# Variation of redistribution of an infused dopant in a multilayer structure with variation of pressure of vapor of the dopant

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**ABSTRACT**

It has recently been shown, that doping of multilayer structures by diffusion or ion implantation and optimization of annealing of dopant and/or radiation defects gives us possibility to increase sharpness of $p$-$n$-junctions (single $p$-$n$-junctions and $p$-$n$-junctions within transistors) and to increase homogeneity of dopant distribution in doped area. In this paper we analyzed influence of pressure of vapor of infusing dopant during doping of multilayer structure on values of optimal parameters of technological process. We also consider an analytical approach to model technological process. In this paper we also consider an analytical approach to model redistribution of dopant.

## 1. INTRODUCTION

In the present time degree of integration of elements of integrated circuits ($p$-$n$-junctions, field and bipolar transistors, thyristors, …) intensively increasing [1–9]. At the same time one can find decreasing dimensions of the elements. To decrease the dimensions different approaches are used. One group of the approaches including into itself laser and microwave types of annealing [10–12]. One can found that during laser and microwave types of annealing inhomogenous distribution of temperature is generated. In this situation dimensions of elements of integrated circuits decreases due to Arrhenius law. To decrease dimensions elements of integrated circuits it could be also used of inhomogeneity of heterostructures [13–15]. However it is necessary to optimize technological process in this case [16,17]. It is known, that radiation damage of semiconductor materials leads to changing of distribution of dopant concentration in $p$-$n$-junctions and transistors [9,13,15,18]. In this situation radiation damage of semiconductor materials attracted an interest [19].

In this paper we consider a heterostructure, which consist of a substrate with known type of conductivity ($p$ or $n$) and epitaxial layer (see Fig. 1). A dopant has been infused in the epitaxial layer from gaseous source to produce required type of conductivity ($n$ or $p$). It is known, that under special conditions sharpness of $p$-$n$-junctions increases [16,17]. Main aim of the present paper is analysis of influence of pressure of vapor in source of dopant on dopant distribution in $p$-$n$-junction.
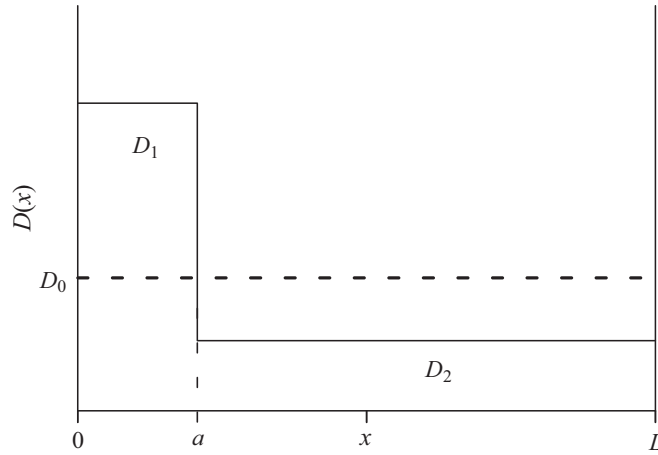
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Figure 1: Heterostructure, which consist of a substrate and an epitaxial layer

## 2. METHOD OF SOLUTION

To solve our aim we determine spatio-temporal distribution of concentration of dopant. We determine spatio-temporal distribution by solving of the second Fick's law [8,9,13]

$$\frac{\partial C(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[ D_C \frac{\partial C(x,t)}{\partial x}\right] \tag{1}$$

with boundary and initial conditions

$$C(0,t) = N, \left.\frac{\partial C(x,t)}{\partial x}\right|_{x=L} = 0, C(x>0,0) = 0. \tag{2}$$

We assume, that dopant infusing from infinite source with near-boundary concentration $N$, which is essentially larger, than limit of solubility of dopant $P$. Here $C(x,t)$ is the spatio-temporal distribution of concentration of dopant; $T$ is the temperature of annealing; $D_C$ is the dopant diffusion coefficient. Value of dopant diffusion coefficient depends on properties of materials of layers of heterostructure, heating and cooling of heterostructure (with account Arrhenius law). Dynamics of redistribution of dopant also depends on level of doping of materials. Dependences of dopant diffusion coefficient on parameters could be approximated by the following relation [8]

$$D_C = D_L(x,T)\left[1 + \xi \frac{C^{\gamma}(x,t)}{P^{\gamma}(x,T)}\right]. \tag{3}$$

Here $D_L(x,T)$ is the spatial (due to inhomogeneity of heterostructure) and temperature (due to Arrhenius law) dependences of dopant diffusion coefficient; $P(x,T)$ is the limit of solubility of dopant; parameter $\gamma$ depends on properties of materials and could be integer in the following interval $\gamma \in [1,3]$ [8]. Concentrational dependence of dopant diffusion coefficient has been described in details in [8].

To solve our aim let us determine solution of Eq. (1) and make analysis of dynamics of dopant. To calculate analytical solution of Eq. (1) we used recently elaborated approach [16,17,19]. Framework the approach we transform approximation of dopant diffusion coefficient to the following form: $D_C = D_{0L}[1 + \varepsilon \eta(x,T)] [1 + \xi C^{\gamma}(x,t)/P^{\gamma}(x,T)]$,

where $0 \le \varepsilon < 1$, $|\eta(x,T)| \le 1$, $D_{0L}$ is the average value of dopant diffusion coefficient. Farther we determine solution of Eq. (1) as the following power series on parameters $\varepsilon$ and $\xi$

$$C(x,t) = \sum_{k=0}^{\infty} \varepsilon^k \sum_{m=0}^{\infty} \xi^m C_{km}(x,t). \tag{4}$$

Functions $C_{km}(x,t)$ could be determine by solution of the following system of equation

$$\frac{\partial C_{00}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{00}(x,t)}{\partial x^2}$$

$$\frac{\partial C_{k0}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{k0}(x,t)}{\partial x^2} + D_{0L} \frac{\partial}{\partial x}\left[\eta(x,T)\frac{\partial C_{k-10}(x,t)}{\partial x^2}\right], k \ge 1$$

$$\frac{\partial C_{0m}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{0m}(x,t)}{\partial x^2} + D_{0L} \frac{\partial}{\partial x}\left\{\left[\frac{C_{00}(x,t)}{P(x,T)}\right]^{\gamma} \frac{\partial C_{0m-1}(x,t)}{\partial x^2}\right\}, m \ge 1 \tag{5}$$

$$\frac{\partial C_{11}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{11}(x,t)}{\partial x^2} + D_{0L} \frac{\partial}{\partial x}\left[\eta(x,T)\frac{\partial C_{01}(x,t)}{\partial x^2}\right] + D_{0L} \frac{\partial^2}{\partial x^2}\left[C_{10}(x,t)\frac{C_{00}^{\gamma}(x,t)}{P^{\gamma}(x,T)}\right]$$

$$+ D_{0L} \frac{\partial}{\partial x}\left[\eta(x,T)\left(\frac{C_{00}(x,t)}{P(x,T)}\right)^{\gamma} \frac{\partial C_{00}(x,t)}{\partial x}\right]$$

with boundary and initial conditions

$$C_{00}(0, t) = N, \frac{\partial C_{km}(x,t)}{\partial x}\bigg|_{x=0} = 0, k \ge 1, m \ge 1; C_{km}(0, t) = 0, C_{km}(x > 0, 0) = 0, k \ge 1, m \ge 1;$$

$$C_{00}(x, 0) = 0, C_{00}(0,0) = N; C_{km}(x, 0) = 0, k \ge 1, m \ge 1. \tag{6}$$

Solutions of the system of equations (5) with account conditions (6) could be obtain by standard approaches [20,21] and could be written as

$$C_{00}(0, t) = N, C_{00}(x > 0,t) = P_0\left[1 + \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{\sin(v_{n+0.5}x)e_{n+0.5}(t)}{n+0.5}\right],$$

where $v_n = \pi n L^{-1}$, $F_n = \int_0^L f(u)\cos(v_n u)\,du$, $e_n(t) = \exp(-v_n^2 D_{0L}t)$, $P_0$ is the average value of limit solubility of dopant

$$C_{10}(x,t) = \frac{2D_0\pi}{L^3}\sum_{n=0}^{\infty}(n+0.5)\sin(v_{n+0.5}x)e_{n+0.5}(t)$$

$$\times \sum_{m=0}^{\infty}\int_0^t e_{n+0.5}(-u)e_{n+0.5}(u)\left[H_{n+m+1}(u)+H_{n-m}(u)\right]du,$$

$$C_{20}(x,t) = -\frac{2}{L^6}\sum_{k=0}^{\infty}(k+0.5)\sin(v_{k+0.5}x)\,e_{k+0.5}(t)\sum_{n=0}^{\infty}(n+0.5)^2\sum_{m=1}^{\infty}\int_0^t e_{k+0.5}(-u)e_{n+0.5}(u)\,[H_{n-k}(u)$$

$$+ H_{n+k+1}(u)]\int_0^u e_{n+0.5}(-\tau)e_{m+0.5}(\tau)[H_{n-m}(\tau)+H_{n+m+1}(\tau)]\,d\tau du\pi^3 D_0^2,$$

where $H_n(t) = \int_0^L \eta(u,T) \; P(u,T) \sin(v_n u)\, du$.

$$C_{01}(x,t) = -\gamma \alpha_1 - \alpha_2,$$

where $\alpha_1 = \dfrac{2}{\pi^3} \sum_{n=0}^{\infty} (n+0.5)^2 \sin(v_{n+0.5} x) \sum_{k=0}^{\infty} \dfrac{1}{k+0.5} \sum_{m=0}^{\infty} \dfrac{e_{n+0.5}(t)\left[ e_{km}(t) - e_{n+0.5}(t) \right]}{(n+0.5)^2 - (m+0.5)^2 - (k+0.5)^2}$

$$\times \left\{ \left[ (m+0.5)^2 - (n-k)^2 \right]^{-1} - \left[ (m+0.5)^2 - (n+k+1)^2 \right]^{-1} \right\}, \; e_{km}(t) = e_{k+0.5}(t) e_{m+0.5}(t),$$

$$\alpha_2 = \begin{cases} 0, \gamma < 3 \\ \dfrac{1}{\pi^5} \sum_{n=0}^{\infty} (n+0.5)^3 \sin(v_{n+0.5}x) e_{n+0.5}(t) \sum_{k=0}^{\infty} \dfrac{1}{k+0.5} \sum_{l=0}^{\infty} \dfrac{1}{l+0.5} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left\{ \left[ nn - (k-l+i-j)^2 \right]^{-1} \right. \end{cases}$$

$$+\left[ (n+0.5)^2 - (k-l-i+j)^2 \right]^{-1} - \left[ (n+0.5)^2 - (k-l+i+j+1)^2 \right]^{-1} - \left[ (n+0.5)^2 - (k-l-i \right.$$

$$\left. -j+1)^2 \right]^{-1} - \left[ (n+0.5)^2 - (k+l+i-j+1)^2 \right]^{-1} - \left[ (n+0.5)^2 - (i-j-k-l+1)^2 \right]^{-1}$$

$$+\left[ (n+0.5)^2 - (i+j-k-l)^{2-1} \right] + \left[ (n+0.5)^2 - (i+j+k+l-2)^2 \right]^{-1} \right\}$$

$$\times \dfrac{(i+0.5)^{-1}(j+0.5)^{-1} \left[ e_{klij}(t) - e_{n+0,5}(t) \right]}{(n+0.5)^2 - (k+0.5)^2 - (l+0.5)^2 - (i+0.5)^2 - (j+0.5)^2}, \gamma = 3,$$

where $\alpha_3 = \dfrac{4}{\pi^5} \sum_{n=0}^{\infty} (n+0.5)^2 \sin(v_{n+0.5}x) e_{n+0.5}(t) \sum_{k=1}^{\infty} \dfrac{1}{k+0.5} \sum_{l=0}^{\infty} \left[ (n-k)^2 - (n+k+1)^2 \right]$

$$\times \left[ (l+0.5)^2 - (n-k)^2 \right]^{-1} \left[ (l+0.5)^2 - (n+k+1)^2 \right]^{-1} \sum_{m=1}^{\infty} \sum_{i=0}^{\infty} \dfrac{1}{i+0.5} \sum_{j=0}^{\infty} \left[ (l+i+1)^2 - (l-i)^2 \right]$$

$$\times \left[ (l+0.5)^2 - (i+0.5)^2 - (j+0.5)^2 \right]^{-1} \left[ (j+0.5)^2 - (l+i+1)^2 \right]^{-1} \left[ (j+0.5)^2 - (l-i)^2 \right]^{-1}$$

$$\times (l+0.5)^3 \sum_{i=0}^{\infty} \dfrac{1}{i+0.5} \sum_{j=0}^{\infty} \dfrac{(l+i+1)^2 - (l-i)^2}{\left[ (l+0.5)^2 - (i+0.5)^2 - (j+0.5)^2 \right] \left[ jj - (l+i+1)^2 \right] \left[ jj - (l-i)^2 \right]}$$

$$\times \left[ \dfrac{e_{klj}(t) - e_{n+0.5}(t)}{(n+0.5)^2 - (k+0.5)^2 - (i+0.5)^2 - (j+0.5)^2} - \dfrac{e_{kl}(t) - e_{n+0.5}(t)}{(n+0.5)^2 - (k+0.5)^2 - (l+0.5)^2} \right],$$

$$\alpha_4 = \begin{cases} 0, \gamma = 1, \gamma = 2 \\ \dfrac{3}{\pi^7} \sum_{n=0}^{\infty} (n+0.5)^4 \sin(v_{n+0.5}x) e_{n+0.5}(t) \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \dfrac{(j+0.5)^2}{i+0.5} \left\{ \left[ (n+0.5)^2 - (i-j)^2 \right]^{-1} - \left[ (n+0.5)^2 \right. \right. \end{cases}$$

$$\left. -(i+j+1)^2 \right]^{-1} \right] \sum_{k=0}^{\infty} \dfrac{1}{k+0.5} \sum_{l=0}^{\infty} \dfrac{1}{l+0.5} \sum_{m_1=0}^{\infty} \dfrac{1}{m_1+0.5} \sum_{m_2=0}^{\infty} \left\{ \left[ (j+0.5)^2 - (k-l+m_1-m_2)^2 \right]^{-1} \right.$$

$$+\left[\,(j+0.5)^2-(k-l-m_1+m_2)^2\,\right]^{-1}-\left[\,(j+0.5)^2-(k-l+m_1+m_2)^2\,\right]^{-1}-\left[\,(j+0.5)^2\right.$$

$$-(k-l-m_1-m_2-1)^2\,\Big]^{-1}-\left[\,(j+0.5)^2-(k+l+m_1-m_2-1)^2\,\right]^{-1}-\left[\,(j+0.5)^2-(k-m_1+m_2\right.$$

$$+l-1)^2\,\Big]^{-1}+\left[\,(j+0.5)^2-(k+l-m_1-m_2)^2\,\right]^{-1}+\left[\,(j+0.5)^2-(k+l+m_1+m_2+2)^2\,\right]^{-1}\Bigg\}$$

$$\times\Bigg\{\frac{e_{iklm_1m_2}(t)-e_n(t)}{(n+0.5)^2-(i+0.5)^2-(j+0.5)^2-(k+0.5)^2-(l+0.5)^2-(m_1+0.5)^2-(m_2+0.5)^2}$$

$$-\frac{e_{ij}(t)-e_n(t)}{(n+0.5)^2-(i+0.5)^2-(j+0.5)^2}\Bigg\}\frac{1}{m_2+0.5}+\frac{4}{\pi^7}\sum_{n_1=0}^{\infty}(n_1+0.5)^3\sin(v_{n_1+0.5}x)\sum_{n_2=0}^{\infty}\frac{1}{n_2+0.5}$$

$$\times\sum_{n_3=0}^{\infty}\frac{1}{n_3+0.5}\sum_{n_4=0}^{\infty}\frac{1}{n_4+0.5}\sum_{n_5=0}^{\infty}(n_5+0.5)^2\Bigg\{\left[\,(n_1+0.5)^2-(n_2-n_3-n_4+n_5)^2\,\right]^{-1}+\left[\,(n_1+0.5)^2\right.$$

$$-(n_2-n_3+n_4-n_5)^2\,\Big]^{-1}-\left[\,(n_1+0.5)^2-(n_2+n_3+n_4-n_5+1)^2\,\right]^{-1}-\left[\,(n_1+0.5)^2-(n_2+n_3\right.$$

$$-n_4+n_5+1)^2+\left[\,(n_1+0.5)^2-(n_2-n_3+n_4-n_5)^2\,\right]^{-1}-\left[\,(n_1+0.5)^2-(n_2+n_3+n_4-n_5+1)^2\,\right]^{-1}$$

$$-\left[\,(n_1+0.5)^2-(n_2+n_3-n_4+n_5+1)^2\,\right]^{-1}-\left[\,(n_1+0.5)^2-(n_2-n_3+n_4+n_5+1)^2\,\right]^{-1}$$

$$-\left[(n_1+0.5)^2-(n_2-n_3-n_4-n_5-1)^2\,\right]^{-1}+\left[\,(n_1+0.5)^2-(n_2+n_3-n_4-n_5)^2\,\right]^{-1}+\left[\,(n_1+0.5)^2\right.$$

$$-(n_2+n_3+n_4+n_5+2)^2\,\Big]^{-1}\Bigg\}\sum_{k=0}^{\infty}\frac{1}{k+0.5}\sum_{m=0}^{\infty}\left[\frac{1}{(m+0.5)^2-(n_5-k)^2}-\frac{1}{(m+0.5)^2-(n_5+k+1)^2}\right]$$

$$\times\Bigg[\frac{e_{n_2n_3n_4km}(t)-e_{n_1}(t)}{(n_1+0.5)^2-(n_2+0.5)^2-(n_3+0.5)^2-(n_4+0.5)^2-(k+0.5)^2-(m+0.5)^2}$$

$$-\frac{e_{n_5}(t)-e_{n_1}(t)}{(n_1+0.5)^2-(n_5+0.5)^2}\Bigg]\frac{1}{(n_5+0.5)^2-(k+0.5)^2-(m+0.5)^2}\,.$$

$$C_{11}(x,t)=\alpha_5+2\alpha_6+2\alpha_7-2\alpha_8+2\alpha_9+\alpha_{10}+4\alpha_{11},$$

where $\alpha_5=\dfrac{2\gamma\,D_{0L}}{\pi\,L^3}\displaystyle\sum_{l=0}^{\infty}(l+0.5)\sin(v_{l+0.5}x)\,e_{l+0.5}(t)\sum_{n=0}^{\infty}(n+0.5)^3\sum_{k=0}^{\infty}\frac{1}{k+0.5}\sum_{m=0}^{\infty}\Bigg\{\left[\,(m+0.5)^2\right.$

$$-(n-k)^2\,\Big]^{-1}-\left[\,(m+0.5)^2-(n+k+1)^2\,\right]^{-1}\Bigg\}\left[\,(n+0.5)^2-(k+0.5)^2-(n+0.5)^2\,\right]^{-1}$$

$$\times\int_0^t\left[H_{l-n}(u)+H_{l+n+1}(u)\right]\left[e_{l+0.5}(-u)e_{k+0.5}(u)e_{m+0.5}(u)-e_{l+0.5}(-u)e_{n+0.5}(u)\right]du,$$

$$\alpha_6=\frac{\gamma D_{0L}^2}{L^5}\sum_{n=0}^{\infty}(n+0.5)^2\sin(v_{n+0.5}x)e_{n+0.5}(t)\sum_{k=0}^{\infty}(k+0.5)^3\sum_{l=0}^{\infty}(l+0.5)^2\Bigg\{\left[\,(l+0.5)^2-(n-k)^2\,\right]^{-1}$$

$$-\left[(l+0.5)^2-(n+k+1)^2\right]^{-1}\right\}\sum_{m=0}^{\infty}\int_0^t e_{n+0.5}(-u)e_{k+0.5}(u)e_{l+0.5}(u)\int_0^u\left[H_{l-m}(\tau)+H_{l+m+1}(\tau)\right]$$

$$\times e_{l+0.5}(-\tau)\,e_{m+0.5}(\tau)\,d\tau du,$$

$$\alpha_7=D_{0L}\frac{\pi}{L^3}\sum_{n=0}^{\infty}(n+0.5)\,e_{n+0.5}(t)\sin(v_{n+0.5}x)\sum_{m=0}^{\infty}\int_0^t e_{n+0.5}(-u)\,e_{m+0.5}(u)\left[H_{m-n}(u)+H_{m+n+1}(u)\right]du,$$

$$\alpha_8=\frac{\gamma\,\pi\,D_{0L}}{L^3}\sum_{n=0}^{\infty}(n+0,5)\sin(v_{n+0.5}x)\,e_{n+0.5}(t)\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{k+0,5}\int_0^t e_{n+0.5}(-u)e_{km}(u)\left[H_{n+k-m),5}(u)+H_{n-k+m),5}(u)\right.$$

$$\left.-H_{n\text{-}k\text{-}m\text{-}0.5}(u)-H_{n+k+m+1,5}(u)\right]d\,u,$$

$$\alpha_9=\begin{cases}0,\gamma=1\\\dfrac{\gamma\,D_{0L}}{\pi\,L^3}\sum_{n=0}^{\infty}(n+0.5)e_{n+0.5}(t)\sin(v_{n+0.5}x)\sum_{k=0}^{\infty}\dfrac{1}{k+0.5}\sum_{l=0}^{\infty}\dfrac{1}{l+0.5}\sum_{m=0}^{\infty}\int_0^t e_m(-\tau)e_{klm}(\tau)\end{cases}$$

$$\times[I_{k-l-n+m}(T)+I_{k-l+n-m}(T)+I_{k-l-n-m-1}(T)+I_{k-l+n+m+1}(T)-I_{k-l-n-m-1}(T)-I_{k+l+n-m+1}$$
$$(T)-I_{m-k-l+n-}(T)-I_{m+k+l+n+2-}(T)]d\tau,\gamma>2,$$

where $\displaystyle I_n(T)=\int_0^L\eta(u,T)\cos(v_nu)\,du,$

$$\alpha_{10}=\begin{cases}0,\gamma<3\\\dfrac{2D_{0L}}{\pi^3L^3}\sum_{i=0}^{\infty}(i+0.5)e_{i+0.5}(t)\sin(v_{i+0.5}x)\sum_{n=0}^{\infty}(n+0.5)^2\int_0^t\left[I_{i-n}(T)+I_{i+n+1}(T)\right]\sum_{k=0}^{\infty}\dfrac{1}{k+0.5}\end{cases}$$

$$\times\sum_{l=0}^{\infty}\frac{1}{l+0.5}\sum_{m_1=0}^{\infty}\frac{1}{m_1+0.5}\sum_{m_2=0}^{\infty}\frac{1}{m_2+0.5}\left\{\left[(n+0.5)^2-(k-l+m_1-m_2)^2\right]^{-1}+\left[(n+0.5)^2\right.\right.$$

$$-(k-l-m_1+m_2)^2]^{-1}-[(n+0.5)^2-(k-l+m_1+m_2+1)]^{-1}-[(n+0.5)^2-(k-l-m_1-m_2-1)]^{-1}$$
$$-[(n+0.5)^2-(k+l+m_1$$
$$-m_2+1)]^{-1}-[(n+0.5)^2-(k+l-m_1+m_2+1)]^{-1}+[(n+0.5)^2-(k+l-m_1-m_2)]^{-1}$$
$$+[(n+0.5)^2-(k+l+m_1+m_2+2)]^{-1}\}[(n+0.5)^2-(k+0.5)^2-(l+0.5)^2-(m_1+0.5)^2-(m_2+0.5)^2]^{-1}$$

$$\times\left[e_i(-u)e_{klm_1m_2}(u)-e_i(-u)e_n(u)\right]\,du,\gamma>3,$$

$$\alpha_{11}=\begin{cases}0,\gamma<3\\\dfrac{D_{0L}^2}{\pi^3L^5}\sum_{i=0}^{\infty}(i+0.5)^3e_{i+0.5}(t)\sin(v_{i+0.5}x)\sum_{j=0}^{\infty}\dfrac{1}{j+0.5}\sum_{k=0}^{\infty}\dfrac{1}{k+0.5}\sum_{l=0}^{\infty}\dfrac{1}{l+0.5}\sum_{n=0}^{\infty}(n+0.5)\end{cases}$$

$$\times\{(i+0.5)^2-(j-k+l-n)^2]^{-1}+[(i+0.5)^2-(j-k-l+n)^2]^{-1}-[(i+0.5)^2-(j+k-l+n+1)^2]^{-1}-[(i+0.5)^2$$
$$-(j+k-l-n+1)^2]^{-1}-[(i+0.5)^2-(j+k-l+n+1)^2]^{-1}+[(i+0.5)^2-(j+k-l-n)^2]^{-1}-[(i+0.5)^2-(j+k+l+n$$

$$+1)^2]^{-1}\right\}\sum_{m=0}^{\infty}\int_0^t e_{i+0.5}(-u)\,e_{jkln}(u)\int_0^u e_{n+0.5}(-\tau)e_m(\tau)\left[I_{n-m}(T)+I_{n+m+1}(T)\right]d\tau du\,,\gamma=3$$

$$\alpha_{11}=\begin{cases}0,\gamma<3\\\dfrac{D_{0L}^2}{\pi^2L^3}\sum_{n=0}^{\infty}(n+0.5)e_{n+0.5}(t)\sin(v_{n+0.5}x)\sum_{k=0}^{\infty}\dfrac{1}{k+0.5}\sum_{l=0}^{\infty}\dfrac{1}{l+0.5}\sum_{m=0}^{\infty}\dfrac{1}{m+0.5}\sum_{i=0}^{\infty}\int_0^t e_{n+0.5}(-u)\end{cases}$$

$\times e_{klmi}(u)\{H_{k-n+i-l+m+0.5}(u)+H_{k-n-i+l-m+0.5}(u)+H_{k-n+l+i-m+0.5}(u)+H_{k+n-i-l+m+0.5}(u)$
$-H_{k+n-i-l+m-0.5}(u)$
$-H_{k-n+I+l-m+1.5}(u)-H_{k-n+i-l+m-0.5}(u)-H_{k+n-i+l-m+1.5}(u)+H_{k+n+i-l+m+1.5}(u)+H_{k-n-i+l-m-0.5}(u)$
$+H_{k-n-i-l+m-0.5}(u)+H_{k+n+i+l-m+1.5}(u)-H_{k+n-i-l-m+0.5}(u)-H_{k-n-i+l+m+0.5}(u)-H_{k-n-i-l-m-1.5}(u)$
$-H_{k+n+I+l+m+2.5}(u)\}du.$

Analysis of spatiotemporal distributions of dopant concentrations has been done analytically by using the second-order approximation of dopant concentration on parameters $\varepsilon$ and $\xi$. Farther the distribution has been amended numerically.

## 3. DISCUSSION

In this section we analyzed dynamics of redistribution of dopant in heterostructure from Fig. 1 based on calculated in previous section relations. Fig. 2 shows spatial distributions of concentration of dopant in the considered heterostructure at fixed value of annealing time and different values of parameter $\varepsilon$, which characterize deviation of approximation of dopant diffusion coefficient from average value $D_{0L}$. The Fig. 2 shows, that increasing of difference between values of dopant diffusion coefficient in the substrate and in the epitaxial layer gives us possibility to increase sharpness of *p-n*-junction and at the same time to increase homogeneity of dopant distribution in doped area. However using this type of doping leads to necessity in optimization of annealing time. Reason of this optimization is following. If annealing time is small, dopant cannot achieves interface between layers of heterostructure. In this situation homogeneity of distribution of concentration of dopant became less, than in heterostructure. If annealing time is large, distribution of concentration of dopant became overly homogenous. We determine optimal annealing time by using recently introduced criterion [16,17,19,22]. Framework the approach we approximate real distribution of
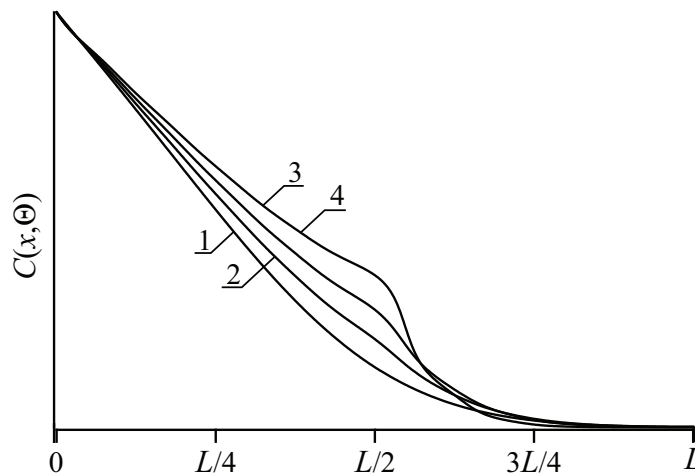


Figure 2: Distributions of concentration of dopant in heterostructure from Fig. 1 in direction, which is perpendicular to interface between layers. Increasing of number of curves corresponds to increasing of difference between values of dopant diffusion coefficient in layers of heterostructure. This curves correspond to situation, when value of dopant diffusion coefficient in the epitaxial layer is larger, than in the substrate
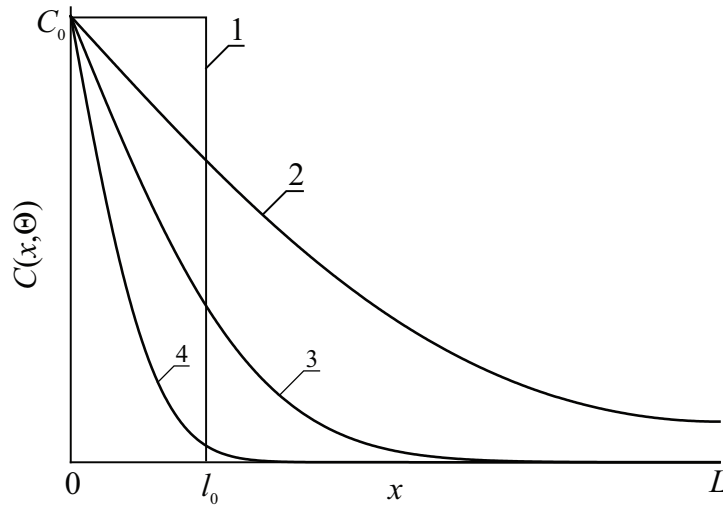
Figure 3: Spatial distributions of concentration of dopant. Curve 1 is required idealized distribution of concentration of dopant. Curve 2-4 are real distributions of concentration of dopant. Increasing of number of curves corresponds to increasing of annealing time

concentration of dopant by step-wise function (see Fig. 3). Farther we determine optimal value of annealing time by minimization of the following mean-squared error

$$U = \frac{1}{L}\int\limits_0^L \left[ C(x,\Theta) - \psi(x) \right] dx, \tag{7}$$

Here $\psi(x)$ is the approximation time. $\Theta$ is the optimal value of annealing time. Dependences of optimal value of annealing time on parameters are presented in Fig. 4.

Farther we analyzed influence of value of pressure of vapor of dopant from infinite source on distribution of concentration of the dopant in the considered heterostructure. We assume, that gaseous source of dopant is ideal gas. In this case pressure of gas and surficial concentration of dopant are correlated with each other by linear law: $pM = R\,TN$, where $M$ is the molar mass, $R = 8.31$ J/(mole$\cdot$K) is the gas constant, $p$ is the pressure of gas. In this situation increasing of pressure of gas in source of dopant leads to proportional increasing of surficial concentration of dopant. However dependence of optimal value of annealing time on pressure is not so simple due to nonlinearity of criterion of estimation of the time [16,17,19,22,23]. Analysis of dynamics of redistribution of dopant shows, that variation of value of dynamics of vapor leads to quantitative variation of distribution of concentration of dopant inside of the considered heterostructure, but not to quantitative variation.

## 4. CONCLUSION

In this paper we analyzed influence of changing of pressure of vapor of dopant from infinite source on distribution of concentration of the dopant in the *p-n*-heterojunction. It has been shown, that the changing of pressure leads to some quantitative variation of distribution of concentration of dopant, but not to quantitative variation.
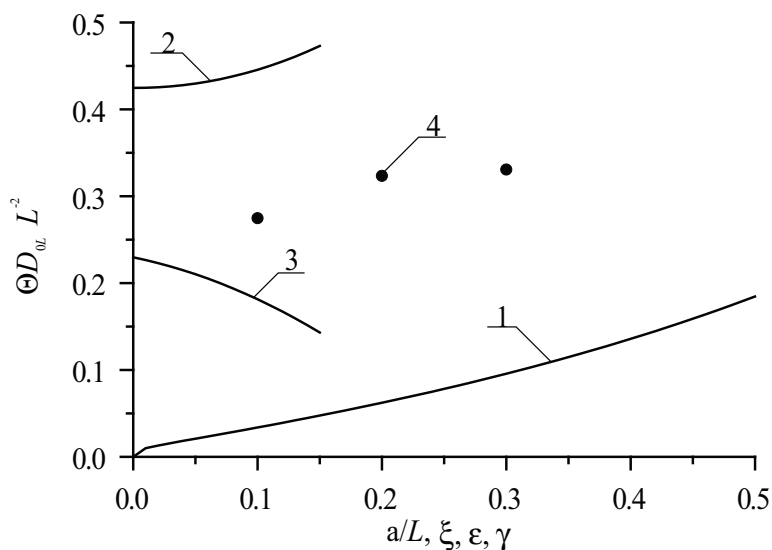
Figure 4: Dependences of dimensionless optimal annealing time, which have been obtained by minimization of mean-squared error (7), on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on the relation $a/L$ and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter $\varepsilon$ for $a/L=1/2$ and $\xi = \gamma = 0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter $\xi$ for $a/L=1/2$ and $\varepsilon = \gamma = 0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter $\gamma$ for $a/L=1/2$ and $\varepsilon = \xi = 0$

## ACKNOWLEDGMENTS

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