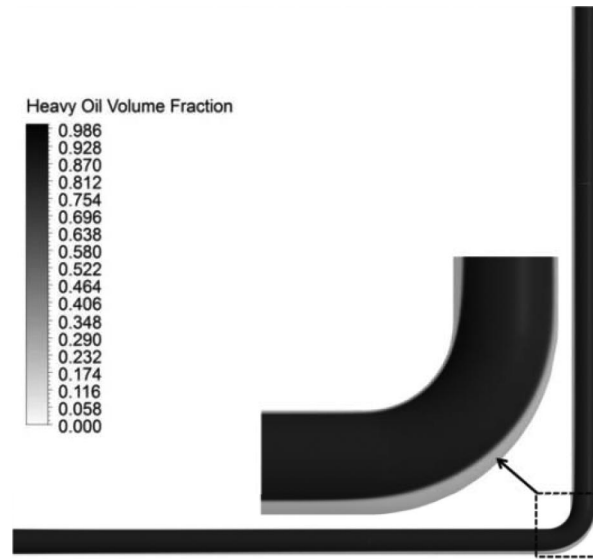


Figure 7: Heavy oil volume fraction in the curved region extension of the curved pipe at the instant 13 seconds ($U_w = 1.8$ m/s)

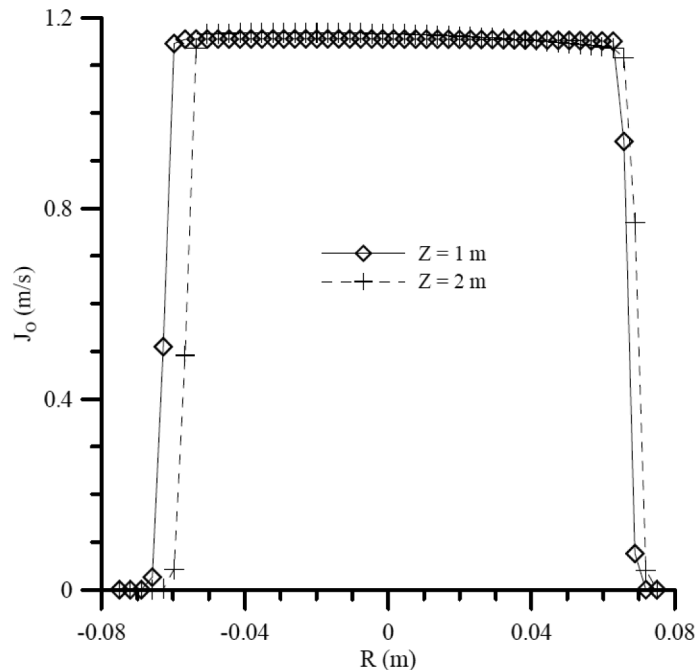
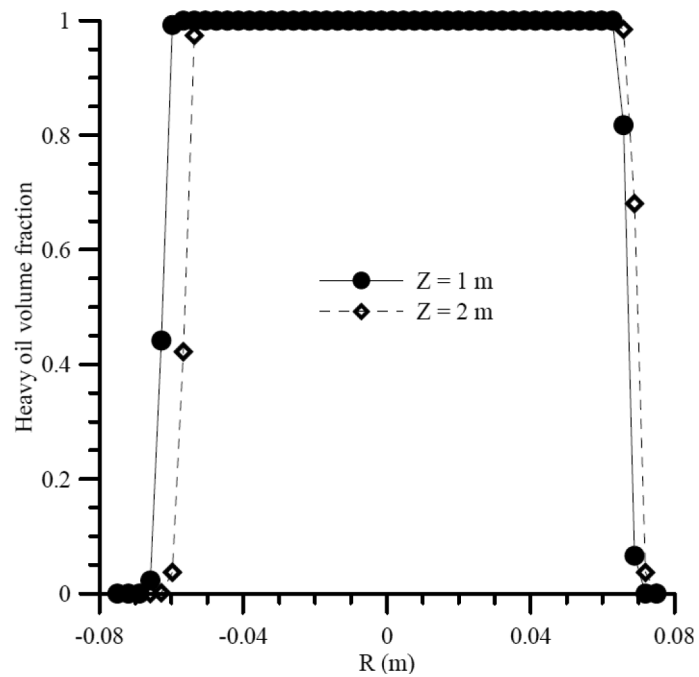
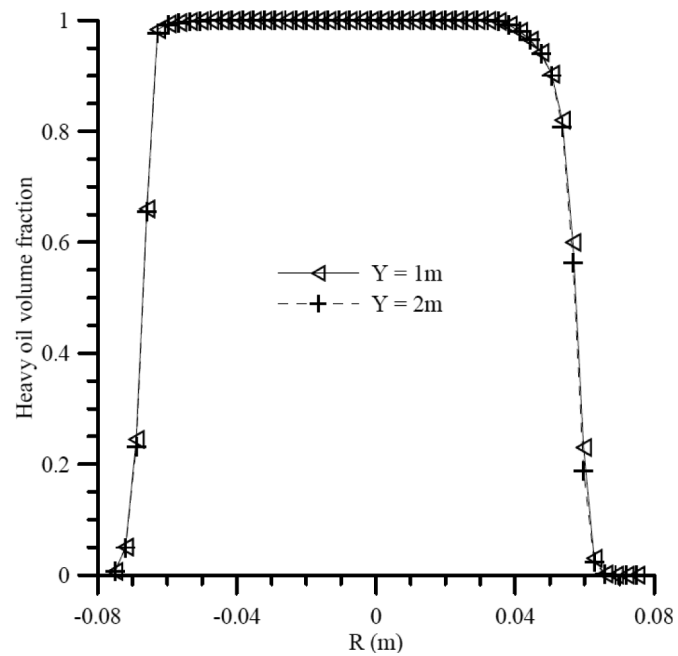


Figure 8: Heavy oil superficial velocity in horizontal pipe ($U_w = 1.8$ m/s)

Figure 10 shows the oil volume fraction at the Z plane for two axial position $Y = 1$ m and $Y = 2$ m (vertical pipe). We can see the eccentricity of the oil core after curvature region due to inertial effect. We notice that this eccentricity tend to be minimized for more distant position of the curvature.

Figure 9: Heavy oil volume fraction in horizontal pipe ($U_w = 1.8$ m/s)Figure 10: Heavy oil volume fraction in vertical pipe ($U_w = 1.8$ m/s)

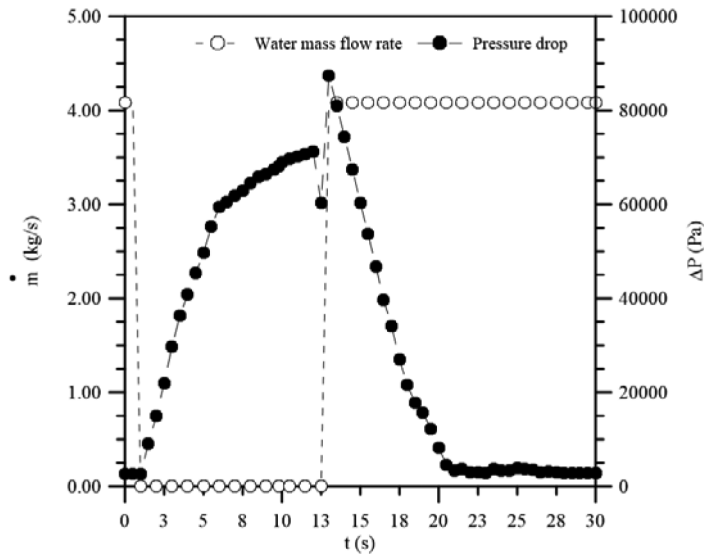


Figure 11: Pressure drop and water mass flow rate as function of time

3.2 INTERMITTENT FLOW

An extremely important factor to be considered in the annular flow using in the core-flow technique is related to the pumps that move the fluids. Because the characteristics of the fluids the pumps for the oil and water are different, so operating as handles these pumps can be dangerous factors. For example, when shutdown in the water pump, the pressure of the system increases rapidly in few seconds because the annular flow pattern is completely unconfigured allowing the oil to touch the pipe wall and, consequently, increasing the friction pressure. This flow behavior may cause serious damage to the equipments, for example, the rupture of valves, flanges, pipes and others. To put in evidence the occurrence of this phenomenon, it was plotted the water mass flow rate and pressure drop as a function of time (see Figure 11). This figure illustrates that, after switching off the water pump, the annular flow pattern is undone, and thus, causing to quickly increase pressure drop. After the water pump off, it stays stopped for 12 seconds, during this period the pressure drop of the system continues to increase until reaching a value of 71240 Pa. After 12 seconds the water pump is turned on again, there is a rapid period of oscillation in pressure drop peaking at $\Delta P = 87421$ Pa and then the annular pattern begins to be restored, thus, pressure drop of the system decreases and it remains practically constant after 20 seconds with a value close to the value of the pressure drop at the beginning of the simulation (1 second $\Delta P = 2727$ Pa). After shutdown and restart the water pump, the pressure drop remained almost constant $\Delta P = 2826$ Pa, one equivalent situation that when $U_w = 1.8$ m/s was used in the continuous the flow.

4. CONCLUSIONS

This study aims to analyze the horizontal and upward flow of heavy oil and water through the curved pipe using the core annular flow technique. For this, 3D numerical simulation was performed using ANSYS CFX® Release 12.0. From the study the following conclusions can be cited:

- a) The mathematical model employed using the software ANSYS CFX® Release 12.0 represented well the physics of annular flow (core-flow type) when applied to

transport of heavy oils. It was found that the utilization of this technique provides significant reduction in pressure drop, which is very useful for the oil industry, since oil reserves of the type Brent are becoming scarcer;

- b) It was possible to observe that in horizontal annular flow due to density difference between the fluids, the oil core tends to occupy a position eccentric to the axis of the pipe;
- c) The water injection of 4.0779 kg/s around 21% of the overall mass flow rate was sufficient to maintain the annular flow along the whole pipe thus reducing pressure loss by friction;
- d) Turning off the water pump during the runoff caused a rapid increase in pressure drop. When the water pump is turned on again a reduction in pressure drop is observed until the pressure drop value before the water pump is turned off. This type of behavior can cause serious damage to the system, such as the rupture of valves, flanges and other accessories, and consequently may lead to accidents causing environmental and economical losses;.
- e) The shutdown and restart the water pump should be done in a planned preventative maintenance as possible, by performing shutdown in the heavy oil pump initially.

ACKNOWLEDGEMENTS

The authors of this work thanks to CNPq, CAPES, FINEP, PETROBRAS, ANP/PRH-25, Cooperative Research Network for Computational Modeling (RPCMOD), Brazil, for the technical and financial support.

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